Batalin-Tyutin Quantization of the Chiral Schwinger Model

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ABSTRACT

We quantize the chiral Schwinger Model by using the Batalin-Tyutin formalism. We show that one can systematically construct the first class constraints and the desired involutive Hamiltonian, which naturally generates all secondary constraints. For $a > 1$, this Hamiltonian gives the gauge invariant Lagrangian including the well-known Wess-Zumino terms, while for $a = 1$ the corresponding Lagrangian has the additional new type of the Wess-Zumino terms, which are irrelevant to the gauge symmetry.

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I. Introduction

Batalin and Fradkin (BF) [1] had proposed a new kind of quantization procedure for second class constraint systems. When combined with Batalin et al. (BFV) [2] formalism for first class constraint systems, the BFV formalism is particularly powerful for deriving a covariantly gauge-fixed action in configuration space. Fujiwara et al. (FIK) [3] have proposed an improved treatment of anomalous gauge theories based on the BF formalism. We have applied the FIK method to the bosonized chiral Schwinger model (CSM) [4]. Recently, Banerjee, Rothe, and Rothe [5] have pointed out that the FIK analyses [3,4] are not a systematic application of the BFV formalism. After their work, Banerjee [6] has systematically applied Batalin-Tyutin (BT) Hamiltonian method [7] to the second class constraint system of the abelian Chern-Simons (CS) field theory [8-10]. As a result, he has obtained the new type of an abelian Wess–Zumino (WZ) action, which is irrelevant to the gauge symmetry. Very recently, we have quantized the nonabelian case [11], and the abelian self-dual massive theory [12] by using the BT formalism. As shown in these works, the nature of second class constraint algebra also originates from the symplectic structure of the CS term as well as the local gauge symmetry breaking effect. There are some other interesting examples in this approach [13].

On the other hand, there has been a great progress in the understanding of the physical meaning of anomalies in quantum field theory through the study of the CSM. Jackiw and Rajaraman [14] showed that a consistent and unitary, quantum field theory is even possible in the gauge non-invariant formulation. Alternatively, a gauge invariant version [15] can be obtained by adding a Wess-Zumino action to the gauge non-invariant original theory, as was proposed by Faddeev and Shatashvili [16]. Since their works, the CSM have been still analyzed by many authors as an archetype of anomalous gauge theory [4,5,17].

In the present paper, we shall apply the BT method to the bosonized CSM having still novel features. In Sec. II, we consider the bosonized CSM with $a > 1$, which has two second class constraints. Through the BT analysis, we will obtain the well-known WZ term to cancel the usual gauge anomaly after we convert the original second class system into the fully first class one. In Sec. III, we consider the bosonized CSM for $a = 1$, which has four second class constraints. In contrast to the $a > 1$ case, we will obtain an additional new WZ action, which cannot be obtained in the usual path-integral framework, as well as the usual WZ action needed to cancel the gauge anomaly. In fact, the usual WZ action is not enough to make the second class system the first class one for $a = 1$. Sec. IV is devoted to a conclusion.

II. CSM in the case of $a > 1$

In this section, we consider the bosonized CSM model in the case of $a > 1$ [16]

$$S_{CSM} = \int d^2x \left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + eA_\nu(\eta^{\mu\nu} - \epsilon^{\mu\nu})\partial_\mu\phi + \frac{1}{2}ae^2 A_\mu A^\mu \right], \quad (1)$$

where $\eta^{\mu\nu} = $ diag.(1,-1), $\epsilon^{01} = 1$, and a is a regularization ambiguity [14], which is defined for calculating the fermionic determinant of the fermionic CSM. The canonical momenta are given by

$$
\begin{aligned}
\Pi^0 &= 0, \\
\Pi^1 &= F_{01} = \dot{A}_1 - \partial_1 A_0, \\
\Pi_\phi &= \dot{\phi} + e(A_0 - A_1),
\end{aligned}
\quad (2)
$$

where the overdot means the time derivative. Following the usual Dirac's standard procedure [19], there are one primary constraint

$$\Omega_1 \equiv \Pi^0 \approx 0, \quad (3)$$

and one secondary constraint

$$\Omega_2 \equiv \partial_1\Pi^1 + e\Pi_\phi + e\partial_1\phi + e^2 A_1 + (a-1)e^2 A_0 \approx 0. \quad (4)$$

This constraint is obtained by conserving Ω_1 with the total Hamiltonian

$$H_T = H_c + \int dx \; u\Omega_1, \quad (5)$$

where H_c is the canonical Hamiltonian as follows

$$
\begin{aligned}
H_c = \int dx \Bigg[&\frac{1}{2}(\Pi^1)^2 + \frac{1}{2}(\Pi_\phi)^2 + \frac{1}{2}(\partial_1\phi)^2 - e(\Pi_\phi + \partial_1\phi)(A_0 - A_1) \\
&-A_0\partial_1\Pi^1 - \frac{1}{2}ae^2\{(A_0)^2 - (A_1)^2\} + \frac{1}{2}e^2(A_0 - A_1)^2 \Bigg],
\end{aligned}
\quad (6)
$$

and we denote a Lagrange multiplier u. Note that by fixing the Lagrange multiplier u as follows

$$u = \partial_1 A_1 - \frac{1}{a-1}\Pi^1, \quad (7)$$

no further constraints are generated via this procedure. Then, the constraints $\Omega_\alpha(\alpha = 1, 2)$ form the second class algebra as follows

$$
\begin{aligned}
\Delta_{\alpha\beta}(x, y) &\equiv \{\Omega_\alpha(x), \Omega_\beta(y)\} \\
&= e^2(a-1) \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \delta(x - y).
\end{aligned}
\quad (8)
$$

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Following the BT approach [7], we introduce new auxiliary fields Φ^α in order to convert the second class constraint Ω_α into the first class in an extended phase space, and assume that the Poisson algebra of the new fields is given by

$$\{\Phi^\alpha(x), \Phi^\beta(y)\} = \omega^{\alpha\beta}(x, y), \tag{9}$$

where $\omega^{\alpha\beta}$ is an antisymmetric matrix. Then, the modified constraints in the extended phase space are given by

$$\tilde{\Omega}_\alpha(\Pi^\mu, A_\mu, \Phi^\alpha) = \Omega_\alpha + \sum_{n=1}^\infty \Omega_\alpha^{(n)}; \qquad \Omega_\alpha^{(n)} \sim (\Phi^\alpha)^n \tag{10}$$

satisfying the boundary condition, $\tilde{\Omega}_\alpha(\Pi^\mu, A_\mu, 0) = \Omega_\alpha$. The first order correction term in the infinite series [7] is given by

$$\Omega_\alpha^{(1)}(x) = \int dy \; X_{\alpha\beta}(x, y)\Phi^\beta(y), \tag{11}$$

and the first class constraint algebra of $\tilde{\Omega}_\alpha$ requires the condition as follows

$$\triangle_{\alpha\beta}(x, y) + \int dw \; dz \; X_{\alpha\mu}(x, w)\omega^{\mu\nu}(w, z)X_{\beta\nu}(z, y) = 0. \tag{12}$$

As was emphasized in Ref. [6,11,12], there is a natural arbitrariness in choosing $\omega^{\alpha\beta}$ and $X_{\alpha\beta}$ from Eq. (9) and Eq. (11), which corresponds to the canonical transformation in the extended phase space [1,7]. Without any loss of generality, we take the simple solutions as

$$\omega^{\alpha\beta}(x, y) = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \delta(x - y),$$

$$X_{\alpha\beta}(x, y) = e\sqrt{a - 1} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \delta(x - y), \tag{13}$$

which are compatible with Eq. (12), and this choice considerably simplifies the algebraic manipulations. As a result, using Eqs. (10), (11) and (13), the new set of constraints is found to be

$$\tilde{\Omega}_\alpha = \Omega_\alpha + e\sqrt{a - 1} \; \Phi^\alpha, \tag{14}$$

which are strongly involutive,

$$\{\tilde{\Omega}_\alpha, \tilde{\Omega}_\beta\} = 0. \tag{15}$$

In other words, we can make the second class constraints the first class by introducing the new fields in the extended phase space. Therefore, we have all the first class constraints in the extended phase space by applying the BT formalism systematically. Observe further that only $\Omega_\alpha^{(1)}$ contributes in the series (10) defining the first class constraint. All higher order terms given by Eq. (10) vanish as a consequence of the proper choice (13).

Next, we derive the corresponding involutive Hamiltonian in the extended phase space. It is given by the infinite series [7],

$$\tilde{H}(\Pi^\mu, A_\mu, \Phi^\alpha) = H_c + \sum_{n=1}^\infty H^{(n)}; \quad H^{(n)} \sim (\Phi^\alpha)^n \tag{16}$$

satisfying the initial condition, $\tilde{H}(\Pi^\mu, A_\mu, 0) = H_c$. The general algebraic form for the involution of $\tilde{H}$ is given by

$$H^{(n)} = -\frac{1}{n} \int dx\,dy\,dz \; \Phi^\alpha(x)\omega_{\alpha\beta}(x,y)X^{\beta\gamma}(y,z)G_\gamma^{(n-1)}(z), \quad (n \geq 1), \tag{17}$$

where the generating functionals $G_\alpha^{(n)}$ are given by

$$\begin{aligned} G_\alpha^{(0)} &= \{\Omega_\alpha^{(0)}, H_c\}, \\ G_\alpha^{(n)} &= \{\Omega_\alpha^{(0)}, H^{(n)}\}_{\mathcal{O}} + \{\Omega_\alpha^{(1)}, H^{(n-1)}\}_{\mathcal{O}} \quad (n \geq 1), \end{aligned} \tag{18}$$

where the symbol $\mathcal{O}$ represents that the Poisson brackets are calculated among the original variables, $i.e.$, $\mathcal{O} = (\Pi^\mu, A_\mu)$. Here, $\omega_{\alpha\beta}$ and $X^{\alpha\beta}$ are the inverse matrices of $\omega^{\alpha\beta}$ and $X_{\alpha\beta}$, respectively. Explicit calculations of $G_\alpha^{(0)}$ yield

$$G_1^{(0)} = \Omega_2, \tag{19}$$

$$G_2^{(0)} = e^2\Pi^1 + e^2(a-1)\partial_1 A_1, \tag{20}$$

which are substituted in Eq. (17) to obtain $H^{(1)}$,

$$H^{(1)} = \frac{1}{e\sqrt{a-1}} \int dx[G_2^{(0)}\Phi^1 - \Omega_2\Phi^2]. \tag{21}$$

This is inserted back in Eq. (18) in order to deduce $G_\alpha^{(1)}$ as follows

$$G_1^{(1)} = e\sqrt{a-1}\,\Phi^2, \tag{22}$$

$$G_2^{(1)} = -e\sqrt{a-1}\,\partial_1^2\Phi^1 + \frac{e^3}{\sqrt{a-1}}\,\Phi^1, \tag{23}$$

which then yield $H^{(2)}$ from Eq. (17),

$$H^{(2)} = \int dx \left[\frac{1}{2}(\partial_1\Phi^1)^2 + \frac{e^2}{2(a-1)}(\Phi^1)^2 - \frac{1}{2}(\Phi^2)^2\right]. \tag{24}$$

Since $G_\alpha^{(n)} = 0 \; (n \geq 2)$, the final expression for the Hamiltonian after the $n = 2$ finite truncations is given by

$$\tilde{H} = H_c + H^{(1)} + H^{(2)}, \tag{25}$$

which is strongly involutive with the first class constraints (14),

$$\{\tilde{\Omega}_\alpha, \tilde{H}\} = 0. \tag{26}$$

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Before performing the momentum integrations to obtain the partition function in the configuration space, it seems to appropriate to comment on the strongly involutive Hamiltonian (25). If we use the above Hamiltonian (25), we cannot naturally generate the first class Gauss' law constraint $\tilde{\Omega}_2$ from the time evolution of the primary constraint $\tilde{\Omega}_1$. Therefore, in order to avoid this problem as the case of the self-dual massive model [12], we use the equivalent first class Hamiltonian without any loss of generality, which only differs from the Hamiltonian (25) by adding a term proportional to the first class constraint $\tilde{\Omega}_2$ as follows

$$\tilde{H}' = \tilde{H} + \int dx \; \frac{1}{e\sqrt{a-1}} \Phi^2 \tilde{\Omega}_2 . \tag{27}$$

Then, this modified Hamiltonian $\tilde{H}'$ consistently generates the Gauss' law constraint such that $\{\tilde{\Omega}_1, \tilde{H}'\} = \tilde{\Omega}_2$ and $\{\tilde{\Omega}_2, \tilde{H}'\} = 0$. Note that when we act this Hamiltonian on physical states, the difference between $\tilde{H}'$ and $\tilde{H}$ is trivial because such states are annihilated by the first class constraint. Similarly, the equations of motion for observables (i.e., gauge invariant variables) will also be unaffected by this difference since $\tilde{\Omega}_2$ can be regarded as the generator of the gauge transformations.

Now we derive the Lagrangian including the WZ term, which describes the first class system, corresponding to the Hamiltonian (27). The first step is to identify the new variables Φ^α occurring in the extended phase space as canonically conjugate pairs in the Hamiltonian formalism,

$$\Phi^\alpha \equiv \sqrt{a-1} \left(\theta, \frac{1}{(a-1)} \Pi_\theta \right) , \tag{28}$$

satisfying Eqs. (9) and (13). Then, the starting phase space partition function is given by the Faddeev formula [20],

$$Z = \int \mathcal{D}A_\mu \mathcal{D}\Pi^\mu \mathcal{D}\phi \mathcal{D}\Pi_\phi \mathcal{D}\theta \mathcal{D}\Pi_\theta \prod_{\alpha,\beta=1}^{2} \delta(\tilde{\Omega}_\alpha)\delta(\Gamma_\beta) \det | \{\tilde{\Omega}_\alpha, \Gamma_\beta\} | e^{iS'}, \tag{29}$$

where

$$S' = \int d^2x \left(\Pi^\mu \dot{A}_\mu + \Pi_\phi \dot{\phi} + \Pi_\theta \dot{\theta} - \tilde{\mathcal{H}}' \right) \tag{30}$$

with the Hamiltonian density $\tilde{\mathcal{H}}'$ corresponding to $\tilde{H}'$, which is now expressed in terms of $\{\theta, \Pi_\theta\}$ instead of Φ^α. The gauge fixing conditions Γ_α are chosen so that the determinant occurring in the functional measure is nonvanishing. Furthermore, Γ_α may be assumed to be independent of the momenta so that these are considered as the Faddeev-Popov type gauge conditions.

Next, we perform the momentum integrations to obtain the configuration space partition function. First, the Π^0 integration is trivially performed by exploiting the delta function $\delta(\tilde{\Omega}_1) = \delta[\Pi^0 + e(a-1)\theta]$. Then, after exponentiating the remaining delta function $\delta(\tilde{\Omega}_2) = \delta[\partial_1\Pi^1 + e\Pi_\phi + e\partial_1\phi + e^2 A_1 + (a-1)e^2 A_0 - e\Pi_\theta]$ with Fourier

variable ξ as $\delta(\tilde{\Omega}_2) = \int \mathcal{D}\xi e^{-i\int d^2x \xi\tilde{\Omega}_2}$, and transforming $A_0 \rightarrow A_0 + \xi$, and integrating the other momentum variables Π_ϕ and Π^1, we obtain the following intermediate action

$$
\begin{aligned}
S &= \int d^2x \left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + eA_\nu(\eta^{\mu\nu} - \epsilon^{\mu\nu})\partial_\mu\phi + \frac{1}{2}ae^2 A_\mu A^\mu \right.\\
&\quad + \theta\{(a-1)e(\partial_1 A_1 - \dot{A}_0 - \dot{\xi}) + \frac{1}{2}(a-1)\partial_1^2\theta - e\epsilon^{\mu\nu}\partial_\mu A_\nu\}\\
&\quad + \left. \Pi_\theta\{\dot{\theta} - e\xi - \frac{1}{2(a-1)}\Pi_\theta\} - \frac{1}{2}(a-1)e^2\xi^2 \right],
\end{aligned}
\tag{31}
$$

and the corresponding measure is given by

$$
[\mathcal{D}\mu] = \mathcal{D}A_\mu \mathcal{D}\phi \mathcal{D}\theta \mathcal{D}\Pi_\theta \mathcal{D}\xi \prod_{\beta=1}^{2} \delta\left(\Gamma_\beta[A_0 + \xi, A_1, \theta]\right) \det | \{\tilde{\Omega}_\alpha, \Gamma_\beta\} | .
\tag{32}
$$

At this stage, the original theory is simply reproduced, if we choose the usual unitary gauge condition

$$
\Gamma_\alpha = (\theta, \Pi_\theta).
\tag{33}
$$

Note that this gauge fixing is consistent because when we take the gauge fixing condition $\theta \approx 0$, the condition $\Pi_\theta \approx 0$ is naturally generated from the time evolution of θ, i.e., $\dot{\theta} = \{\theta, \tilde{H}'\} = \frac{1}{(a-1)}\Pi_\theta \approx 0$. Then, one can easily realize that the new fields Φ^α are nothing but the gauge degrees of freedom, which can be removed by utilizing the gauge symmetry.

Finally, we perform the Gaussian integration over Π_θ. Then all terms including ξ in the action are canceled out, the resultant action is obtained as follows

$$
\begin{aligned}
S &= S_{CSM} + S_{WZ} \ ;\\
S_{CSM} &= \int d^2x \left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + eA_\nu(\eta^{\mu\nu} - \epsilon^{\mu\nu})\partial_\mu\phi + \frac{1}{2}ae^2 A_\mu A^\mu \right],\\
S_{WZ} &= \int d^2x \left[\frac{1}{2}(a-1)\partial_\mu\theta\partial^\mu\theta - e\theta\{(a-1)\eta^{\mu\nu} + \epsilon^{\mu\nu}\}\partial_\mu A_\nu \right],
\end{aligned}
\tag{34}
$$

where S_{WZ} is the well-known WZ term, which is needed to cancel the gauge anomaly. On the other hand, the corresponding Liouville measure just comprises the configuration space variables as follows

$$
[\mathcal{D}\mu] = \mathcal{D}A_\mu \mathcal{D}\phi \mathcal{D}\theta \mathcal{D}\xi \prod_{\beta=1}^{2} \delta\left(\Gamma_\beta[A_0 + \xi, A_1, \theta]\right) \det | \{\tilde{\Omega}_\alpha, \Gamma_\beta\} | .
\tag{35}
$$

Starting from the Lagrangian (34), we can easily reproduce the same set of all the first class constraints (14) and the modified Hamiltonian (27) effectively equivalent to the strongly involutive Hamiltonian (25).

Now, it seems appropriate to comment on the momentum integration by taking the different order. After integrating Π^0, exponentiating $\delta(\tilde{\Omega}_2)$, transforming $A_0 \rightarrow A_0 + \xi$,

let us integrate Π_θ and Π^1 in order. Then we obtain another intermediate action as follows

$$
\begin{aligned}
S \;=\; \int d^2 x \; \Bigg[&-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2}(a-1)\partial_\mu \theta \partial^\mu \theta - e\{(a-1)\eta^{\mu\nu} + \epsilon^{\mu\nu}\}\theta \partial_\mu A_\nu \\
&+\frac{1}{2} a e^2 A_\mu A^\mu + \phi\{\frac{1}{2}\partial_1^2 \phi + e(\epsilon^{\mu\nu} - \eta^{\mu\nu})\partial_\mu A_\nu\} \\
&-\frac{1}{2}\{\Pi_\phi - e(A_0 - A_1)\}\{\Pi_\phi - e(A_0 - A_1) - 2\dot\phi\} \Bigg] ,
\end{aligned}
\tag{36}
$$

and the corresponding measure is given by

$$
[\mathcal{D}\mu] = \mathcal{D}A_\mu \mathcal{D}\phi \mathcal{D}\Pi_\phi \mathcal{D}\theta \mathcal{D}\xi \prod_{\beta=1}^{2} \delta\left(\Gamma_\beta[A_0 + \xi, A_1, \phi, \theta]\right) \det \mid \{\tilde\Omega_\alpha, \Gamma_\beta\} \mid .
\tag{37}
$$

In this case, if we choose the matter gauge fixing condition $\phi \approx 0$ [18], then we obtain the consistency condition from the time evolution of ϕ as follows

$$
\dot\phi = \{\phi, \ \tilde{H}'\} = \Pi_\phi - e(A_0 - A_1) \approx 0.
\tag{38}
$$

Therefore, if we choose the above unitary gauge as follows

$$
\Gamma_\alpha = (\phi, \Pi_\phi - eA_0 + eA_1),
\tag{39}
$$

we easily reproduce the equivalent anomalous CSM [18]. Note that the WZ field θ becomes a dynamical field instead of the original matter field ϕ in this equivalent model. Furthermore, if we perform the Gaussian integration over Π_ϕ without choosing this gauge at this stage, we obtain the same resultant action (34). Therefore, we have explicitly shown that the final desired result is independent of the order of the momentum integration. Furthermore, if we add a term proportional to the constraint $\tilde\Omega_1$, which is trivial when acting on the physical Hilbert space, to the Hamiltonian (27) as follows

$$
\tilde{H}_c = \tilde{H}' + \int dx \; u\tilde\Omega_1,
\tag{40}
$$

one can exactly reproduce the BFV Hamiltonian $\tilde{H}_c$ of the CSM obtained in Ref. [4] with the exactly same first class constraints (14) with the new fields (28). Then, one can easily reconstruct the covariant effective action of the CSM, which is invariant under the BRST transformation [4].

III. CSM in the case $a = 1$

In this section, we consider the CSM in the case of $a = 1$, which is given by

$$
S \;=\; \int d^2 x \; \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2}\partial_\mu \phi \partial^\mu \phi + e A_\nu (\eta^{\mu\nu} - \epsilon^{\mu\nu}) \partial_\mu \phi + \frac{1}{2} e^2 A_\mu A^\mu \right] .
\tag{41}
$$

8

The canonical momenta are given by

$$
\begin{aligned}
\Pi^0 &\approx 0, \\
\Pi^1 &= F_{01} = \dot{A}_1 - \partial_1 A_0, \\
\Pi_\phi &= \dot{\phi} + e(A_0 - A_1).
\end{aligned}
\tag{42}
$$

There are one primary constraint

$$
\Omega_1 \equiv \Pi^0 \approx 0,
\tag{43}
$$

and three secondary constraints

$$
\begin{aligned}
\Omega_2 &\equiv \partial_1 \Pi^1 + e\Pi_\phi + e\partial_1\phi + e^2 A_1, \\
\Omega_3 &\equiv e^2 \Pi^1, \\
\omega_4 &\equiv -e^3 \Pi_\phi - e^3 \partial_1 \phi + e^4 A_0 - 2e^4 A_1.
\end{aligned}
\tag{44}
$$

Note that these constraints are obtained by conserving the constraints with the total Hamiltonian,

$$
H_T = H_c + \int dx \; u\Omega_1,
\tag{45}
$$

where H_c is the canonical Hamiltonian,

$$
\begin{aligned}
H_c = \int dx \Big[& \frac{1}{2}(\Pi^1)^2 + \frac{1}{2}(\Pi_\phi)^2 + \frac{1}{2}(\partial_1\phi)^2 - e(\Pi_\phi + \partial_1\phi)(A_0 - A_1) \\
& A_0 \partial_1 \Pi^1 - e^2 A_0 A_1 + e^2 (A_1)^2 \Big],
\end{aligned}
\tag{46}
$$

and we denote the Lagrange multiplier u. By fixing the Lagrange multiplier u as follows

$$
u = \frac{1}{e}\partial_1 \Pi_\phi + \frac{1}{e}\partial_1^2 \phi + 2\Pi^1 + 2\partial_1 A_1,
\tag{47}
$$

no further constraints are generated via this procedure. We find that all the constraints are fully second class constraints. However, in order to carry out the simple algebraic manipulations, it is essential to redefine ω_4 by using Ω_1 as follows

$$
\begin{aligned}
\Omega_4 &\equiv \omega_4 + e^2 \partial_1 \Omega_1 \\
&= -e^3 \Pi_\phi - e^3 \partial_1 \phi + e^4 A_0 - 2e^4 A_1 + e^2 \partial_1 \Pi^0,
\end{aligned}
\tag{48}
$$

although the redefined constraints are still completely the second class in contrast to the CS theories [6,11]. Otherwise, one will have a complicated constraint algebra including the derivative terms which are difficult to handle. Then, the simplified second class constraint algebra for $\Omega_\alpha (\alpha = 1, \cdots, 4)$ is given by

$$
\begin{aligned}
\Delta_{\alpha\beta}(x, y) &\equiv \{\Omega_\alpha(x), \Omega_\beta(y)\} \\
&= e^4 \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 2e^2 \\ 1 & 0 & -2e^2 & 0 \end{pmatrix} \delta(x - y).
\end{aligned}
\tag{49}
$$

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Following the BT approach [7], we introduce the matrix (9), which is compatible with the new fields Φ^α as follows

$$\omega^{\alpha\beta}(x,y) = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \delta(x-y). \tag{50}$$

Then the other matrix $X_{\alpha\beta}$ in Eq. (11) is easily obtained by solving Eq. (12) with $\Delta_{\alpha\beta}$ given by Eq. (49),

$$X_{\alpha\beta}(x,y) = e^2 \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ e^2 & 0 & 1 & 0 \\ 0 & e^2 & 0 & -1 \end{pmatrix} \delta(x-y). \tag{51}$$

Similar to the $a > 1$ case, there is also an arbitrariness in choosing $\omega^{\alpha\beta}$, which would naturally be manifested in Eq. (50). However, as has also been evidenced in other calculations [6,11,14], these choices of Eqs. (50) and (51) give remarkable algebraic simplifications for the case of $a = 1$. Using Eqs. (10), (11), (50) and (51), the new set of constraints is found to be

$$\begin{aligned} \tilde{\Omega}_1 &= \Omega_1 - e^2 \Phi^1 , \\ \tilde{\Omega}_2 &= \Omega_2 - e^2 \Phi^2 , \\ \tilde{\Omega}_3 &= \Omega_3 + e^4 \Phi^1 + e^2 \Phi^3 , \\ \tilde{\Omega}_4 &= \Omega_4 + e^4 \Phi^2 - e^2 \Phi^4 , \end{aligned} \tag{52}$$

which are strongly involutive as those should be

$$\{\tilde{\Omega}_\alpha, \tilde{\Omega}_\beta\} = 0. \tag{53}$$

Recall the Φ^α are the new variables satisfying the algebra (9) with $\omega^{\alpha\beta}$ given by Eq. (50). Therefore, we obtain the fully first class constraint system in the extended phase space.

The next step is to obtain the involutive Hamiltonian including the new fields Φ^α. It is noteworthy that there are only two terms Ω_α and $\Omega_\alpha^{(1)}$ in the expansion (52) due to the intuitive choices (50) and (51). The generating functionals $G_\alpha^{(n)}$ are obtained from Eq. (18) as follows,

$$\begin{aligned} G_i^{(0)} &= \Omega_{i+1} \qquad (i = 1, 2) , \\ G_3^{(0)} &= \Omega_4 - e^2 \partial_1 \Omega_1 , \\ G_4^{(0)} &= e^2 \partial_1^2 \Pi^1 - 2e^4 \Pi^1 - e^4 \partial_1 A_1, \end{aligned} \tag{54}$$

which are substituted in Eq. (17) to obtain $H^{(1)}$,

$$H^{(1)} = -\frac{1}{e^2} \int dx \ [\Phi^1(e^2\Omega_3 + G_4^{(0)}) - \Phi^2(e^2\Omega_2 + \Omega_4 - e^2\partial_1\Omega_1) - \Phi^3\Omega_3 - \Phi^4\Omega_2]. \tag{55}$$

This is inserted back in Eq. (18) to deduce $G_\alpha^{(1)}$ as follows

$$
\begin{aligned}
G_1^{(1)} &= -e^2\Phi^2, \\
G_2^{(1)} &= e^4\Phi^1 + e^2\Phi^3, \\
G_3^{(1)} &= e^4\partial_1\Phi^1 + e^4\Phi^2 - e^2\Phi^4, \\
G_4^{(1)} &= -2e^6\Phi^1 + 2e^4(\partial_1)^2\Phi^1 + e^4\partial_1\Phi^2 - 2e^4\Phi^3,
\end{aligned}
\tag{56}
$$

which then yield $H^{(2)}$

$$
H^{(2)} = \int dx \left[\frac{1}{2}e^4(\Phi^1)^2 + e^2\Phi^1\Phi^3 + e^2(\partial_1\Phi^1)^2 - e^2\Phi^1\partial_1\Phi^2 - \Phi^2\Phi^4 + \frac{1}{2}(\Phi^3)^2 \right]. \tag{57}
$$

Since $G_\alpha^{(n)} = 0$ $(n \geq 2)$, after the $n = 2$ finite truncations the final expression for the desired Hamiltonian is given by

$$
\tilde{H} = H_c + H^{(1)} + H^{(2)}, \tag{58}
$$

which is involutive,

$$
\{\tilde{\Omega}_\alpha, \tilde{H}\} = 0. \tag{59}
$$

According to the usual BT formalism, this formally completes the operatorial, abelian conversion of the original second class system with the Hamiltonian H_c and the constraints Ω_α into the first class with the Hamiltonian $\tilde{H}$ and the constraints $\tilde{\Omega}_\alpha$.

However, similar to the $a > 1$ case, if we use the above Hamiltonian, we cannot naturally generate the first class constraints $\tilde{\Omega}_i(i = 2,3,4)$ from the time evolution of the primary constraint $\tilde{\Omega}_1$. In order to avoid this situation, we also use another equivalent first class Hamiltonian without any loss of generality, which differs from the involutive Hamiltonian (58) by adding terms proportional to the first class constraint $\tilde{\Omega}_\alpha$ as follows

$$
\tilde{H}' = \tilde{H} + \Phi^1\tilde{\Omega}_3 - \frac{1}{e^2}\Phi^2(\tilde{\Omega}_3 + e^2\tilde{\Omega}_2) - \frac{1}{e^2}\Phi^3\tilde{\Omega}_3 - \frac{1}{e^2}\Phi^4\tilde{\Omega}_2, \tag{60}
$$

which is easily found through the simple algebraic manipulations of the new fields Φ^α with $\omega^{\alpha\beta}$. Then, this Hamiltonian $\tilde{H}'$ automatically generates the first class constraints such that $\{\tilde{\Omega}_i, \tilde{H}'\} = \tilde{\Omega}_{i+1}$ $(i = 1,2,3)$ and $\{\tilde{\Omega}_4, \tilde{H}'\} = 0$.

We now extract out the Lagrangian corresponding to the Hamiltonian (60). The first step is to identify the new variables Φ^α as canonically conjugate pairs in the Hamiltonian formalism,

$$
\Phi^\alpha \equiv (\frac{1}{e}\theta, -\frac{1}{e}\rho, -e\Pi_\rho, e\Pi_\theta) \tag{61}
$$

satisfying Eqs. (9) and (50). The starting phase space partition function is then given by the Faddeev formula [20],

$$
Z = \int \mathcal{D}A_\mu \mathcal{D}\Pi^\mu \mathcal{D}\phi \mathcal{D}\Pi_\phi \mathcal{D}\theta \mathcal{D}\Pi_\theta \mathcal{D}\rho \mathcal{D}\Pi_\rho \prod_{\alpha,\beta=1}^{4} \delta(\tilde{\Omega}_\alpha)\delta(\Gamma_\beta) \det | \{\tilde{\Omega}_\alpha, \Gamma_\beta\} | e^{iS'}, \tag{62}
$$

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where

$$S' = \int d^2x \left(\Pi^\mu \dot{A}_\mu + \Pi_\phi \dot{\phi} + \Pi_\theta \dot{\theta} + \Pi_\rho \dot{\rho} - \tilde{\mathcal{H}}' \right) \tag{63}$$

with the Hamiltonian density $\tilde{\mathcal{H}}'$ corresponding to $\tilde{H}'$, which is now expressed in terms of $\{\rho, \Pi_\rho, \theta, \Pi_\theta\}$ instead of Φ^α. As in the previous section, the gauge fixing conditions Γ_α may be also assumed to be independent of the momenta so that these are considered as the Faddeev-Popov type gauge conditions.

Next, to obtain the partition function in the configuration space, we perform the momentum integrations by taking the proper order for more simpler calculation without any loss of generality. First, the Π^0, Π_θ and Π_ρ integrations are trivially performed by exploiting the delta functions $\delta(\tilde{\Omega}_1) = \delta[\Pi^0 - e\theta]$, $\delta(\tilde{\Omega}_4) = \delta[-e^3\Pi_\phi - e^3\partial_1\phi + e^4 A_0 - 2e^4 A_1 - e^3\rho - e^3\Pi_\theta + e^3\partial_1\theta]$, and $\delta(\tilde{\Omega}_3) = \delta[e^2\Pi^1 + e^3\theta - e^3\Pi_\rho]$. Then, after exponentiating the remaining delta function $\delta(\tilde{\Omega}_2) = \delta[\partial_1\Pi^1 + e\Pi_\phi + e\partial_1\phi + e^2 A_1 + (a-1)e^2 A_0 - e\Pi_\theta]$ with Fourier variable ξ as $\delta(\tilde{\Omega}_2) = \int \mathcal{D}\xi e^{-i\int d^2x \xi\tilde{\Omega}_2}$ and transforming $A_0 \to A_0 + \xi$, we obtain the action as follows

$$
\begin{aligned}
S =\ & \int d^2x \left[-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}e^2 A_\mu A^\mu + e(\eta^{\mu\nu} - \epsilon^{\mu\nu})A_\nu\partial_\mu\phi \right. \\[4pt]
& + \theta\partial_1^2\theta + \rho\partial_1\theta - \dot{\theta}\partial_1\phi - eA_1\rho - \frac{1}{2}\rho^2 - \rho\partial_1\phi - eA_1\dot{\theta} + \dot{\theta}\partial_1\theta + \dot{\rho}\theta \\[4pt]
& \left. + e\epsilon^{\mu\nu}\theta\partial_\mu A_\nu + \frac{1}{2}\dot{\theta}^2 - \dot{\phi}\dot{\theta} - \rho\dot{\phi} + \Pi^1\left(\dot{A}_1 - \partial_1 A_0 + \frac{1}{e}\dot{\rho} + \frac{1}{e}\partial_1^2\theta\right) \right], \quad (64)
\end{aligned}
$$

and the corresponding measure is given by

$$[\mathcal{D}\mu] = \mathcal{D}A_\mu \mathcal{D}\phi \mathcal{D}\theta \mathcal{D}\rho \mathcal{D}\Pi^1 \mathcal{D}\xi \prod_{\beta=1}^{4} \delta\left(\Gamma_\beta[A_0 + \xi, A_1, \phi, \theta, \rho]\right) \det |\, \{\tilde{\Omega}_\alpha, \Gamma_\beta\}\, | . \tag{65}$$

Finally, we perform the Gaussian integration over Π^1. The resultant action is obtained as follows

$$
\begin{aligned}
S_{tot} =\ & S_{CSM} + S_{WZ} + S_{NWZ}\ ; \\[4pt]
S_{CSM} =\ & \int d^2x \left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + eA_\nu(\eta^{\mu\nu} - \epsilon^{\mu\nu})\partial_\mu\phi + \frac{1}{2}e^2 A_\mu A^\mu \right], \\[4pt]
S_{WZ} =\ & -\int d^2x [e\theta\epsilon^{\mu\nu}\partial_\mu A_\nu], \\[4pt]
S_{NWZ} =\ & \int d^2x \left[-(\partial_1\phi + eA_1 + \dot{\phi} - \partial_1\theta)(\dot{\theta} + \rho) + \frac{1}{2}\{(\dot{\theta})^2 - \rho^2\} \right. \\[4pt]
& \left. - \frac{1}{2e^2}(\dot{\rho} + \partial_1^2\theta)^2 + e\theta\partial_\mu A^\mu \right], \quad (66)
\end{aligned}
$$

where S_{WZ} is a usual WZ term needed to cancel the gauge anomaly, and S_{NWZ} is a new type of WZ term, which is irrelevant to the gauge symmetry. Note that the new type of WZ term S_{NWZ} as well as the well–known WZ term S_{WZ} should be needed to

make the second class system into the first class. On the other hand, the corresponding nontrivial Liouville measure just comprises the configuration space variables as follows

$$
\begin{aligned}
[\mathcal{D}\mu] &= \mathcal{D}A_\mu \mathcal{D}\phi \mathcal{D}\theta \mathcal{D}\rho \mathcal{D}\xi \delta[F_{01} + \frac{1}{e}(\dot{\rho} + \partial_1^2 \theta)] \\
&\quad \prod_{\beta=1}^{4} \{\delta(\Gamma_\beta[A_0 + \xi, A_1, \phi, \theta, \rho])\} \det \mid \{\tilde{\Omega}_\alpha, \Gamma_\beta\} \mid,
\end{aligned}
\tag{67}
$$

where $\delta[F_{01} + e^{-1}(\dot{\rho} + \partial_1^2 \theta)]$ is expressed by $\int \mathcal{D}\Pi^1 e^{-i \int d^2 x [F_{01} + e^{-1}(\dot{\rho} + \partial_1^2 \theta)]\Pi^1}$. Note that although the Maxwell term is disappeared as the cases of the conventional phase space approach [2,21] through the momentum integration due to the constraint Ω_3 in Eq. (44), we reintroduce this term into the action (64) because we have the δ-function related to F_{01} in the measure part. In contrast to the case of $a > 1$, from the action S_{tot} including the term of the phase space variable Π^1 due to the appearance of the δ-function in the measure instead of the Lagrangian (64) including only configuration space variables, i.e., at the level of action (66), we can only reproduce the same set of all the first class constraints (52) and the modified Hamiltonian (58). However, although we have succeeded to obtain the first class system for the case of $a = 1$, the final theory has not the gauge symmetry due to the presence of the non-trivial δ–function in the measure part. Therefore, through this analysis we have learned that in general the first class system do not always need to have the gauge symmetry. However, similar to the $a > 1$ case, if we add terms proportional to the constraints $\tilde{\Omega}_i$, i.e.,

$$
\tilde{H}_c = \tilde{H}' + \int dx \, [(u - \frac{1}{e^2}\partial_1 \Omega_2)\tilde{\Omega}_1 - \frac{1}{e^2}\partial_1 \Omega_1 \tilde{\Omega}_2],
\tag{68}
$$

which is trivial when acting on the physical Hilbert space, to the Hamiltonian (58), we can exactly reproduce the BFV Hamiltonian $\tilde{H}_c$ and the corresponding BRST invariant Lagrangian of the CSM obtained in Ref. [4].

IV. Conclusion

We have quantized the bosonized CSM having the different algebra of the constraints depending on the regularization parameter a by using the BT formalism. We have shown that one can systematically construct the first class constraints and the desired involutive Hamiltonian, which naturally generates all the secondary constraints including the Gauss constraint. For $a > 1$, this Hamiltonian gives the gauge invariant Lagrangian including the well-known WZ terms, while for $a = 1$ the corresponding Lagrangian has the new type of the WZ terms, which are irrelevant to the gauge symmetry and cannot be obtained in the usual path-integral framework, as well as the usual WZ term to cancel the gauge anomaly.

Acknowledgements

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References

1. I. A. Batalin, E. S. Fradkin: Phys. Lett. B180 (1986) 157; Nucl. Phys. B279 (1987) 514

2. E. S. Fradkin, G. A. Vilkovisky: Phys. Lett. B55 (1975) 224; I. A. Batalin, G. A. Vilkovisky: Phys. Lett. B69 (1977) 309

3. T. Fujiwara, Y. Igarashi, J. Kubo: Nucl. Phys. B341 (1990) 695

4. Y.-W. Kim et al.: Phys. Rev. D46 (1992) 4574

5. R. Banerjee, H. J. Rothe, K. D. Rothe: Phys. Rev. D49 (1994) 5438

6. R. Banerjee: Phys. Rev. D48 (1993) R5467

7. I. A. Batalin, I. V. Tyutin: Int. J. Mod. Phys. A6 (1991) 3255

8. Edited by S. Treiman et el.: Topological Investigations of Quantized Gauge Theories, Singapore: World Scientific 1985

9. G. Semenoff: Phys. Rev. Lett. 61 (1988) 517; G. Semenoff, P. Sodano: Nucl. Phys. B328 (1989) 753

10. R. Banerjee: Phys. Rev. Lett. 69 (1992) 17; Phys. Rev. D48 (1993) 2905

11. W. T. Kim, Y. -J. Park: Phys. Lett. B336 (1994) 376

12. Y.-W. Kim et al.: Phys. Rev. D51 (1995) 2943; E.-B. Park et al.: Mod. Phys. Lett. A10 (1995) to appear

13. N. Banerjee, S. Ghosh, R. Banerjee: Nucl. Phys. B417 (1994) 257; Phys. Rev. D49 (1994) 1996; R. Banerjee, H. J. Rothe, K. D. Rothe: Nucl. Phys. B426 (1994) 129

14. R. Jackiw, R. Rajaraman: Phys. Rev. Lett. 54 (1985) 1219; Phys. Rev. Lett. 54 (1985) 2060(E); N. K. Falck, G. Kramer: Ann. Phys. 176 (1987) 330; Z. Phys. C37 (1988) 321; K. Shizuya: Phys. Lett. B213 (1988) 298; K. Harada: Phys. Rev. Lett. 64 (1990) 139; Phys. Rev. D42 (1990) 4170

15. F. Schaposnik, C. Viallet: Phys. Lett. B177 (1986) 385; K. Harada, I. Tsutsui: Phys. Lett. B183 (1987) 311

16. L. D. Faddeev, S. S. Shatashvili: Phys. Lett. B167 (1986) 225

17. R. Rajaraman: Phys. Lett. B154 (1985) 305; H. O. Girotti, H. J. Rothe, K. D. Rothe: Phys. Rev. D34 (1986) 592; H. J. Rothe, K. D. Rothe: Phys. Rev. D40 (1989) 545; J. Sladkowski: Phys. Lett. B296 (1992) 361; J. -G. Zhou, Y. -G. Miao, Y. -Y. Liu: Mod. Phys. Lett. A9 (1994) 1273

18. W. T. Kim et al.: Mod. Phys. Lett. A7 (1992) 411

19. P. A. M. Dirac: Lectures on quantum mechanics, New York: Yeshiba University Press 1964

20. L. D. Faddeev, V. N. Popov: Phys. Lett. B25 (1967) 29

21. I. Batalin, E. S. Fradkin: Phys. Lett. B128 (1983) 303