

Real and Virtual Strange Processes

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Abstract

Following notions of quantum mechanics as interpreted by the Copenhagen School, we make a distinction between measurements involving one or two virtual K mesons. New predictions result for the period of K oscillations at the Φ factory.

1 Introduction

Our purpose is to discuss the insight that may be gained from Φ factory experiments ($\Phi \rightarrow K^0 + \bar{K}^0$) concerning a central issue of quantum measurements; i.e. *What is virtual and what is real?* Before discussing our concrete predictions (i.e. numbers!) for the outcome of some Φ factory experiments, we wish first to discuss qualitatively that *not all* aspects of quantum measurements have simple agreed upon answers. Briefly, we have a great deal of sympathy with Einstein who lost the debate with Bohr. Known experiments do fit into Bohr's scheme and mere "thought experiments" get boring after a while. Einstein and Bohr did agree upon what the quantum mechanical scheme entailed, but disagreed on whether this scheme was the complete story [1, 2, 3].

A primary rule of the scheme is that all experimental data are classical. Suppose that experimental data are stored on (say) a computer disk. Now

imagine that the magnetic grains on the disk were in superposition (an amplitude for “0” and an amplitude for “1” on all of the little magnetic bits in some big binary file). If the computer sends to the standard output the message “disk unreadable”, then you know you have no data. Real data are “classical” to a sufficient degree of accuracy. (A few coherent quantum bits here or there might appear in an “error bar”, but if you are really in quantum superposition, you have no data.) We call data *real*. The primary rule is that data are classical. The classical part of physics is essential to quantum mechanics in the Bohr scheme. Superposition of amplitudes applies to those parts of the physical system that you *do not see*. There is no data on a quantum object interacting with the classical measurement apparatus. If you have data, then it is from the classical part of the physical system. What you do not see, i.e. the quantum part of the physical system, is what we call *virtual*. The apparatus is real, and the quantum object is virtual. According to the dictates of Bohr, a measurement is an interaction between a classical apparatus and a quantum object. That was decided by Bohr, Heisenberg, and other assorted friends of the Copenhagen School. That is the conventional scheme of things. We have reviewed all this because over the years many workers have distorted what others previously agreed constituted the quantum mechanical analysis of measurements. For example, we offer a \$100.00 reward to the first reader who finds for us a manuscript with Bohr as an author which includes the so-called “collapse of the wave function” as a part of quantum mechanical theory.

Briefly, in the canonical “electron moves through two slits thought experiment” you get quantum interference when there are no data on which slit the electron passed through. The electron path is then “virtual”. If data exist showing which slit the electron chooses, then electron is real and there is no interference. The electron is real when you see it and virtual when you do not see it.

Bohr found no problems. The outcome of an experiment depends on how you set up the real (classical) apparatus, and PLEASE do not ask for data from a virtual (quantum) object. JUST DO NOT ASK WHAT THE QUANTUM OBJECT IS DOING! If you get the data, then the object is *not quantum*. Do not ask because there will be no data! Einstein had some problems. He asked questions. Einstein looked at the moon and it was real. Einstein stopped looking. Then what? We think the moon is still real, but the contrary has been proposed by some perfectly competent physicists. So

it is our *prejudice* that the moon is real! And we really do not care who looks at it.

2 What is Virtual and What is Real in Particle Physics?

In particle physics there are theoretical and experimental views of the matter. From the theoretical view we would take as “virtual” the somewhat technical but conventional definition of those processes in the “internal parts” of the Feynman diagrams. Consider the Feynman diagram in Fig.1. The “external legs” (solid lines) are real. In the laboratory they show up as data, e.g. particle tracks. You see them in an experiment. The dotted line is internal. That is a “virtual” particle. Bohr has warned us not ask about the virtual quantum process. You are not meant to know (dear reader) about the virtual. Do not ask! For example, there are two (Bohr) complimentary views: (i) four momentum space amplitudes and (ii) space-time amplitudes.

(i) In momentum space the “virtual particle” dotted line in Fig.1 has a nice four momentum label $P_{virtual}$. That seems simple enough. The only problem is that

$$[P_{virtual}^2 + (Mc)^2] \neq 0. \text{ (possibly off mass shell).} \quad (1a)$$

Virtual particles may not be on the “mass shell”. Real particles are on the mass shell. Virtual particles are not real. Bohr warned us not to ask.

(ii) The space-time “virtual particle” in the words of Feynman “...does anything it likes...”. It goes forward in time. It goes backward in time. It goes space-like. Any old speed said Feynman. Anything it likes,

$$x_{virtual}^2 = r_{virtual}^2 - c^2 t_{virtual}^2 > 0 \text{ (possibly superluminal speed).} \quad (1b)$$

STOP IT! Bohr dictates the if it is virtual, then do not ask. Virtual particles are not real. There will be no data.

(iii) But experimentalists tell us every day that there are data on virtual particles you do not actually see. Look again at the Feynman diagram of Fig.1. Suppose the external legs are measured tracks shown in postscript colors on an experimentalist’s transparency (right after we see a small person standing next to a huge machine). *We do not see the dotted line.* The right

tracks lead to a primary vertex and left tracks emerge from the secondary vertex. The dotted line is a particle that moved from the primary vertex to the secondary vertex. *Only nobody sees it!* It had no charge! It left no track! But we all know it was there. This, by our definition, is an *experimental virtual particle*. It moved from vertex to vertex and nobody saw it. We are not talking about the moon. We are talking about an elementary particle. If you do not see it, then it is virtual. But the experimentalists tell us that this virtual particle moved from the primary vertex y to the secondary vertex x along a classical path at constant (four velocity) v and on the mass shell,

$$x^\mu = y^\mu + v^\mu \tau, \quad (\text{experimental virtual particle}), \quad (2a)$$

$$P_{\text{virtual}}^2 + (Mc)^2 = M^2(v^2 + c^2) = 0, \quad (\text{experimental virtual particle}). \quad (2b)$$

Comparing the experimentally employed Eqs.(2) to the exciting theoretical possibilities in Eqs.(1), we find that the experimentalist's virtual particle is rather dull and classical, i.e. *it looks real (not virtual) even if you do not see it.*

Bohr had all the answers. He threw in the correspondence principle along with the complimentarity of momentum space and space-time. Only Einstein was worried. The correspondence principle says that when the action of the particle is large on the scale of \hbar the particle is classical *even if you do not look at it.* The particle going from the primary to the secondary vertex had to move so far that the action was very large compared with \hbar . The particle may do “anything it likes”, but when the action is large it “likes” to be classical with overwhelming probability. To do otherwise would have such small probability as to be a miracle. Miracles can happen, but they are hard to duplicate. There are no good statistics on miracles.

The mathematics is that the propagator from vertex to vertex

$$[-\partial_\mu \partial^\mu + \kappa^2]D(x - y) = \delta(x - y), \quad \kappa = (Mc/\hbar), \quad (3)$$

has the Dirac-Feynman path integral representation

$$D(x - y) = \int_{z(0)=y}^{z(\tau)=x} \prod_\tau \left(\frac{Dp Dz}{2\pi\hbar} \right) \exp \frac{i}{\hbar} \int_y^x [p_\mu dz^\mu - \mathcal{H}(p)d\tau], \quad (4)$$

where

$$\mathcal{H}(p) = \frac{1}{2M} [p^2 + (Mc)^2]. \quad (5)$$

If the particle were classical, then the free particle Hamilton-Jacobi equation [4]

$$-\frac{\partial S}{\partial \tau} = \mathcal{H} \left(p = \frac{\partial S}{\partial x} \right), \quad (6a)$$

$$S(x - y, \tau) = -\frac{Mc^2}{2} \left(\tau - \frac{(x - y)^2}{c^2 \tau} \right), \quad (6b)$$

would yield the classical experimental Eqs.(2) via the equations

$$-\frac{\partial S}{\partial \tau} = 0, \quad S = -Mc\sqrt{-(x - y)^2}, \quad M \frac{dx^\mu}{d\tau} = \partial^\mu S. \quad (6c)$$

The actual space-time propagator in the Schwinger “proper time” representation is given by

$$D(x - y) = \frac{M}{8\pi^2 \hbar} \int_0^\infty \left(\frac{d\tau}{\tau^2} \right) \exp \frac{i}{\hbar} S(x - y, \tau), \quad (7)$$

where the proper time τ internal to the particle need not be the laboratory proper time $\sqrt{-(x - y)^2/c^2}$ between the vertex events. The particle has all the proper time in the world to do what it likes; forward in time, backward in time, spacelike ... and so forth. But the Bohr correspondence principle tells us that when the action in Eq.(7) obeys $|S| \gg \hbar$, then the stationary phase evaluation of the proper-time integral in Eq.(7) yields $\tau = \sqrt{-(x - y)^2/c^2}$ and what the particle then likes to do is to be classical. The experimentalist has every right to suppose that the “virtual particle” is classical even if nobody detects the path, and likewise for the moon. If Bohr’s correspondence holds true, then all is right with this boring world. But when a particle shows up that exhibits virtual quantum interference, in *spite* of the correspondence principle, everybody says that this particle must be very strange.

3 The “ $K^o \bar{K}^o$ ” Particle

The “ $K^o \bar{K}^o$ ” virtual particle is not merely strange, it has some deep psychological problems. It cannot decide whether to exhibit itself as a K^o going forward in time or as \bar{K}^o going backward in time. (The language is that of Stückelberg and Feynman.) If the dotted line in Fig.1 is “ $K^o \bar{K}^o$ ” as a virtual

particle, it will never be classical even for $|S| \gg \hbar$ because *any decent free classical particle* (or classical anti-particle) will make up its mind once and for all which direction in time it wants to go and then it learns to live in psychological peace with its decision. Hence, the Bohr correspondence principle works very well except for those cases where it does *not* work very well.

If a “ $K^o\bar{K}^o$ ” T violating particle (we assume TCP=1) goes from a primary to a secondary vertex its inner turmoil refuses the classical limit of the Bohr correspondence principle and remains truly virtual, oscillating with quantum interference phase factors. Very good. Now the Bohr dictate is that we cannot ask what the “ $K^o\bar{K}^o$ ” is doing. But it is really hard not to try and form a mental picture such as maybe the particle path is still classical but “wobbles” a little bit.

The mathematics is that the (two by two) non-Hermitian mass matrix of the “ $K^o\bar{K}^o$ ” [5, 6]

$$\mathcal{M} = M - i(\hbar\Gamma/2c^2), \quad (8)$$

produces the (two by two) “wobbly” propagator

$$\mathcal{D}(x - y) = \frac{\mathcal{M}}{8\pi^2\hbar} \int_0^\infty \left(\frac{d\tau}{\tau^2} \right) \exp \frac{i\mathcal{M}}{2\hbar} \left[-c^2\tau + \frac{(x - y)^2}{\tau} \right]. \quad (9)$$

The Bohr correspondence principle tells us to evaluate the proper time integral by stationary phase yielding the particle proper time as the laboratory proper-time so that asymptotically the particle has the laboratory proper-time

$$\mathcal{D}(x - y) \sim e^{-i\mathcal{M}c^2\tau/\hbar}, \quad -c^2\tau^2 \approx (x - y)^2, \quad (10)$$

and we are allowed (if we choose) to think of the propagator Eqs.(9) and (10) as a proper time Schrödinger equation

$$i\hbar \frac{\partial}{\partial \tau} \begin{pmatrix} A_{K^o}(\tau) \\ A_{\bar{K}^o}(\tau) \end{pmatrix} = \mathcal{M}c^2 \begin{pmatrix} A_{K^o}(\tau) \\ A_{\bar{K}^o}(\tau) \end{pmatrix}, \quad (11)$$

but we are not allowed to think of the “ $K^o\bar{K}^o$ ” traveling from a primary vertex to a secondary vertex as real (i.e. classical). It is truly an experimental virtual particle. Typical of this kind of experiment is the CPLEAR project [7] where the Feynman diagram of a typical event is shown in Fig.2. The “ $K^o\bar{K}^o$ ” is not observed (there is no track) and it is truly virtual because

you cannot Bohr correspond when you need a Schrödinger equation. Bohr has warned us about getting data from a quantum object.

One notes that the four velocity of the “ $K^o\bar{K}^o$ ” particle

$$v^\mu = \frac{(x^\mu - y^\mu)}{\tau}, \quad \tau = \sqrt{\frac{-(x - y)^2}{c^2}}, \quad (12)$$

is determined if we know the space-time positions of the vertex events in which the classical paths (legs of the Feynman diagrams) are located. That the “ $K^o\bar{K}^o$ ” particle is still virtual is due to the fact that it has two possible (“long” or “short”) four momenta

$$p_L = M_L v \quad \text{or} \quad p_S = M_S v, \quad (13)$$

so that a typical amplitude

$$A(\tau) = A_L e^{-\Gamma_L \tau/2} e^{-iM_L c^2 \tau/\hbar} + A_S e^{-\Gamma_S \tau/2} e^{-iM_S c^2 \tau/\hbar}, \quad (14)$$

has two “plane wave” fixed momentum states going from vertex y to vertex x , e.g. in Eqs.(12), (13) and (14)

$$\exp(-iM_j c^2 \tau/\hbar) = \exp(ip_j \cdot (x - y)/\hbar), \quad j = L \text{ or } S. \quad (15)$$

The notion that the single virtual “ $K^o\bar{K}^o$ ” has no trouble making up its mind as to its velocity but becomes confused about what should be its momentum, due to its mass splitting $\Delta M = (M_L - M_S)$, is crucial to the conventional analysis of single $\mathcal{D}(x, y)$ propagator experiments (even when a regenerator is included as part of the propagator). While theoretically this seems all right, e.g. the value of ΔM found in CPLEAR should agree with ΔM in regenerator experiments [8], we still feel somewhat uncomfortable with the result.

If the single virtual (large spreading wave packet) “ $K^o\bar{K}^o$ ” is in a superposition of two plane wave momentum states, as in Eqs.(14) and (15), and this particle scatters off other particles, should not some other particles also have trouble deciding in which momentum states *they are*? After all, no matter what the state of mind of each particle may be, there should be four momentum conservation. In other words, if an initial state is in a superposition of two different incoming four momenta (P_1 and P_2), i.e.

$$|\Psi\rangle = c_1 |in, P_1\rangle + c_2 |in, P_2\rangle, \quad (16a)$$

and if the scattering operator conserves four momenta, i.e.

$$S|\Psi \rangle = c_1|out, P_1 \rangle + c_2|out, P_2 \rangle, \quad (16a)$$

then shouldn't the outgoing particles (with the same amplitudes c_1 and c_2) also be in superposition? The confused particles would also be virtual, since Bohr dictates that those particles which leave tracks are real and classical and thereby do not maintain "superpositions". The superposition principle has to be relegated to the virtual processes that you do not observe, and don't ask what a virtual particle is doing because there will be no data!

Let us now proceed to "two propagator" experiments wherein there are at least two virtual particles. These include the proposed Φ factory experiments, and here momentum conservation has some unusual implications.

4 Two Propagator Experiments

Shown in Fig.3 (below) are typical Feynman diagrams of a previous two propagator experiment [9] (which involves a virtual Λ) as well as a virtual " $K^0 \bar{K}^0$ ". Also shown is a typical two propagator Feynman diagram for a process anticipated at the Φ factory [10]. If the " $K^0 \bar{K}^0$ " internal propagators are forced to have a superposition of two four momentum states, then we have predicted that even the Λ would be forced to be in a superposition of two momentum states. " Λ oscillations" [11] would be the result of such a superposition, born by a combination of the mass splitting $\Delta M = (M_L - M_S)$ and of momentum conservation at the $\bar{K}^0 + p^+ \rightarrow \Lambda + \pi^+$ vertex. The inner turmoil of the " $K^0 \bar{K}^0$ " spreads to any other virtual particle connected to it in the Feynman diagram.

To see that a discussion of the real and the virtual is not trivial, consider the Φ factory two virtual K meson process. The total four momentum is determined by the real electron positron pair producing the Φ ,

$$P_\Phi = p_{e^+} + p_{e^-} = p_{L,left} + p_{S,right} = p_{S,left} + p_{L,right}. \quad (17)$$

Now, from the two propagators you try to put together a simple two particle wave function (not quite a Schrödinger wavefunction because there are too many "times"),

$$\Psi_{simple}(\tau_{right}, \tau_{left}) = A_L(\tau_{right})A_S(\tau_{left}) - A_L(\tau_{left})A_S(\tau_{right}), \quad (18)$$

where

$$A_j(\tau) = e^{-\Gamma_j\tau/2} e^{-iM_j c^2 \tau/\hbar} \quad (j = L \text{ or } S). \quad (19)$$

But we find that the total momentum conservation Eq.(17) cannot be maintained by Eq.(18); e.g., see Eq.(15),

$$A_L(\tau_{right})A_S(\tau_{left}) \sim \exp\frac{i}{\hbar}(p_L \cdot x_{right} + p_S \cdot x_{left}), \quad (20a)$$

$$A_L(\tau_{left})A_S(\tau_{right}) \sim \exp\frac{i}{\hbar}(p_L \cdot x_{left} + p_S \cdot x_{right}), \quad (20b)$$

but the two plane waves in Ψ_{simple} will not conserve total momentum unless $p_L = p_R$ (*false!*). This ultimately violates the one velocity and two momenta inner turmoil Eq.(13), and would thereby afford an undue psychological relief to the mass difference induced $K^o \bar{K}^o$ oscillations. So, the simple wave function in Eq.(18) appears inadequate for describing the two propagator result, if conservation of total momentum is strictly applied [12].

What is needed for conservation of total momentum is precisely what is written in Eq.(17), which *actually describes conservation of momentum*. Only one needs more proper times in the amplitude, because you need more momenta in the plane waves

$$\begin{aligned} \Psi(\tau_{L,right}, \tau_{L,left}, \tau_{S,right}, \tau_{S,left}) = \\ A_L(\tau_{L,right})A_S(\tau_{S,left}) - A_L(\tau_{L,left})A_S(\tau_{S,right}). \end{aligned} \quad (21)$$

What can we do with all of those times (don't ask!)? Here are the numbers we promised you (dear reader) in Sec.1. Actually it is one (singular) number but it is predicted to occur very often in the data.

When you absolute square amplitudes with a wave function that has two terms, e.g. Eq.(21), the cross terms give you interference. For the problem at hand the interference phase is given by

$$\theta = \frac{c^2}{\hbar}[M_L(\tau_{L,left} - \tau_{L,right}) - M_S(\tau_{S,left} - \tau_{S,right})]. \quad (22)$$

Defining averages and differences

$$\tau_i = (1/2)(\tau_{L,i} + \tau_{S,i}), \quad (i = left \text{ or } right), \quad (23a)$$

$$\Delta\tau_i = \tau_{L,i} - \tau_{S,i}, \quad (i = left \text{ or } right), \quad (23b)$$

$$\bar{M} = (1/2)(M_L + M_S), \quad \Delta M = (M_L - M_S), \quad (23c)$$

we find from Eqs.(22) and (23),

$$(\hbar\theta/c^2) = \bar{M}(\Delta\tau_{left} - \Delta\tau_{right}) + \Delta M(\tau_{left} - \tau_{right}). \quad (24)$$

The first term on the left hand side Eq.(24) would not be present if we employed Eq.(18). But again by conservation of momentum we find the two terms on the right hand side of Eq.(24) approximately equal, and thus the factor of two in our final result

$$\theta = 2(c^2\Delta M/\hbar)(\tau_{left} - \tau_{right}). \quad (25)$$

The factor of “2” in Eq.(25) is the central result of this work.

5 Conclusion

We are *emotional* about a factor of “2” because we believe it says something unexpected about what is virtual and what is real. Others are *very emotional* about it because they calculate the factor to be “1”!. But the nice thing about it, is that the answer will surely be found at the Φ factory, with the experimentalists as the final referees. In Italy, where Galileo taught us that experimental data are the final arbiters in a theoretical dispute, what else could happen?

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