Cheshire Cat Scenario in a $3 + 1$ dimensional Hybrid Chiral Bag

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Abstract

The total energy in the two-phase chiral bag model is studied, including the contribution due to the bag (Casimir energy plus energy of the valence quarks), as well as the one coming from the Skyrmion in the external sector.

A consistent determination of the parameters of the model and the renormalization constants in the energy is performed.

The total energy shows an approximate independence with the bag radius (separation limit between the phases), in agreement with the Cheshire Cat Principle.

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The hybrid chiral bag [1, 2] is an effective model to describe the behavior of strongly interacting baryons. In this model, color degrees of freedom are confined to a bounded region and coupled to a bosonic external field (skyrmion) through boundary conditions.

These two phase models are intermediate between two successful descriptions of baryons: bag models [3, 4] – with QCD degrees of freedom at short distances – and Skyrme model [5, 6, 7], an effective (non renormalizable) nonlinear sigma model, useful when the low energy properties of baryons are considered.

An interesting feature of Chiral Bag Models (CBM) is the appearance of the so called Cheshire Cat Principle (CCP) [1, 8], according to which fermionic degrees of freedom can be replaced by bosonic ones in certain regions of space, the resulting position of the limit of separation between the two phases having no physical consequences.

In 1+1-dimensions, the Cheshire Cat behavior follows from the bosonization of fermionic fields [1]. In the $3 + 1$ case, topological quantities, such as the baryonic number, have a similar behavior [9] but, for non topological ones, the CCP is expected to be only approximately valid.

In what follows, we will study the energy of a four-dimensional hybrid model consisting of quarks and gluons confined to a spherical bag plus a truncated exterior Skyrme field in a hedgehog configuration. It is our aim to study the dependence of the total energy on the size of the bag, thus testing the Cheshire Cat hypothesis.

In bag models, quarks and gluons are confined to a bounded region that, in our case, will be taken as an static sphere of radius R. Adequate boundary conditions are imposed on the field so as to guarantee the vanishing of the flux of color current to the external sector.

In the MIT bag model [3] the boundary conditions for fermions and gluons are

$$B\,\Psi\big|_B = \frac{1}{2}\left(\mathcal{I} + i\ \not{n}\right)\Psi\bigg|_B = 0 \tag{1}$$

$$n_\mu F^{\mu\nu}\big|_B = 0\ . \tag{2}$$

The Dirac operator for the fermionic field, together with the boundary condition (1) define an elliptic boundary value problem [10]. Moreover, (1) gives a vanishing current at the boundary, $\overline{\Psi}n_\mu\Psi\Big|_B = 0$.

But the axial symmetry, present in QCD with massless quarks, is broken at the boundary when condition (1) is satisfied. To avoid such unwelcome behavior, CBM [4] have been proposed, which amount to imposing on the fermionic field the boundary condition

$$A\Psi|_B = \frac{1}{2}\left(\mathcal{I} + i\ \not{n}e^{-i\theta(\vec{\tau}\cdot\vec{n})\gamma^5}\right)\Psi\bigg|_B = 0\,, \tag{3}$$

where τ_i are Pauli matrices, and adding an external sector to the bag.

Through this boundary condition, the fermionic field is connected with the external field represented by the "chiral angle" $\theta(R)$. As it will be shown later, the external phase can be described using the Skyrme model.

To obtain the energy in the chiral bag model we will proceed in steps, using some results previously obtained in references [11, 12].

In the first place, we will introduce the difference between Casimir energies of chiral and MIT fermionic bags. It corresponds to the zero temperature limit, $T \to 0$, of the results presented in [11]. As a second step, we will study the reference vacuum energy, i.e., the MIT Casimir energy, which is the $T \to 0$ limit of the cases analized in [12] for the fermionic and gauge fields. An external Skyrme field will then be introduced, so as to complete the two phase model (TPM). The caracteristic parameters in its Lagrangian, together with the renormalization constants in the bag energy, will be determined through physical considerations, thus obtaining the total energy of the TPM.

Determinants of quotients of elliptic differential operators under different boundary conditions can be expressed as p-determinants of quotients of Forman's operators [13, 14, 15]. This leads, for a bounded euclidean time, to the study of differences of free energies of the physical system subject to two different boundary conditions. In such a way, in reference [11] the difference between the free energies of an $SU(2)$ chirally symetric system of massless fermions, confined to a spherical region and subject to chiral and MIT boundary conditions respectively, was calculated.

To construct Forman's operator, which is totally defined by its action over functions in the kernel of $i\ \not{\partial}$, a discrete basis of the space of solutions was considered. The differential operator and the boundary conditions are invariant under the diagonal subgroup of $SU(2)_{rotation} \otimes SU(2)_{isospin}$, and leave

invariant the subspaces caracterized by the quantum numbers $\{k, j, l, m\}$, associated with the eigenvalues of $\{\mathbf{K}^2, \mathbf{J}^2, \mathbf{L}^2, K_z\}$, where $\mathbf{K} = \mathbf{L} + \mathbf{S} + \mathbf{I}$. In particular, k takes non-negative integer values.

The $T \to 0$ limit of the above mentioned diference of free energies leads to

$$\Delta e_c(\theta) = R\left[E_c(R, \theta) - E_{c,MIT}\right]$$

$$= 3\left\{ \frac{1}{4\pi} \left[4\pi K_Q \sin^2 \theta + 0.463 \sin^4 \theta + 0.023 \sin^6 \theta\right]\right.$$

$$-\frac{1}{2\pi} \sum_{k=1}^{\infty} \int_0^{\infty} dx \left[2\nu \log\left(1 + C_k(x)\sin^2 \theta + D_k(x)\sin^4 \theta - \Delta(k, x)\right)\right]$$

$$+\frac{1}{4\pi} \left\{ \begin{array}{cc} \theta^2 & 0 < \theta \leq \pi/2 \\ (\pi - \theta)^2 & \pi/2 < \theta < \pi \end{array} \right\}$$

$$-\frac{1}{2\pi} \int_0^{\infty} dx \, \log\left[1 + \frac{\frac{\alpha(x)}{x} \frac{-4 + \left[2(1 - a^2(x))^2 + \frac{\alpha(x)}{x}\right]\cos^2 \theta}{4a^2(x) + (1 - a^2(x))^2 \cos^2 \theta}}{\left(1 - \frac{\alpha(x)}{x} \frac{1}{1 + a^2(x)}\right)^2}\right]\right\} , \tag{4}$$

where the Casimir energies have been adimensionalized. The first and second terms in the r.h.s. of (4) correspond to the subspaces $k \geq 1$. The third and fourth ones, to the $k = 0$ case. We have used the definitions

$$C_k(x) = \frac{-2}{\left[4x^2 d_k^2(x) + (\rho^2 - d_k^2(x))^2\right]^2} \times$$

$$\left\{ (\rho^2 - d_k^2(x))^2 \left[4x^2 d_k^2(x) + (\rho^2 - d_k^2(x))^2\right]\right.$$

$$+ (\rho^2 - \nu^2) \left[4x_n^2 d_k^2(x) - (\rho^2 - d_k^2(x))^2\right]\right\} \tag{5}$$

$$D_k(x) = \frac{\left[(\rho^2 - d_k^2(x))^2 - (\rho^2 - \nu^2)\right]^2}{\left[4x^2 d_k^2(x) + (\rho^2 - d_k^2(x))^2\right]^2} \tag{6}$$

$$\nu = k + 1/2 \qquad \rho = \sqrt{x^2 + \nu^2}$$

4

$$d_k(x) = x \frac{d}{dx} \ln I_\nu(x) \,.$$

$$a(x) = \coth x \qquad \alpha(x) = 2a(x) - \frac{1}{x} \,,$$

where $I_\nu(x)$ is the modified Bessel function. In (4), $\Delta(k, x)$ represents the first few terms[1] in the asymptotic (Debye) expansion of the other terms in the argument of the logarithm, required to isolate the divergent pieces in the Casimir energy.

In obtaining (4), an analytic regularization of non-absolutely convergent series has been performed through the introduction of the factor ρ^{-s}, for $\Re(s)$ large enough, and then taking the finite part at $s = 0$ [11], giving rise to the first term in the r.h.s. This procedure leaves an arbitrary finite part proportional to $\sin^2(\theta)$, which requires the introduction of the undetermined constant K_Q. This amounts to the introduction, in the Lagrangian of the external Skyrme field, of the counterterm [16]

$$\frac{K_0}{16\pi R} \int_{r=R} d^2x \, \mathrm{tr} \left\{ L_\alpha L_\alpha - (n_\alpha L_\alpha)^2 \right\} = \frac{K_0}{R} \mathrm{sen}^2 \theta, \tag{7}$$

where $L_\alpha = U^\dagger \partial_\alpha U = e^{-i\theta(\vec{\tau}\cdot\hat{r})} \partial_\alpha e^{i\theta(\vec{\tau}\cdot\hat{r})}$.

The second and fourth terms of (4) require numerical calculations. In the first case, the sum over the index k has been cutted when the tail of the series becomes negligible. Integrations in the x variable have been numerically solved in both cases.

As it was said before, the reference (MIT) Casimir energy for fermions and that corresponding to gluons have been studied in a complementary way. In reference [12], the free energy for a fermionic and an abelian gauge field (enough for the 1-loop description of the free energy for gluons) was studied. Such evaluations have been made using analytically regularized traces which involve the Green functions of the boundary problems considered.

The $T \to 0$ limit of the results in [12] has a simple structure. As it can be understood by dimensional analysis, the Casimir energy (once singularities

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[1]Note that, in order to isolate divergences, it is enough to retain the first three terms in the asymptotic expansion [11]. However, in the present calculation, we have retained the first six terms in the Debye expansion for computational convenience.

have been removed by the renormalization of the zero-loop energy) takes the form

$$e_{c,MIT} = RE_{c,MIT} = K_{MIT} \, , \qquad (8)$$

where K_{MIT} is the arbitrary finite part left by the renormalization procedure.

We thus have now the total Casimir energy of the bag, including the correction due to the interaction of fermions with the external Skyrme field, represented by the chiral angle θ,

$$e_c(\theta) = \Delta e_c(\theta) + e_{c,MIT} \, , \qquad (9)$$

up to the knowledge of the constants K_Q and K_{MIT}. As we will show later, they can be determined imposing physical conditions in the framework of the two phase chiral bag model.

A further contribution has to be included if the total inner bag energy (energy of the defect, in the remaining of the paper) is studied.

In reference [9], it has been proved that the valence quark contribution must be taken into account in the $0 \leq \theta \leq \pi/2$ region, to obtain the baryon number, $\mathcal{B}$, in the TPM. In fact, when adding the contributions coming from the truncated Skyrme model, the Dirac sea and the valence quarks, one obtains $\mathcal{B} = 1$ for any value of θ. This can be understood by studying the energy of the fundamental eigenstate of the Dirac Hamiltonian [17]. When $\theta < \pi/2$, the fundamental energy becomes positive and valence quarks leave the Dirac sea. In this case, they must be explicitly included. As regards the bag energy, the valence quark contribution can be written as

$$e_q = 3\epsilon(\theta)H(\pi/2 - \theta) \, , \qquad (10)$$

where $\epsilon(\theta)$ is the fundamental eigenvalue and $H(x)$ is the Heaviside step function.

Note that our regularization prescription lead to a finite bag energy (depending on K_Q and K_{MIT}). All derived quantities will also be finite. Such is the case of the axial flux through the boundary of the sphere,

$$\phi^f(R,\theta) = \int_{r=R} d\Omega \, n_\mu n_a \, \langle j_5^{\mu,a} \rangle = \frac{1}{R}\frac{d}{d\theta}e_t(\theta) \, , \qquad (11)$$

where $e_t(\theta) = [\Delta e_c(\theta) + e_q(\theta) + K_{MIT}]$. The last term, coming from the MIT bag, is θ-independent and does not contribute to the axial flux.

As it was proposed in reference [18], the vanishing of the flux when $R \to 0$ determines the K_Q renormalized constant. This physical imposition is consistent with the analysis we will perform later, when treating the TPM.

The external phase of the TPM is described by the Skyrme model [5, 6]. The lagrangian can be written as

$$\mathcal{L} = \frac{1}{16} F_\pi^2 \mathrm{Tr} \left(\partial_\mu U \partial^\mu U^\dagger \right) + \frac{1}{32 e^2} \mathrm{Tr} \left[\left(\partial_\mu U \right) U^\dagger, \left(\partial_\nu U \right) U^\dagger \right]^2 , \qquad (12)$$

where the scalar field $U(x)$ takes its values in the $SU(2)$ group.

Two parameters have been introduced here: F_π, associated with the pion's decay constant (experimental value $F_\pi^{exp} = 186 \, \mathrm{MeV}$) and e^2, which represents the strength of the stabilizing term. These parameters will be adjusted later in the framework of the two phase model [2].

The Skyrmion is a topologically stable classical solution of the Lagrangian in (12), when the whole space is considered. It is given by

$$U_0(\vec{x}) = e^{i\theta(r)(\vec{\tau} \cdot \vec{x})} , \qquad (13)$$

where $\theta(r)$ is what we are calling the chiral angle. Spatial and isospinorial indexes are linked in the argument of the exponential.

By the imposition of the boundary conditions

$$\theta(r = 0) = \pi \qquad \theta(r) \to_{r \to \infty} 0 , \qquad (14)$$

an skyrmion of topological baryonic number (winding number) $\mathcal{B} = 1$ is obtained (in the pure Skyrme model).

Replacing the Skyrme ansatz in the Lagrangian (12), the equation of motion is obtained as a nonlinear differential equation for $\theta(r)$ [7].

The Skyrme lagrangian is invariant under $SU(2)_L \otimes SU(2)_R$ chiral transformations

$$U \to AUB^{-1} , \qquad (15)$$

where A and B belong to $SU(2)$. When $A = B^{-1}$, we are in the case of axial symmetry leading to the locally conserved axial current. Chiral boundary conditions guarantee its conservation even at the boundary, when the TPM is considered.

The flux of the axial current through an sphere of radius r, in terms of the adimensionalized radius $\hat{r} = \frac{eF_\pi}{2} r$, is given by

$$\phi^{Sk}(\hat{r}) = \frac{2\pi F_\pi}{e} \frac{d\theta}{d\hat{r}} \hat{r}^2 \left[1 + \frac{2}{\hat{r}^2} \sin^2 \theta \right] . \qquad (16)$$

It is not difficult to show that, when $R \to 0$ ($\theta \to \pi, \theta' < \infty$) $\phi^{Sk} \to 0$. It is reasonable to extend such behavior to the flux from the inner fermionic phase, using this criterium to determine the value of the renormalized constant K_Q.

In the TPM, the Skyrmion is truncated to the exterior of an sphere of radius R. In the $R \to 0$ limit, the pure skyrmion should describe the baryon properties. The contribution K_{MIT}/R to the Casimir energy is forbidden in such scheme. Then, the validity of the hybrid chiral model, even in the $R \to 0$ limit, imposes $K_{MIT} = 0$.

Once our renormalization scheme has been established, a fit can be proposed for the numerically evaluated Casimir energy. Following [18] we propose

$$e_c(\theta) - 3K_Q \sin^2 \theta = 3 \left\{ \frac{3}{4\pi} \left(\left\{ \begin{array}{ll} \theta^2 & 0 \leq \theta \leq \frac{\pi}{2} \\ (\pi - \theta)^2 & \frac{\pi}{2} \leq \theta \leq \pi \end{array} \right\} - \sin^2 \theta \right) \right.$$

$$\left. + C_2 \sin^2 \theta + C_4 \sin^4 \theta + C_6 \sin^6 \theta + C_8 \sin^8 \theta \right\} , \qquad (17)$$

where the coefficients take the values

$$\begin{array}{llll} C_2 = & -0.13381 & C_4 = & 0.05085 \\ C_6 = & -0.01257 & C_8 = & 0.01241 \end{array} . \qquad (18)$$

The required vanishing of the axial flux of fermions in the $R \to 0$ limit inmediately leads to

$$K_Q = -C_2 ,$$

thus eliminating the contribution proportional to $(\pi - \theta)^2$ for $R \to 0$.

Having fixed the renormalized constants, the energy of the inner phase is as shown in Figure 1. Our results are totally consistent with those presented in [18]. The symmetry of the Casimir energy (dashed line) about $\theta = \pi/2$ is

evident. Such property was a priori expectable from simple properties of the eigenvalues of the Dirac hamiltonian of the model.

In the same figure, the solid line represents the total energy of the inner phase (when the valence quark contribution is also taken into account). As in reference [18], the resulting interior energy is a smooth function.

In the TPM picture, the axial flux through the boundary of the defect should be continuous for all values of the bag radius. So, a fine tunning of the strength of the stabilization term $e(R)$ must be performed to ensure

$$\phi^f(R) = \phi^{Sk}(R) \qquad \text{for} \quad \text{all} \quad \text{R}.$$ (19)

To impose this condition, the knowledge of the chiral angle as a function of R, $\theta(R)$, is necessary. As it was said, its value can be obtained numerically, by solving a nonlinear differential equation. But, following the proposition of M. Atiyah and N. Manton [19], $U(\vec{x})$ configurations can be constructed by evaluating the holonomy of Yang-Mills fields with topological charge k, in the time direction. This is derived from a t'Hooft instanton of width λ.

For the $k = 1$ case, the resulting chiral angle is

$$\theta(\hat{r}) = \pi \left[1 - \frac{1}{\sqrt{1 + \left(\frac{\hat{\lambda}}{\hat{r}}\right)^2}} \right].$$ (20)

Replacing $\theta(\hat{r})$ in the energy of the skyrmion[7]

$$M_{Sk} = -\int d^3x \, \mathcal{L}_{Sk}$$

$$= 4\pi \int_0^\infty \left\{ \frac{F_\pi^2}{8} \left(r^2 \theta'^2 + 2\sin^2\theta \right) + \frac{1}{2e^2} \frac{\sin^2\theta}{r^2} \left\{ 2r^2\theta'^2 + \sin^2\theta \right\} \right\},$$ (21)

and minimizing with respect to $\hat{\lambda}$, the value of this parameter is fixed to $\hat{\lambda} = 1.452$ [19]. The resulting profile is very close to the one obtained numerically. For computational convenience, we will use this aproximation for the rest of the paper.

With the Atiyah-Manton profile, equation (19) gives

$$\frac{1}{e^2(\theta)} = -8 \frac{3\pi}{32\hat{\lambda}^2} \frac{1}{(\pi - \theta)^3 \left[1 + \frac{2}{\hat{\lambda}^2} \frac{\theta(2\pi - \theta)}{(\pi - \theta)^2} \sin^2\theta\right]} \frac{de_t(\theta)}{d\theta},$$ (22)

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Figure 1: (Dimensionless) Inner energy of the chiral bag ; – – : Casimir energy ; — : Total inner energy

when parametrized by the chiral angle θ.

When the $\theta \to \pi$ ($R \to 0$) limit is taken, the stabilization strength for the pure skyrmion is obtained

$$e(R = 0) = 4.216\,, \qquad (23)$$

to be compared with the value $e = 5.45$ obtained by G. S. Adkins, C. R. Nappi and E. Witten by fitting the masses of the nucleon and the Δ particle.

Now, the value of F_π can be fixed in the scheme proposed in [7]. Having e and the nucleon mass $M_n^{exp} = 938\mathrm{MeV}$, expression (9) of [7] (with λ and M calculated for the Atiyah-Manton profile) leads to $F_\pi = 99.59\,\mathrm{MeV}$. This value, far from the experimental one, is, however, near $F_\pi = 129\,\mathrm{MeV}$ obtained in [7]. As a consistency check, the Δ particle mass has been calculated with our parameters, giving $M_\Delta = 1206\,\mathrm{MeV}$ (experimental value: $M_\Delta^{exp.} = 1230\,\mathrm{MeV}$).

The calculations just detailed complete the determination of the parameters of the Skyrme model, in a consistent way with the TPM under study. Now, recovering the dimensional variable r, the strength $e(R)$ is as shown in Figure 2.

To complete our task, the energy contained in the Skyrme external sector must be evaluated. To do this calculation, the integral in (21), with the lower limit truncated to the radius of the defect, must be studied. We have performed the numerical evaluation of this quantity using the Atiyah-Manton profile and the parameters of the Skyrme model F_π and $e(R)$, thus obtaining the energy in the Skyrme sector as a function of the defect radius.

Figure 3 is the main result of this paper. It shows in dashed lines, from top to bottom, the energy of the external Skyrme phase and the energy of the defect, as functions of the position of the limit between the phases. Also shown, in solid line, is the total energy of the hybrid chiral bag model. This last shows a remarkable independence with the bag radius (for $0 \le R \le 1\,\mathrm{fm}$), as suggested by the CCP.

Summarizing, we have employed the results presented in [11, 12] for the internal Casimir energies in a chiral bag model. In these papers, by the use of analytical regularizations, a renormalized Casimir energy, dependent on the K_Q and K_{MIT} constants, was obtained. In the present paper, a TPM

Figure 2: Strength of the stabilization term $e(R)$

Figure 3: Two Phase model energy — : Complete model ; – · – : Skyrmion's sector ; – – : Bag's sector

was completed, by introducing an external Skyrme phase. The renormalized constants K_Q and K_{MIT}, as well as the parameters F_π and $e(R)$ of the truncated Skyrme Lagrangian were determined according to physical conditions, suitable for the TPM.

In reasonable agreement with the Cheshire Cat hypothesis, the total energy of this model shows an approximate independence with the bag radius (separation limit between the phases) in the range of $0 \leq R \leq 1\,\text{fm}$.

The study of CBM at finite temperature [20, 21] is an interesting effective approach to the analysis of deconfinement transitions. In those references, succesive approximations to the problem, based on the validity of the Cheshire Cat hypothesis at $T = 0$ have been made. The present results give a ground to such hypothesis, thus making it sensible to look for the presence of deconfinement transitions only in the temperature-dependent contributions to the free energy of the bag.

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