Space-time dimension, Euclidean action and signature change *

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Abstract

This talk is devoted to the problem how to compute relative nucleation probabilities of configurations with different topology and dimension in quantum cosmology. Assuming the semiclassical approximation, the usual formula for the nucleation probability induced by the no-boundary wave function is $P_{NB} \approx \exp(-I)$, where I is the Euclidean action, evaluated at a solution of the effective Euclidean field equations. In the simplest case, these are just Einstein's field equations with a cosmological constant Λ. Relative probabilities of different configurations are usually compared at equal values of Λ. If Λ is an effective vacuum energy density arising from, say, a massive scalar field ϕ (i.e. $\Lambda \sim \phi^2$), one thus compares probabilities at equal values of this field. When configurations with different dimensions are admitted (the n-dimensional gravitational constant being subject to a rather mild restriction), as e.g. $\mathbf{S}^n$ for any n, this procedure leads to the prediction that the space-time dimension tends to be as large as possible, $n \to \infty$. In this contribution, I would like to propose an alternative scheme, namely to compare the probabilities $P_{NB} \approx \exp(-I)$

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at equal values of the *energy* E, instead of the *energy density* Λ. As a result, the space-time dimension settles at $n = 4$. Attempts to predict the topology of the spacelike slices lead to the candidates $\mathbf{S}^3$ and $\mathbf{S}^1 \times \mathbf{S}^2$. Since the "process" of nucleation (possibly connected with decoherence) is not well known in detail, we expect that either *both* configurations may be realized with roughly equal probability, or the *latter* one is favoured. Finally, we comment on the analogous situation based on the tunneling wave function.

1 Euclidean quantum cosmology, signature change and nucleation probabitilies

The no-boundary wave function of the universe for a model whose variables are the metric $g_{\mu\nu}$ and some matter fields (denoted as Φ and ϕ) has as its arguments the values (h_{ij}, Φ, ϕ) of the spatial metric and the matter fields, evaluated at some spacelike hypersurface Σ. It is symbolically given by a path integral [1]

$$\psi_{NB}[h_{ij}, \Phi|_\Sigma, \phi|_\Sigma] = \int \mathcal{D}g\, \mathcal{D}\Phi\, \mathcal{D}\phi\, e^{-I} \tag{1.1}$$

over compact Euclidean metrics (i.e. Riemannian metrics: signature $+ + \dots +$) and according Euclidean matter configurations. A standard procedure to approximate this object is to replace the path integral over generic configurations by a sum over configurations that solve the effective Euclidean field equations [2]. Here, by "effective", we refer to a division of the matter variables into two groups, denoted by Φ and ϕ, such that the field equations become

$$R_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu} R = -\Lambda\, g_{\mu\nu} + 8\pi G_n\, T_{\mu\nu}(\Phi)\,, \tag{1.2}$$

together with the field equation for Φ. $\Lambda \equiv \Lambda(\phi)$ is an effective cosmological constant. In case of ϕ being a minimally coupled scalar field with potential $V(\phi)$, it becomes $\Lambda(\phi) = 8\pi G_n V(\phi)$ and defines a regime in which the approximation $\phi \approx const$ is imposed (cf. Ref. [3]). G_n is the n-dimensional gravitational constant.

The division of the matter variables into Φ and ϕ is somewhat arbitrary and corresponds to the concrete goals one has in mind. For fixed Λ, the wave function behaves exponential for "small" spatial geometries (small spatial volumes) and oscillatory for "large" spatial geometries. In between these regimes the universe is thought to become "real" or classical. ψ_{NB} develops into a WKB-type wave function peaked around a family of classical Lorentzian (i.e. Pseudo-Riemannian) time evolutions (signature $-++\ldots+$) [4]. This is usually refered to as "nucleation" of the universe. However, there must be an additional step that explains why only one member of this family is observed. This may either be viewed as a property emerging dynamically ("decoherence"; see e.g. Refs. [5]) or (at least to some extent) be achieved by means of the physical interpretation of the wave function (*à la* the observer herself is part of just one classical history).

In the semiclassical approximation any nucleation scenario may be encoded in terms of a "real tunneling configuration" [6]. By this we mean a manifold $\mathcal{M}_{\rm sig\ ch}$, divided into two parts $\mathcal{M}_{\rm Eucl}$ and $\mathcal{M}_{\rm Lor}$ by a hypersurface Σ such that the Euclidean field equations for $(g_{\mu\nu}, \Phi)$ with cosmological constant $\Lambda(\phi)$ are satisfied on $\mathcal{M}_{\rm Eucl}$ (which is assumed to be compact), and the according Lorentzial (physical) field equations are satisfied on $\mathcal{M}_{\rm Lor}$. The junction conditions at the hypersurface Σ (at which the metric signature changes discontinuously) are that the extrinsic curvature K_{ij} as well as the (affine) time derivative $\partial_t\Phi$ of the matter fields vanish and that Σ is spacelike with respect to the Lorentzian part (see Fig. 1). The hybrid configuration as a whole thus satisfies the Einstein equations with matter (which are *à priorily* defined without reference to the metric signature) in a distibutional sense [7]. Likewise, it can be regarded as a stationary point of the action if the latter is defined by using an imaginary time variable in the Euclidean domain (Wick rotation). This situation is sometimes denoted as "strong" signature change [8]. The simplest example occurs if there are no fields Φ, $\mathcal{M}_{\rm Eucl}$ being half of $\mathbf{S}^n$ with radius a $(2\Lambda a^2 = (n-1)(n-2))$, and $\mathcal{M}_{\rm Lor}$ the corresponding half of n-dimensional de Sitter space, joined together along the equator $\Sigma = \mathbf{S}^{n-1}$ of $\mathbf{S}^n$ (see Ref. [9]).

The information stored in such a configuration is threefold: At the initial hypersurface Σ the universe starts its classical evolution with some initial values (h_{ij}, Φ), such that $K_{ij} = \partial_t\Phi = 0$. The subsequent classical period is given by the manifold $\mathcal{M}_{\rm Lor}$ and the field configuration on it. The probability for this scenario (relative to others of the same type) is given by

$$P_{NB} = |\psi_{NB}|^2 \approx e^{-2I_{\rm Eucl}} \tag{1.3}$$

where $I_{\rm Eucl}$ is the Euclidean action, evaluated over the Euclidean part $\mathcal{M}_{\rm Eucl}$. Hence, $P_{NB} \equiv P_{NB}(\phi)$ may in principle distinguish between different candidate nucleation configurations. Since $\mathcal{M}_{\rm Eucl}$ is half of a compact configuration $\mathcal{M}$ admitting a reflection symmetry (in the above example this is just the whole of $\mathbf{S}^n$), one may write as well $P_{NB} \approx \exp(-I)$, with I the Euclidean action evaluated over $\mathcal{M}$.

The standard pocedure to analyze a particular model within the range of validity of the approximations imposed consists of (i) considering all solutions to the Euclidean field equations which are regular and admit a hypersurface Σ along which they may be joined to their Lorentzian counterparts; (ii) minimizing I (i.e. maximizing P_{NB}) at constant Λ and (iii) comparing the maximized values of $P(\phi)$ at different ϕ. The interpretation of the third step depends on the details and the goals connected with the particular model. In the case of a massive scalar field ϕ, the prediction is that the nucleation value (i.e. initial value) of ϕ tends to be as small as possible (thus getting thrown out of the range of applicability of the approximation $\phi \approx const$). Nevertheless, in the literature configurations with different topology (as e.g. $\mathbf{S}^4$ and $\mathbf{S}^2 \times \mathbf{S}^2$) are frequently compared by means of P_{NB} at equal (finite) values of ϕ (respectively Λ; see e.g. Refs. [6] and [10]). A somewhat different sort of models emerges if small scale topological fluctuations (wormholes) are admitted (ϕ representing the "wormhole parameters"). Then the procedure outlined above leads to Coleman's argument that Λ is infinitely peaked at 0 [11]. In the following we are mainly interested in the question of "nucleation". Moreover, we will leave apart the matter fields denoted by Φ, so the field equations reduce to Einstein's equations with a (positive) effective cosmological constant $\Lambda(\phi)$. In the case of a minimally coupled massive scalar field, $\Lambda(\phi) = 4\pi G_n m^2 \phi^2$.

2 Nucleation energy

The effective cosmological constant plays the role of a vacuum *energy density*. To be more precise, at the nucleation hypersurface Σ, the non-gravitational energy density is given by the expression $\Lambda/(8\pi G_n)$. Hence (omitting any further field Φ), the corresponding amount of *energy* with which the universe is born, is given by

$$E = \int_\Sigma d^{n-1}x\, \sqrt{h}\, \frac{\Lambda}{8\pi G_n} = \mathcal{V}_\Sigma\, \frac{\Lambda}{8\pi G_n}, \tag{2.1}$$

where $\mathcal{V}_\Sigma$ is the volume of Σ as an $(n-1)$-manifold. However, note that the total energy contained in a closed universe is in some sense identical to zero (since the gravitational field carries precisely the energy $-E$).

Now suppose we are given several possible nucleation configurations at equal values of Λ, each one sloppily denoted by $(\mathcal{M}, \Sigma)$. Minimizing I (maximizing P_{NB}) at constant Λ (and constant n) corresponds to the question for the most probable classical initial configuration, given that the energy density at nucleation is $\Lambda/(8\pi G_n)$. Since for (low dimensional) products of spheres the radii appearing are of the order $a \sim \Lambda^{-1/2}$, this is somewhat related to the question for the most probable nucleation configuration at a given "size" of the initial hypersurface Σ. Thereby the underlying idea is that the wave function describes an ensemble of universes that "probe" for competing configurations at equal values of the energy density or of the size. This idea is related to the use of the "configuration representation" $\psi_{NB}[h_{ij}, \ldots]$. Once a configuration has beeen singled out (e.g. by overwhelming probability or by some process of decoherence), we may naively think about the universe being created at some initial energy density or at some initial size.

It is however conceivable that some sort of energy E is included in the variables, as $\psi_{NB}[E, \ldots]$, such that for the nucleation configuration E coincides with the quantity defined above. This would amount to a different question: Which of several competing configurations is most probable, when the comparison is carries out at equal values of E? When some configuration is realized, we may naively think about the universe being created at some initial energy. In a formal sense, this would relate the emergence of classical time with its conjugate quantity, energy.

One would expect both questions to give the same answers, at least as far as large scale variables such as topology and dimension are concerned. However, this is not the case, the formal reason being that the relation (2.1) between Λ and E contains the volume $\mathcal{V}_\Sigma$ as well as the dimension n.

Since we do not know much about the nature of the underling structures (we do not even know to what extent these structures are physical "processes" that depend on the particular model, or fundamental issues related to the interpretation of wave function of the universe), it is worth investigating the structure of the relative probabilities for competing configurations if the energy E is kept fixed, instead of the cosmological constant.

From now on we admit configurations at arbitrary topology and dimension, hence we consider a "multiple-dimensional" model, rather than just a "multi-dimensional" one. (Different approaches to the problem of space-time dimension in Euclidean quantum cosmology may be found in Ref. [12]).

The n-dimensional gravitational constant is written as

$$G_n = \left(\frac{\kappa_n}{m_P}\right)^{n-2} \tag{2.2}$$

where we just know $\kappa_4 = 1$. In the main part of my talk I will use $\kappa_n \approx 1$ for all n, although this condition may be relaxed without changing much of the results.

3 Probabitities at equal energy

Minimizing the Euclidean action at constant Λ leads to the problem of arbitrarily large dimensions. Inserting round spheres $\mathcal{M} = \mathbf{S}^n$ with radius a, the Euclidean Einstein equations imply $a^2 = (n-1)(n-2)/(2\Lambda)$, and for large n

$$I \sim -\left(\frac{n}{\Lambda}\right)^{n/2} . \tag{3.1}$$

Hence, there is no finite minimizing dimension n.

In order to test the proposal formulated above, I would like to admit a larger set of configurations. Consider arbitrary products of round spheres

$$\mathcal{M} = \mathbf{S}^{n_1} \times \mathbf{S}^{n_2} \times \ldots \times \widetilde{\mathbf{S}^{n_A}} \times \ldots \times \mathbf{S}^{n_m} \tag{3.2}$$

with radii $(a_1, \ldots, a_m)$ and total dimension $n = \sum_{B=1}^{m} n_B$. In the A-th sphere (denoted by a $\tilde{}$) the change of signature occurs, i.e. this sphere is joined along its equator to half of the n_A-dimensional de Sitter space, whereas the other spheres remain unaffected. (In other words, the Lorentzian time coordinate emerges from an angular coordinate on the A-th factor sphere by a Wick rotation). The nucleation hypersurface Σ is thus the product of $\mathbf{S}^{n_A-1}$ with all the remaining spheres $\mathbf{S}^{n_B}$. The Euclidean Einstein equations reduce to $2\Lambda a_B^2 = (n_B - 1)(n-2)$, and hence require all $n_B > 1$ and $n \geq 3$. As a consequence, all the radii a_B are completely determined by Λ and the dimensions n_B. The Euclidean action has the form

$$I = -\frac{1}{16\pi G_n} \int_{\mathcal{M}} d^n x \, \sqrt{g} \, (R - 2\Lambda) \tag{3.3}$$

(note that no boundary term is needed here). This can be evaluated on the set of solutions specified above. Our proposal consists of eliminating Λ in terms of E, which yields, after some tedious manipulations,

$$I = -\left(\frac{8\pi}{F}\right)^{1/(n-3)} \left(\frac{2\,\kappa_n}{n-2}\,\frac{E}{m_P}\right)^{(n-2)/(n-3)}, \tag{3.4}$$

where

$$F = \left(\frac{v_{n_A-1}}{v_{n_A}(n_A-1)^{1/2}}\right)^{n-2} \prod_{B=1}^{m} v_{n_B}(n_B-1)^{n_B/2}\,, \tag{3.5}$$

v_q being the volume of the unit-$\mathbf{S}^q$. If $n = 3$, the energy is fixed by $E = 1/(6G_3)$, and since we expect E to play the role of a generic quantity, we ignore this case and set $n \geq 4$. We analyze this model in three steps, the mathematical details of which have originally been presented in Ref. [13].

Step 1: Minimize I at E and n fixed. This amounts to minimize F, and we do not have to know the κ_n during this first step. The analysis for small n may be carried out by explicitly by computing the quantity F. At any n, we find a Euclidean configuration $\mathcal{K}_n$ that minimizes the action I. The first few of these configurations are

$$\mathcal{K}_4 = \mathbf{S}^2 \times \widetilde{\mathbf{S}^2} \tag{3.6}$$

$$\mathcal{K}_5 = \mathbf{S}^2 \times \widetilde{\mathbf{S}^3} \tag{3.7}$$

$$\mathcal{K}_6 = \mathbf{S}^2 \times \widetilde{\mathbf{S}^4} \tag{3.8}$$

$$\mathcal{K}_7 = \mathbf{S}^2 \times \mathbf{S}^2 \times \widetilde{\mathbf{S}^3}\,. \tag{3.9}$$

It is quite surprising that at $n = 4$ the favoured configuration is not the round $\mathbf{S}^4$. The nucleation hypersuface associated with $\mathcal{K}_4$ is $\Sigma = \mathbf{S}^1 \times \mathbf{S}^2$. For large n one obtains that $\mathcal{K}_n$ is a product of a bunch of two-spheres with $\mathbf{S}^p$, where $p \approx 1.277\sqrt{n} + 1$.

Hence, the favoured topology of the spatial sections in $n = 4$ is $\mathbf{S}^1 \times \mathbf{S}^2$, which implies (as far as it is actually realized) that the universe is of Kantowski-Sachs type. This topology may as well be interpreted as representing an $\mathbf{S}^3$ with a (primordial) black hole [10].

Step 2: Minimize $I(\mathcal{K}_n)$ at fixed E. For large n we find

$$I(\mathcal{K}_n) = -\sqrt{2}\left(\frac{\kappa_n}{n-2}\right)^{(n-2)/(n-3)} \frac{E}{m_P}\left(1 + O(\frac{1}{n})\right) \sim -\frac{1}{n}\,\frac{E}{m_P} \tag{3.10}$$

where the $\sim$ sign is for $\kappa_n \approx 1$. This may be relaxed to the condition $\kappa_n/n \rightarrow 0$ and possibly some monotonicity requirement without changing much. As a consequence, with each energy E we may associate a well-defined dimension n minimizing (3.10). Asymptotically one finds, $n \sim \ln(m_P/E)$. Hence, large E corresponds to small n. For large E, the exponent structure in (3.4) implies that the minimizing dimension is $n = 4$. At some value $E = E_4$, defined by the equality $I(\mathcal{K}_4) = I(\mathcal{K}_5)$, the minimizing dimension becomes $n = 5$. Defining by analogous equality of adjacent probabilities a sequence of energy levels E_n, we find n to be the minimizing dimension in the interval $E_n < E < E_{n-1}$. The first two value (for $\kappa_5 = \kappa_6 = 1$) are $E_4 \approx 0.287 m_P$ and $E_5 \approx 0.143 m_P$. For large n we find $E_n \sim m_P \exp(-n)$. Fig. 2 shows the minimizing dimension n as a function of E (the scale being $m_P = 1$). The regime of small energies can be thought of as representing the multiple-dimensional quantum state of the universe.

Step 3: Minimize $I(\mathcal{K}_{\text{minimizing n}})$ with respect to E? Formally, the action is minimized for $E \to \infty$, which implies $n = 4$. However, it is not quite clear what this means physically. One may imagine that – in some semiclassical picture – the universe "evolves" from small to large energies. At $E > E_4 \approx m_P$, the minimizing configuration $\Sigma = \mathbf{S}^1 \times \mathbf{S}^2$ may be thought of "freezing out" by some decoherence effect. However, at $E \approx E_4$, several other configurations will still have comparable probability. Hence we estimate the relative probabilities with which higher dimensions and alternative topologies are suppressed. For $E \gg E_4$, we find

$$p_{\text{dim}}(E) \equiv \frac{P(\mathcal{K}_5)}{P(\mathcal{K}_4)} \approx \exp\left(-\frac{2E^2}{\pi m_P^2}\right) \approx \left(\frac{P(\mathbf{S}^4)}{P(\mathcal{K}_4)}\right)^9 \equiv p_{\text{top}}(E)^9 . \qquad (3.11)$$

Fig. 3 shows these two curves (the scale is $m_P = 1$; for small E, the formulae above are not very accurate, hence the slight mismatch between (3.11) and Fig. 3). Thus we estimate that at a scale E_{dim} of several m_P the dimensions greater that 4 become suppressed, while at a larger scale $E_{\text{top}} \approx 3E_{\text{dim}}$ the competing sphere $\mathbf{S}^4$ becomes suppressed as compared to $\mathcal{K}_4$.

I can imagine three possibilities for a decoherence process to work: The first one is based on the idea that the relaxation of the dimension $n = 4$ at E_{dim} somehow "induces" one of the configurations to "nucleate". In this case $\Sigma = \mathbf{S}^3$ and $\Sigma = \mathbf{S}^1 \times \mathbf{S}^2$ will be realized at probabilities of the same order of magnitude. Alternatively, the relaxation of dimension and topology might be "decoupled" from one another and occur at E_{dim} and E_{top}, respectively. (Naively, "first" the dimension becomes classical, while the topology is still quantum). Such a mechanism could result into a considerable suppression of the isotropic configuration and predict $\Sigma = \mathbf{S}^1 \times \mathbf{S}^2$. In both cases, a scalar field ϕ with mass $m \approx 10^{-5} m_P$ (according to the bound set by the microwave anisotropy) would nucleate at $\phi \approx m_P^2/m$, which is quite enough to ensure sufficient inflation of the subsequent classical time evolution. A third possibility is that nucleation and decoherence occur at higher energy scales, induced by some other mechanism. Since there is no charactistic scale above E_4 in our approach, this could mean that we predict $\phi \to 0$, and reproduce the usual problems related with finding the most probable initial scalar field value for the no-boundary wave function [14]. This third possibility will throw us out of the range of our approximations, but the dominance of $\Sigma = \mathbf{S}^1 \times \mathbf{S}^2$ might still survive.

We should thus study in a conceptually deeper way the relations between energy, dimensional and topological "relaxation", nucleation and decoherence, emergence of time and the interpretation of the wave function. For those who do not consider higher dimensions to be of theoretical relevance at all, there is still the open question of the competition between $\mathbf{S}^4$ and $\mathbf{S}^2 \times \mathbf{S}^2$.

4 Tunneling wave function

The tunneling wave function of the universe [9], [15] differs from the no-boundary wave function in the semiclassical Euclidean regime only by a sign in the exponent. Thus the nucleation probabilities may be approximated by $P_T \approx \exp(I)$. Now recall that the probabilities due to the no-boundary wave function $P_{NB} \approx \exp(-I)$ can predict $n = 4$ only if the n-dimensional gravitational constant is such that $\kappa_n/n \to 0$ as $n \to \infty$ (recall (2.2) and (3.10)). In the case $\kappa_n/n \to \infty$, there is a well-defined dimension n maximizing the action at a given value of E. Hence, in this case, the tunneling wave function may be invoked. As opposed to the no-boundary case, small energies correspond to low dimensions. For sufficiently small E ($E < 3\sqrt{2}\,\kappa_5^3\, m_P/16$ if some monotonicity in n is assumed), the favoured dimension is $n = 4$, and the according Euclidean configuration is $\mathbf{S}^4$, thus $\Sigma = \mathbf{S}^3$. The multiple-dimensional quantum state of the universe is associated with large values of E, as if the coming-into-existence is related with some energy minimization. The Euclidean configurations maximizing the action at given E and n are, for $n > 4$, just $\mathbf{S}^{n-2} \times \widetilde{\mathbf{S}^2}$ (as opposed to $\mathcal{K}_n$ in the no-boundary case). Again, the details of the interpretation would require more knowledge about the nucleation process.

The choice $\kappa_n \sim n$ for large n may be considered as a limiting case that potentially neutralizes the question for the space-time dimension.

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