

# COSMOLOGY OF THE NEXT-TO-MINIMAL SUPERSYMMETRIC STANDARD MODEL

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We discuss the domain wall problem in the Next-to-Minimal Supersymmetric Standard Model, with particular attention to the usual solution of explicit breaking of the discrete symmetry by non-renormalisable operators. This “solution” leads to a contradiction between the requirements of cosmology and those of avoiding the destabilisation of the hierarchy.

## 1 Introduction

One of the most popular extensions of the Minimal Supersymmetric Standard Model (MSSM) is that of the Next-to-Minimal Supersymmetric Standard Model (NMSSM)<sup>1</sup> where the usual Higgs sector of two doublets is supplemented by a gauge singlet superfield coupling only through the Higgs sector superpotential. Apart from the important phenomenological consequences, this allows the elimination of the  $\mu$  term from the MSSM superpotential by invoking a  $\mathbf{Z}_3$  symmetry. The  $\mu$  term presents a problem because one might expect it to be of order the Planck mass or at least the GUT scale, and yet reasonable phenomenology forces it to be of order the supersymmetry breaking scale  $m_{3/2}$ <sup>2</sup>

The purpose of this talk is to briefly review one of the most important cosmological implications of the NMSSM, namely that of the domain walls which inevitably form at the weak scale when the  $\mathbf{Z}_3$  is spontaneously broken<sup>3</sup>. The requirements that these walls do not destroy the predictions of standard cosmology forces us to explicitly break the  $\mathbf{Z}_3$ , which can be done with non-renormalisable operators (NROs). This in turn leads to problems with the destabilisation of the hierarchy, and we shall discuss how the relative sizes of the constraints affect the model. A more complete version of this work is contained in Reference 4.

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<sup>a</sup>Talk presented by P.L. White at Valencia 95

## 2 The NMSSM

In the MSSM the Higgs sector contains two doublets  $H_1$  and  $H_2$  coupling to down- and up-type quarks respectively, with a superpotential containing a term  $\mu H_1 H_2$ . The origin of the  $\mu$  term is obscure, but we must avoid taking it much larger than the supersymmetry breaking scale, which is typically assumed to be less than around 1TeV.

One possible solution to the  $\mu$  problem, which has the additional advantage of giving a radically different low energy Higgs phenomenology, is that of the NMSSM<sup>1</sup>. Here we add a gauge singlet superfield  $N$ , and replace the  $\mu$  term in the superpotential with the Higgs superpotential

$$W_{Higgs} = \lambda N H_1 H_2 - \frac{k}{3} N^3 \quad (1)$$

while the usual soft breaking terms are supplemented by another mass and two more trilinear terms instead of the bilinear term  $B\mu H_1 H_2$  of the MSSM. It is then possible to arrange the parameters of the model in a natural way so that the singlet gets a vacuum expectation value (VEV) which is of the same order as those of the other Higgs fields, generating an effective  $\mu$  parameter of form  $\lambda x$  where  $x = \langle N \rangle$ . The terms which we have left out of the potential consistent with gauge symmetry, namely  $\mu H_1 H_2$ ,  $\mu' N^2$ , and  $\mu'' N$ , can all be banned by invoking a  $\mathbf{Z}_3$  symmetry under which all chiral superfields have the same charge so that only trilinear terms are allowed in the superpotential. The inclusion of the  $N^3$  term is necessary both because it is consistent with  $\mathbf{Z}_3$  and because without it there is a Peccei-Quinn symmetry which gives a phenomenologically unacceptable axion.

## 3 Domain Walls

### 3.1 Formation and Structure

When a discrete symmetry is spontaneously broken, it generates domain walls<sup>3</sup>. This is simple to understand, since the discrete symmetry imposes that there are multiple degenerate vacua, and there is no way in which causally disconnected regions of the universe can conspire to all undergo a phase transition to the same vacuum (so long as the fields acquiring VEVs are in thermal equilibrium, which will always be true for the NMSSM). The universe must thus evolve to a state in which there are domains of different vacua, separated by “domain walls”.

On purely dimensional grounds, or by analogy with analytically solvable toy models, we expect the domain walls to have a thickness  $\delta$  and surface

energy  $\sigma$  given by

$$\delta \sim \frac{1}{\nu} \quad \sigma \sim \nu^3 \quad (2)$$

where  $\nu$  is a typical VEV of the fields. In the NMSSM wall solutions may be found numerically<sup>5</sup>, and we find that the above equations hold very well, with  $\nu$  being replaced by some typical VEV of the singlet which is of roughly the same magnitude as that of the Higgs which gives the W and Z bosons their masses, *i.e.* 174GeV.

### 3.2 Dynamics

Once a wall network has formed, it will not remain static. There are three important forces acting on the network of domain walls, namely surface tension, friction, and pressure. The first of these is simply the effect of the constant surface energy density, which makes it energetically favourable for small bubbles of wall to disappear, for walls to smooth themselves out, and so on. This removes the smaller scale structure, and given enough time will remove the wall network entirely. However, by causality it is clear that the largest possible correlation length, by which we mean typical domain size, cannot be larger than the horizon scale, and so the best we can expect is that there is typically one wall per horizon volume throughout the evolution of the universe. As will be discussed later, even if the entire visible universe were to consist of only part of one domain, this still has unwelcome cosmological consequences.

The second force is that of friction. As the wall moves through space it will interact with the thermalised plasma, dissipating its energy and slowing its evolution. This effect can be shown to be insignificant for very high temperatures, when the particles in the plasma have very small reflection coefficients and so do not exert a large force, and also at low temperatures when the density of the plasma is low<sup>4</sup>. We can thus neglect it, as it will only slow down the wall evolution in its early stages.

A final force which we may consider is that of pressure. If one of the vacua is slightly deeper than the others by some amount  $\varepsilon$ , as a consequence of some slight *explicit* violation of the  $\mathbf{Z}_3$  symmetry in the potential, then there will be a force per unit area on the walls of order  $\varepsilon$ . Since the magnitude of the force due to surface tension is given by  $\sigma/R$ , where  $R$  is the curvature scale (effectively the correlation length), we see that the pressure will dominate the dynamics when

$$\varepsilon > \frac{\sigma}{R} \quad (3)$$

Since  $R$  is steadily increasing as a result of surface tension (and the expansion of the universe), we see that for large enough values of  $\varepsilon$  the pressure will come to

dominate the dynamics before the present day. In this case the evolution of the wall network will begin with the curvature scale increasing such that it is always of order the horizon size, until ultimately pressure takes over, the true vacuum comes to dominate, and the wall network is completely eliminated. Numerical simulations suggest that once the pressure comes to dominate, disappearance of the walls is almost as fast as causality will allow<sup>6</sup>.

### 3.3 Cosmological Implications

We now turn to the cosmological implications of a domain wall network. As the universe expands, the energy density due to the walls will fall as  $a^{-1}$  (for static walls) or  $a^{-2}$  (for ultra-relativistic walls in a radiation dominated universe), where  $a$  is the cosmological scale parameter. Since the energy densities of matter and radiation fall as  $a^{-3}$  and  $a^{-4}$  respectively, it is clear that the walls will ultimately dominate the dynamics of the universe, and we can check that this will have happened long before the present day for weak scale walls, and so the wall network must have long since disappeared.

In fact we may draw tighter constraints from other cosmological observations. If the walls decay after nucleosynthesis, then they will release a vast amount of energy which can be shown to cause photodestruction of the light elements whose abundances can be accurately measured<sup>7</sup>. Thus we require that the walls decay before a temperature of around 0.1 to 1MeV, giving a minimum value of  $\varepsilon$  to be

$$\varepsilon \gtrsim \lambda' \sigma M_W^2 / M_{Pl} \quad (4)$$

where  $\lambda'$  is of order  $10^{-7}$ . We conclude that a tiny contribution to the superpotential from an NRO of form, say,  $\lambda' N^4 / M_{Pl}$  is sufficient to evade the cosmological constraints. Such an NRO will have negligible effect on the low energy phenomenology, and so we conclude that an NRO suppressed by at most one power of the Planck mass is sufficient to eliminate the walls in time to avoid cosmological problems.

## 4 Destabilising Divergences

The primary motivation for supersymmetry is the hierarchy problem. If the standard model is an effective theory valid to a large scale  $\Lambda$  then we expect the hierarchy to be destabilised by quadratic divergences and the standard model masses to be driven up to order  $\Lambda$ . Supersymmetry prevents this happening, since softly broken supersymmetry does not have quadratic divergences.

There is one important exception to this statement, in that tadpole diagrams in supersymmetry may be quadratically divergent<sup>8</sup> so long as we have

NROs in our theory. These diagrams may be ruled out for one of three reasons: gauge invariance, which cannot save us from singlet tadpoles;  $\mathbf{Z}_3$  symmetry, which our NROs must break to save us from the cosmological implications; and supersymmetry which is obviously broken for phenomenological reasons.

It is straightforward to check that every dimension five NRO which we can introduce in our superpotential allows the construction of at least one diagram at three loops or less which will be quadratically divergent. For example, the least dangerous such operator is  $\lambda'(H_1 H_2)^2/M_{Pl}$ , where the first such diagram occurs at three loops. These diagrams generate a term in the superpotential of form  $\mu'' N$ , where

$$\mu'' \sim \frac{\lambda' \lambda}{(16\pi^2)^2} m_{3/2} M_{Pl} \quad (5)$$

The presence of one factor of  $m_{3/2}$  is guaranteed by the remark above that in the absence of supersymmetry breaking the non-renormalisation theorem prevents the generation of such terms, while the factor  $M_{Pl}$  is generated by taking the cut-off scale  $\Lambda \sim M_{Pl}$ .

The effects of such terms on the hierarchy is catastrophic, since they generate terms in the low energy Higgs potential which may be as large as those from radiative breaking at each order, and so destroy any reason for us to expect the electroweak scale to be very much less than the GUT or Planck scales. Requiring that the electroweak scale is not driven up by orders of magnitude in scale requires  $\lambda' \lesssim 10^{-11}$ , in clear contrast to the constraint from cosmology given above.

## 5 Conclusions

We have thus seen that the standard solution to the domain wall problem in the NMSSM must always introduce destabilising divergences which will destroy all predictive power in the Higgs sector and drive the electroweak scale to be many orders of magnitude too large. One may still solve the domain wall problem by introducing renormalisable terms which break the  $\mathbf{Z}_3$ , such as a  $\mu$  term, but one then has a far more severe naturalness problem than is usually the case in the MSSM, since we must somehow ban all the dangerous NROs which can cause quadratically divergent singlet tadpoles, while simultaneously breaking every symmetry which could ban them.

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