The Return of the Prodigal Goldstone Boson

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Abstract

We propose that the mass of the η' meson is a particularly sensitive probe of the properties of finite energy density hadronic matter and quark gluon plasma. We argue that the mass of the η' excitation in hot and dense matter should be small, and therefore that the η' production cross section should be much increased relative to that for pp collisions. This may have observable consequences in dilepton and diphoton experiments.

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1 Introduction

One of the great mysteries in the quark model was why there is no ninth Goldstone boson whose mass is somewhere between that of the pion and the kaon. Roughly stated, the problem is that in the limit of massless quarks, the quark model has a $U(3)$ chiral symmetry. This chiral symmetry, when broken, predicts the existence of 9 massless Goldstone bosons. In nature, however, there are only 8 light mesons.

The problem is resolved by the Adler-Bell-Jackiw $U(1)$ anomaly [1]-[3]: the $U(1)$ part of the $U(3)$ symmetry is explicitly broken by interactions. It is possible to show explicitly that instantons [4, 5] dynamically induce the $U(1)$ chiral symmetry breaking. This results in giving the ninth Goldstone boson a mass. The would–be ninth Goldstone boson is presumably the η', which has a mass of nearly one GeV.

As the density of matter is increased, it is expected that the effects induced by the tunneling between different topological vacua of QCD will rapidly disappear [6]-[8]. Let us briefly recall the origin of this belief, based on the example of the instanton solution realizing this tunneling. The amplitude $\mathcal{T}$ of the tunneling transition, calculated in the quasiclassical approximation of instanton configurations, is

$$\mathcal{T}_{\text{instanton}} \sim e^{-S_E} \sim e^{-2\pi/\alpha_S} \,, \tag{1}$$

where S_E is the Euclidean action of the instanton solution. It is expected that the effects of finite energy density will make α_S density dependent such that for large energy densities

$$\alpha_S \sim \frac{24\pi}{(33 - 2N_f)\ln(\epsilon/\Lambda^4)} \,, \tag{2}$$

where ϵ is the energy density and $\Lambda \sim 200$ MeV. As the energy density increases, the effects of instantons rapidly decrease. Note that $\Lambda^4 \sim 200$ MeV/fm^3 is a relatively low energy density.

We therefore expect that as the energy density of hadronic matter is increased, the mass of the η' will be a rapidly falling function of energy density. In the quark

gluon plasma, we expect that excitations with the quantum numbers of the η' will become almost mass–degenerate, modulo current quark mass corrections, with excitations with quantum numbers of the octet of pseudoscalar Goldstone bosons. This is manifest in the quark model since there will be no penalty for making an isospin singlet configuration of quarks relative to an isotriplet configuration.

The plan of this paper is as follows. In Section 2 we recall the mechanism responsible for the large mass of the η' in QCD, and argue about the properties of the η' at high densities. In Section 3 we discuss the dynamics of η' production and annihilation in hot and dense matter. In Section 4 we discuss several expected signals of the proposed enhancement of η' production in dense matter and claim possible experimental evidence in favor of our scenario.

2   Axial anomaly, ghost, and η' at high densities

Consider a quark–antiquark pseudoscalar flavor–singlet field

$$|\eta_0\rangle = \frac{1}{\sqrt{3}}|\bar{u}u + \bar{d}d + \bar{s}s\rangle.$$ (3)

The divergence of the corresponding flavor–singlet axial current acquires an anomalous part, due to the interaction with gluon fields, which does not disappear in the chiral limit $m \to 0$ of massless quarks:

$$\partial^\mu J^0_{5\mu} = 2i \sum_f m_f \bar{q}_f \gamma_5 q_f + 2N_f \frac{g^2}{16\pi^2} \text{Tr}\left(G_{\mu\nu}\tilde{G}^{\mu\nu}\right).$$ (4)

This anomalous part may be written as the full divergence of the gauge–dependent topological current

$$K_\mu = 2N_f \frac{g^2}{16\pi^2} \epsilon_{\mu\nu\lambda\rho} \text{Tr}\left(G^{\nu\lambda} A^\rho\right),$$ (5)

so that in the chiral limit one has the Adler–Bardeen relation

$$\partial^\mu J^0_{5\mu} = \partial^\mu K_\mu.$$ (6)

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It is possible to introduce a new axial current

$$J_{5\mu} = J_{5\mu}^0 - K_\mu \,, \tag{7}$$

which is explicitly conserved in the chiral limit.

$$\partial^\mu J_{5\mu} = 2i \sum_f m_f \bar{q}_f \gamma_5 q_f \tag{8}$$

The corresponding charge

$$Q_5 = \int d^3x \, J_{50} \,, \tag{9}$$

is naively expected to be conserved. Since this charge is the generator of the $U(1)_A$ symmetry, and this symmetry is not observed in the hadron spectrum (no parity doublets exist), we have to presume that the symmetry is spontaneously broken. This would lead to the appearance of a nearly massless Goldstone boson field (3). In nature, however, the physical η' meson has a large mass of almost one GeV and therefore cannot be considered a Goldstone boson.

To check if the charge (9) is really conserved, one can integrate the divergence of the current $J_{5\mu}$ over Euclidean 4–space. After the spatial integration is performed, the result can be represented as

$$\int_{-\infty}^{+\infty} dt \frac{dQ_5}{dt} = 2N_f \nu[G] \,, \tag{10}$$

where

$$\nu[G] = 2N_f \frac{g^2}{32\pi^2} \int d^4x \, \mathrm{Tr} \left(G_{\mu\nu} \tilde{G}^{\mu\nu} \right) \tag{11}$$

is the so–called topological charge. It is equal to zero in Abelian theories, but in QCD $\nu[G] \neq 0$: the one–instanton solution, for example, yields $\nu[G] = 1$. Therefore the charge (9) is not a conserved quantity, and going from (Euclidean) $t = -\infty$ to $t = +\infty$ it changes by

$$\Delta Q_5 = 2N_f \nu[G] \,. \tag{12}$$

Therefore the existence of non–trivial topological solutions explicitly breaks the $U(1)_A$ symmetry, resulting in the vanishing of the corresponding Goldstone mode.

As we have already mentioned in the Introduction, the instanton density vanishes in the high energy density limit as $g^2 = 4\pi\alpha_S \to 0$. We therefore expect that in dense matter the ensemble averaged axial charge Q_5 will be conserved.

$$\frac{d\langle Q_5 \rangle}{dt} = 0 \tag{13}$$

If the $U(1)_A$ symmetry is still spontaneously broken at very high densities, it would imply the return of the ninth Goldstone boson!

Even though the arguments presented above explain on a qualitative level why the physical η' is not a Goldstone excitation, and under what circumstances can it again become one, it is instructive for our purposes to establish the actual relation between the properties of the vacuum and the mass of the η'. To do this we follow the approach developed by Witten [9] and Veneziano [10]. They noticed that the non-vanishing of the topological charge (11) implies the existence of an unphysical massless pole, introduced earlier by Kogut and Susskind [11], in the correlator of the topological current (5). Such a pole means the existence of a massless excitation, or "ghost", which should reflect some fundamental symmetry of the theory. As was shown by Dyakonov and Eides [12], the origin of this excitation in QCD is the periodicity of the potential energy of the vacuum with respect to the collective coordinate

$$X = \int d^3x \, K_0(x) \,. \tag{14}$$

The potential barriers separating different vacua are penetrable, by instantons for example, and the massless ghost just corresponds to this degree of freedom in the theory. If one introduces the propagator $\langle a_\alpha a_\beta \rangle$ of the ghost field a_α, the residue of the ghost contribution λ can be defined as

$$\langle 0| K_\alpha K_\beta |0 \rangle = \lambda^4 \, \langle 0| a_\alpha a_\beta |0 \rangle \,, \tag{15}$$

so that as $q^2 \to 0$

$$q_\alpha q_\beta \langle 0| K_\alpha K_\beta |0 \rangle = \langle 0| \nu\nu |0 \rangle = -\lambda^4 \neq 0 \,. \tag{16}$$

Note that, apart from the ghost contribution, the propagator of the topological current also contains the normal gluon part.

The field (3) can now mix with the ghost, the amplitude of mixing being of order $\lambda^2/f_{\eta'}$, where $f_{\eta'}$ is the η' decay constant. As a result of this mixing the physical η' acquires an additional mass

$$\Delta m \simeq \lambda^2/f_{\eta'}\,, \tag{17}$$

so that the mass of the η' does not vanish in the chiral limit.

$$m_{\eta'}^2 = m_0^2 + (\Delta m)^2 \tag{18}$$

The mass of the bare η' field (3) can simply be estimated in the free quark model as

$$m_0^2 = \frac{1}{3}(2m_K^2 + m_\pi^2)\,. \tag{19}$$

At high energy densities we expect that the density of instantons will diminish, the ghosts will disappear, and the η' will be (almost) entirely described by the field (3), whose mass will then be given by (19).

Of course in nature the situation is likely to be a bit more complicated. Indeed, the mass eigenstates in the isosinglet channel are not the η and η', but the nonstrange and strange states $|\eta_{\mathrm{NS}}\rangle = |\bar{u}u + \bar{d}d\rangle/\sqrt{2}$ and $|\eta_{\mathrm{S}}\rangle = |\bar{s}s\rangle$. These states can only mix if one allows for intermediate gluon states. The extreme assumption that the only allowed gluonic states are non-perturbative ghost-like states would lead to the conclusion that at high densities, when ghosts disappear, the physical isosinglet excitations will be η_{NS} and η_{S}. Their masses will then be $m_{\mathrm{NS}}^2 = m_\pi^2$ and $m_{\mathrm{S}}^2 = 2m_K^2 - m_\pi^2$. However, normal gluonic states certainly contribute, and we expect that the states η_{NS} and η_{S} will mix even at high densities, even though this mixing will probably not yield the states with the η and η' quark wave functions. We expect also that as a consequence of the effects discussed above the $\eta - \eta'$ mixing will be strongly dependent on energy density, and the physical η mass will decrease

too. Nevertheless, since the topological and perturbative gluonic effects are very difficult to separate, for the sake of argument we will assume in the rest of this paper that the η' quark content at any density is given by (3).

3 Dynamics

Production cross sections for light mesons are typically of the order predicted by the Hagedorn model,

$$\sigma_i \sim g_i (M/2\pi)^{3/2} \, e^{-M/T_H} \, , \tag{20}$$

when the particle mass is large compared to $T_H \sim 160$ MeV. The quantity g_i is the number of internal degrees of freedom of the i'th particle species. For pions this same model gives

$$\sigma_\pi \sim g_i / \pi^2 \, . \tag{21}$$

Using this rather simple model we see that the expected cross section of η' production is quite small, $\sigma_{\eta'}/\sigma_{\pi^0} \sim 2 \times 10^{-2}$.

Now suppose that the η' is made in a dense environment. Here we expect that the mass of the η' is small, and the particle ratio $N_{\eta'}/N_{\pi^0} \sim 1$. If the η' becomes a Goldstone boson we might get a factor of up to 50 enhancement in the production cross section!

After an η' is produced it must survive subsequent hadronic interactions until it has escaped the matter. The η' lifetime in vacuum is about 1000 fm/c; if there were no interactions with surrounding particles, it would certainly survive the time it takes for the hadronic matter produced in heavy ion collisions to dissipate.

It is amazing that the results presented in the previous Section imply that the η' should decouple from high density matter and therefore most likely cannot be absorbed. To see this, we will follow the line of reasoning developed in refs. [13] and [14].

Let us first note that the Adler–Bardeen relation (6), and an analog of the PCAC for the η' field,

$$\eta'(x) = \frac{1}{m_{\eta'}^2 f_{\eta'}^2} \partial^\mu J_{5\mu}^0 , \qquad (22)$$

suggest the existence of a relation between the matrix elements of the η' field and of the topological charge (11). With this in mind, we consider a nonsymmetric matrix element of the topological current (5) between some hadronic states[1]. For definiteness we consider nucleons explicitly here. It has the following general form:

$$\langle p'|K_\nu|p\rangle = \bar{u}(p')[\gamma_\nu \gamma_5 G_1(q^2) + q_\nu \gamma_5 G_2(q^2)] u(p) , \qquad (23)$$

where $q = p - p'$, $\bar{u}$, u are the nucleon wave functions, and $G_{1,2}$ are the form factors. Consider the matrix element $\langle 0|\partial^\nu K_\nu|\bar{N}N\rangle$ in the cross channel. Saturating it by the η' pole, one obtains

$$q^2 G_2(q^2) = \langle 0|\nu|\eta'\rangle \frac{1}{q^2 - m_{\eta'}^2} \langle \eta'|\bar{N}N\rangle , \qquad (24)$$

where the last matrix element is just the η' coupling constant $g_{\eta'NN}$. The first matrix element can be evaluated by using the Lehmann–Symanzik–Zimmerman reduction formula in the following form:

$$
\begin{aligned}
\langle 0|\nu|\eta'\rangle &= \int d^4x \, e^{iq \cdot x} (-\partial_x^4 + m^2)\langle 0|T\{\nu \eta'(x)\}|0\rangle \\
&= -\frac{-q^2 + m_{\eta'}^2}{m_{\eta'}^2 f^2} \langle 0|T\{\nu\nu\}|0\rangle .
\end{aligned}
\qquad (25)
$$

As $q^2 \to 0$ we get, from (24), (25) and (16), that

$$q^2 G_2(q^2) \sim \frac{\lambda^4 g_{\eta'NN}}{m_{\eta'}^2 f_{\eta'}} = f_{\eta'} g_{\eta'NN}, \qquad (26)$$

where at the last step we used the relation $m_{\eta'} \simeq \lambda^2/f_{\eta'}$, valid in the chiral limit (see (17), (18)).

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[1] In principle, one can consider the matrix elements taken over the ensemble as a whole.

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In the absence of ghosts, which we expect is the case in high density matter, the form factor $G_2(q^2)$ does not possess a zero–mass pole, and the l.h.s. of (26) is equal to zero at $q^2 = 0$. Therefore, since $f_{\eta'} \neq 0$, we are led to the conclusion that at high densities the coupling of the η' vanishes and it decouples from (nonGoldstonic) matter. A parallel discussion for the coupling of an η' with two ρ mesons [15] can be given with a similar conclusion.

Next, consider moderate to low energy density matter where pions are the most abundant constituents. Then we need to know the cross section for the annihilation reaction $\pi^+ + \eta' \rightarrow \pi^+ + \rho^0$, which is exothermic, and the isospin-related cross sections. The rate can be calculated in the low temperature limit using a low energy effective Lagrangian.

The cross section for $\pi(p_1) + \eta'(p_2) \rightarrow \pi(p_1') + \rho(p_2')$ is dominated by the exchange of a ρ-meson in the t-channel. The $\rho\pi\pi$ vertex is well-known, and the $\eta'\rho\rho$ vertex is the anomalous one [16, 17]. The matrix element is

$$\mathcal{M} = g_{\eta'\rho\rho}\, p_{2\alpha}\, p_{2\beta}'\, \epsilon^{\alpha\beta\nu\tau} \left[-\frac{g_{\mu\nu}}{q^2 - m_\rho^2} + \frac{q_\mu q_\nu}{(q^2 - m_\rho^2)m_\rho^2} \right] g_{\rho\pi\pi}\, (p_1 + p_1')^\mu\, \varepsilon_\tau(p_2') , \quad (27)$$

where $q = p_1' - p_1$. The total cross section for one charge configuration works out to be

$$\sigma_0(s) = \frac{g_{\rho\pi\pi}^2 g_{\eta'\rho\rho}^2}{16\pi p_{cm}^2} \left\{ (t_+ - t_-) + \left(t_+ + t_- - 2m_\rho^2 \right) \ln\left(\frac{m_\rho^2 - t_-}{m_\rho^2 - t_+} \right) + \right.$$

$$\left. \frac{(t_+ - t_-)}{(m_\rho^2 - t_-)(m_\rho^2 - t_+)} \left[-m_\rho^2(t_+ + t_-) + m_\rho^4 + m_\pi^2(m_{\eta'}^2 - m_\rho^2)^2/s \right] \right\} . \quad (28)$$

Here t_+ and t_- are the kinematic limits of t.

From the decay rate for $\rho \rightarrow \pi\pi$ we know that $g_{\rho\pi\pi}^2/4\pi = 2.90$. From the decay rate for $\eta' \rightarrow \rho\gamma$ [18], together with vector meson dominance [16, 17], we get $g_{\eta'\rho\rho} = 3.96 \times 10^{-3}/\text{MeV}$ or, more usefully, $g_{\eta'\rho\rho}^2 = 6.10$ mb. It may be noted that this value is consistent with that predicted by gauging the Wess-Zumino term, which is

$$g_{\eta'\rho\rho} = \frac{g_{\rho\pi\pi}^2}{16\pi^2 f_\pi} \left(\sqrt{6}\cos\theta_P + \sqrt{3}\sin\theta_P \right) ,$$

where θ_P is a pseudoscalar mixing angle with a value of about -20 ± 5 degrees [16, 17, 19].

The annihilation cross section vanishes at threshold and rises monotonically with s. Although thermal averaging can be done numerically to obtain the rate, we shall be content with the following simple estimate. For a collision between an η' and a pion the average value of s at temperature T is easily found to be

$$\langle s \rangle = (m_{\eta'} + m_\pi)^2 + 6 m_{\eta'} T \,.$$

At $T = 150$ MeV, $\sqrt{\langle s \rangle} = 1.44$ GeV. At this value, $\sigma_0 = 2.6$ mb. The mean free path l for η' annihilation is estimated from

$$l^{-1} = \sum_{ij} \sigma_{ij} n_i = 2\sigma_0 n \,, \tag{29}$$

where the sum is over all channels, n is the total pion number density, and σ_0 is evaluated at the average $\sqrt{s}$. For temperatures comparable to or greater than the pion mass the number density is approximately $0.365\ T^3$. At $T = 150$ MeV the mean free path for annihilation is 12 fm. It gets even bigger as the temperature decreases. Since the η' decouples near the phase transition temperature, where the present estimate is not valid, we may conclude that η's will not annihilate to any appreciable degree at any temperature during the expansion.

It might seem paradoxical to argue that the η' decouples at high density yet is produced in roughly equal abundance with the pion. Actually, there is no paradox. Suppose that quark gluon plasma is formed initially. When it hadronizes, all Goldstone bosons will be produced in roughly equal numbers by condensation of the quark and gluon fields. Suppose that high density hadronic matter is formed initially, not quark gluon plasma. Then the initial state is formed via meson production in elementary nucleon-nucleon collisions. Many pions will be produced. In this environment, the η' mass will be low. Since there is no suppression of transitions among the Goldstone bosons themselves, the η' mesons will come to, or at least

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approach, chemical equilibrium with pions, kaons and η mesons.

4 Signals

During the expansion and cooling phase, the η' propagates in the background field of the surrounding hadronic matter. This background field increases the η' mass as the hadronic matter becomes more dilute. Due to energy conservation, any motion of the η' relative to this medium will be damped, and the η' will come to rest. As a consequence, the η' will be strongly coupled to any collective flow of matter, and the p_T distribution of η' may be strongly distorted relative to that in pp collisions.

When the matter is at high energy density there will be mixing between the collective excitations which will become the η and η' in the vacuum, so an enhancement of the η' will lead to an enhancement of the η too. In addition, an important decay mode of the vacuum η' is into η, leading to an enhancement of η after the breakup of hadronic matter occurs.

There are several places where one might see the effects of the return of the ninth Goldstone boson. First, one might study low mass dileptons in the region above the π^0 Dalitz pairs and below the ρ. If the η'/π^0 ratio is of order one, there would be an enhancement due to the $\eta' \rightarrow e^+ e^- \gamma$ decay mode. In Figure 1 we display the enhancement measured in the CERES experiment [20], which was first seen by the HELIOS experiment [21]. To get the curve shown we have scaled the computed η' contribution by a factor so that it corresponds $\eta'/\pi^0 = 1$. No attempt has been made to compute the effects due to a changing shape of the p_T spectrum caused by collective flow. Although the result is a little low compared to the experimental result, it has the correct shape and goes through the experimental points plus error bars. We should caution the reader that this agreement is undoubtedly fortuitous since we have assumed the maximum possible production of η'. A realistic computation might very well give too small a rate. On

the other hand, the η' has a branching ratio of 65% into η, which itself can decay into dileptons. In addition, distortions of the p_T spectrum folded into detection biases might have the effect of artificially enhancing or suppressing the η' contribution. Additional contributions come from dileptons produced in hadron-hadron collisions during the expansion and cooling phase, which help to fill-in not only the low mass region but also the region between the ϕ and the J/ψ mesons [22].

Perhaps the most convincing demonstration of the return of the η' would be a direct measurement. This might be possible for the two photon decay mode, especially if the production cross section is as strongly enhanced as we suggest. It would also be important to have a direct measurement of the η since we expect an enhancement there too.

We should emphasize that unlike the case for the ρ meson, and to a lesser degree for the ω and ϕ, the η' and the η mesons almost always decay after the surrounding hadronic matter has blown apart. Therefore one cannot expect to directly see the effect of the mass shift of the η' or the η meson: the only effect will be due to an enhanced production cross section.

Acknowledgments

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Figure Caption

Yield of low mass dileptons as measured by CERES [20]. Included in the plot are their assumed resonance contributions. The heavy solid curve is the result of multiplying their assumed η' contribution by a factor of 50, corresponding to the extreme limit that $\eta'/\pi^0 \approx 1$.