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# $S U(3)_{\text {FLAVOR }}$-ANALYSIS OF NONFACTORIZABLE CONTRIBUTIONS TO $D \rightarrow P P$ DECAYS 

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#### Abstract

We study charm D - meson decays to two pseudoscalar mesons in Cabibbo favored mode employing $\mathrm{SU}(3)$-flavor for the nonfactorizable matrix elements. Using $D \rightarrow \bar{K} \pi$ and $D_{s} \rightarrow \bar{K} K$ to fix the reduced matrix elements, we obtain a consistent fit for $\eta$ and $\eta^{\prime}$ emitting decays of $D$ and $D_{s}$ mesons.


It is now fairly established that the naive factorization model does not explain the data on weak hadronic decays of charm mesons. On one hand large $N_{c} \rightarrow \infty$ limit, which apparently was thought to be supported by Dmeson phenomenology [1,2], has failed to explain B-meson decays, as B-meson data clearly demands [3] a positive value of the $a_{2}$-parameter. On the other hand even in D-meson decays, the two body Cabibbo favored decays of $D^{0}$ and $D_{s}^{+}$involving $\eta$ and $\eta^{\prime}$ in their final state have proven to be problematic for a universal choice of $a_{1}$ and $a_{2}$ [4]. Annihilation terms, if used to bridge the discrepancy between theory and experiment, require large form factors, particularly for $D \rightarrow \bar{K}^{0}+\eta / \eta^{\prime}$ and $D^{0} \rightarrow \bar{K}^{* 0}+\eta$ decays [4]. Further, factorization also fails to relate $D_{s}^{+} \rightarrow \eta / \eta^{\prime}+\pi^{+} / \rho^{+}$decays with semileptonic decays $D_{s}^{+} \rightarrow \eta / \eta^{\prime}+e^{+} \nu[4,5]$ consistently.

Recently, there has been a growing interest in studying nonfactorizable terms for weak hadronic decays of charm and bottom mesons [6]. In an earlier work [7], we have searched for a systematics in the nonfactorizable contributions for various decays of $D^{0}$ and $D^{+}$mesons involving isospin $1 / 2$ and $3 / 2$ final states. We observe that the nonfactorizable isospin $1 / 2$ and $3 / 2$ amplitudes have nearly the same ratio for $D \rightarrow \bar{K} \pi / \bar{K} \rho / \bar{K}^{*} \pi / \bar{K} a_{1} / \bar{K}^{*} \rho$ decay modes. In order to realize the full impact of isospin symmetry, and to relate $D_{s}^{+}$-decays with those of the nonstrange charm mesons, we generalize it to the SU(3)-flavor symmetry.

We analyze Cabibbo favored decays of $D^{0}, D^{+}$and $D_{s}^{+}$mesons to two pseudoscalar mesons. Determining the $\mathrm{SU}(3)$ reduced matrix elements from $D^{+} \rightarrow \bar{K}^{0} \pi^{+}$and $D_{s}^{+} \rightarrow \bar{K}^{0} K^{+}$, we obtain a consistent fit for $D^{0} \rightarrow \bar{K}+\pi / \eta / \eta^{\prime}$ and $D_{s}^{+} \rightarrow \pi+\eta / \eta^{\prime}$ decays.

We start with the effective weak Hamiltonian

$$
\begin{equation*}
H_{w}=\tilde{G}_{F}\left[c_{1}(\bar{u} d)(\bar{s} c)+c_{2}(\bar{s} d)(\bar{u} c)\right], \tag{1}
\end{equation*}
$$

where $\tilde{G}_{F}=\frac{G_{F}}{\sqrt{2}} V_{u d} V_{c s}^{*}$ and $\bar{q}_{1} q_{2} \equiv \bar{q}_{1} \gamma_{\mu}\left(1-\gamma_{5}\right) q_{2}$ represents color singlet $V-A$ current and the QCD coefficients at the charm mass scale are

$$
\begin{equation*}
c_{1}=1.26 \pm 0.04, \quad c_{2}=-0.51 \pm 0.05 \tag{2}
\end{equation*}
$$

Separating the factorizable and nonfactorizable parts, the matrix element of the operator $(\bar{u} d)(\bar{s} c)$ in eq. (1) between initial and final states can be written as

$$
\begin{gather*}
<P_{1} P_{2}|(\bar{u} d)(\bar{s} c)| D>=<P_{1}|(\bar{u} d)| 0><P_{2}|(\bar{s} c)| D> \\
+<P_{1} P_{2}|(\bar{u} d)(\bar{s} c)| D>_{n o n f a c} \tag{3}
\end{gather*}
$$

Using the Fierz identity

$$
\begin{equation*}
(\bar{u} d)(\bar{s} c)=\frac{1}{N_{c}}(\bar{s} d)(\bar{u} c)+\frac{1}{2} \sum_{a=1}^{8}\left(\bar{s} \lambda^{a} d\right)\left(\bar{u} \lambda^{a} c\right) \tag{4}
\end{equation*}
$$

where $\bar{q}_{1} \lambda^{a} q_{2} \equiv \bar{q}_{1} \gamma_{\mu}\left(1-\gamma_{5}\right) \lambda^{a} q_{2}$ represents color octet current, the nonfactorizable part of the matrix element in eq.(3) can be expanded as

$$
\begin{gather*}
<P_{1} P_{2}|(\bar{u} d)(\bar{s} c)| D>_{\text {nonfac }}=\frac{1}{N_{c}}<P_{2}|(\bar{s} d)| 0><P_{1}|(\bar{u} c)| D> \\
+\frac{1}{2}<P_{1} P_{2}\left|\sum_{a=1}^{8}\left(\bar{s} \lambda^{a} d\right)\left(\bar{u} \lambda^{a} c\right)\right| D>_{n o n f a c}+\frac{1}{N_{c}}<P_{1} P_{2}|(\bar{s} d)(\bar{u} c)| D>_{\text {nonfac }} . \tag{5}
\end{gather*}
$$

Performing a similar treatment to the other operator $(\bar{s} d)(\bar{u} c)$ in eq. (1), the decay amplitude becomes

$$
\begin{gathered}
<P_{1} P_{2}\left|H_{w}\right| D>=\tilde{G}_{F}\left[a_{1}<P_{1}|(\bar{u} d)| 0><P_{2}|(\bar{s} c)| D>\right. \\
+a_{2}<P_{2}|(\bar{s} d)| 0><P_{1}|(\bar{u} c)| D> \\
+c_{2}\left(<P_{1} P_{2}\left|H_{w}^{8}\right| D>+<P_{1} P_{2}\left|H_{w}^{1}\right| D>\right)_{\text {nonfac }}
\end{gathered}
$$

$$
\begin{equation*}
\left.+c_{1}\left(<P_{1} P_{2}\left|\tilde{H}_{w}^{8}\right| D>+<P_{1} P_{2}\left|\tilde{H}_{w}^{1}\right| D>\right)_{\text {nonfac }}\right], \tag{6}
\end{equation*}
$$

where

$$
\begin{gather*}
a_{1,2}=c_{1,2}+\frac{c_{2,1}}{N_{c}},  \tag{7}\\
H_{w}^{8}=\frac{1}{2} \sum_{a=1}^{8}\left(\bar{s} \lambda^{a} d\right)\left(\bar{u} \lambda^{a} c\right), \tilde{H}_{w}^{8}=\frac{1}{2} \sum_{a=1}^{8}\left(\bar{u} \lambda^{a} d\right)\left(\bar{s} \lambda^{a} c\right) ; \\
H_{w}^{1}=\frac{1}{N_{c}}(\bar{s} d)(\bar{u} c), \quad \tilde{H}_{w}^{1}=\frac{1}{N_{c}}(\bar{u} d)(\bar{s} c) . \tag{8}
\end{gather*}
$$

Thus nonfactorizable effects arise through the Hamiltonian made up of coloroctet currents ( $H_{w}^{8}$ and $\tilde{H}_{w}^{8}$ ) and also of color singlet currents ( $H_{w}^{1}$ and $\tilde{H}_{w}^{1}$ ).

Matrix elements of the first and the second terms in eq. (6) can be calculated using the factorization scheme [1]. These are given in Table I. So long as one restricts to the color singlet intermediate states, remaining terms in eq.(6) are ignored and one usually treats $a_{1}$ and $a_{2}$ as input parameters in place of using $N_{c}=3$ in reality. It is generally believed $[1,8]$ that the $D \rightarrow \bar{K} \pi$ decays favour $N_{c} \rightarrow \infty$ limit, i.e.,

$$
\begin{equation*}
a_{1} \approx 1.26, \quad a_{2} \approx-0.51 \tag{9}
\end{equation*}
$$

However, it has been shown that this does not explain all the decay modes of charm mesons $[4,5]$. For instance, the observed $D^{0} \rightarrow \bar{K}^{0} \eta$ and $D^{0} \rightarrow \bar{K}^{0} \eta^{\prime}$ decay widths are considerably larger than those predicted in the spectator quark model. Also in $D \rightarrow P V$ mode, measured branching ratios for $D^{0} \rightarrow \overline{K^{\star 0}} \eta$, $D_{s}^{+} \rightarrow \eta / \eta^{\prime}+\rho^{+}$, are higher than those predicted by the spectator quark diagrams. For $D_{s}^{+} \rightarrow \eta / \eta^{\prime}+\pi^{+}$, though factorization can account for substantial part of the measured branching ratios, it fails to relate them to corresponding semileptonic decays $D_{s}^{+} \rightarrow \eta / \eta^{\prime}+e^{+} \nu$ consistently [4,5]. In addition to the spectator quark diagram, factorizable W -exchange or W -annihilation diagrams may contribute to the weak nonleptonic decays of D mesons. However,
for $D \rightarrow P P$ decays, such contributions are helicity suppressed [1]. For $D$ meson decays, these are futher color-suppressed as these involve QCD coefficient $c_{2}$, whereas for $D_{s}^{+} \rightarrow P P$ decays these vanish [4] due to the conserved vector (CVC) nature of isovector current ( $\bar{u} d)$. Therefore, it is desirable to investigate nonfactorizable contributions more seriously.

It is well known that nonfactorizable terms cannot be determined unambiguiously without making some assumptions [6] as these involve nonperturbative effects arising due to soft-gluon exchange. We thus employ $\mathrm{SU}(3)$-flavorsymmetry [9] to handle these matrix elements. In the $\mathrm{SU}(3)$ framework, the weak Hamiltonians $H_{w}^{8}, \tilde{H}_{w}^{8}, H_{w}^{1}$ and $\tilde{H}_{w}^{1}$ for Cabibbo-enhanced mode behave like $H_{13}^{2}$ component of $6^{*}$ and 15 representations of the $\mathrm{SU}(3)$. Since $H_{w}^{8}$ and $\tilde{H}_{w}^{8}$ transform into each other under interchange of $u$ and $s$ quarks, which forms V-spin subgroup of the $\operatorname{SU}(3)$, we assume the reduced amplitudes to follow

$$
\begin{equation*}
<P_{1} P_{2}\left\|\tilde{H}_{w}^{8}\right\| D>=<P_{1} P_{2}\left\|H_{w}^{8}\right\| D>. \tag{10}
\end{equation*}
$$

Then, the matrix elements $<P_{1} P_{2}\left|H_{w}^{8}\right| D>$ can be considered as weak spurion $+D \rightarrow P+P$ scattering process, whose general structure can be written as

$$
\begin{align*}
<P_{1} P_{2}\left|H_{w}^{8}\right| D> & =b_{1}\left(P_{a}^{m} P_{m}^{c} P^{b}\right) H_{[b, c]}^{a}+d_{1}\left(P_{a}^{m} P_{m}^{c} P^{b}\right) H_{(b, c)}^{a} \\
& +e_{1}\left(P_{m}^{b} P_{a}^{c} P^{m}\right) H_{(b, c)}^{a}+f_{1}\left(P_{m}^{m} P_{a}^{b} P^{c}\right) H_{(b, c)}^{a} \tag{11}
\end{align*}
$$

where $P^{a}$ denotes triplet of D-mesons $P^{a} \equiv\left(D^{0}, D^{+}, D_{s}^{+}\right)$and $P_{b}^{a}$ denotes $3 \otimes 3$ matrix of uncharmed pseudoscalar mesons,

$$
P_{b}^{a}=\left(\begin{array}{ccc}
P_{1}^{1} & \pi^{+} & K^{+}  \tag{12}\\
\pi^{-} & P_{2}^{2} & K^{0} \\
K^{-} & \bar{K}^{0} & P_{3}^{3}
\end{array}\right)
$$

with

$$
P_{1}^{1}=\frac{\pi^{0}}{\sqrt{2}}+\frac{\eta_{8}}{\sqrt{6}}+\frac{\eta_{0}}{\sqrt{3}},
$$

$$
\begin{aligned}
P_{2}^{2} & =-\frac{\pi^{0}}{\sqrt{2}}+\frac{\eta_{8}}{\sqrt{6}}+\frac{\eta_{0}}{\sqrt{3}}, \\
P_{3}^{3} & =-\frac{2 \eta_{8}}{\sqrt{6}}+\frac{\eta_{0}}{\sqrt{3}} .
\end{aligned}
$$

Particle data group [10] defines the physical $\eta-\eta^{\prime}$ mixing as

$$
\begin{align*}
& \eta=\eta_{8} \cos \phi-\eta_{0} \sin \phi, \\
& \eta^{\prime}=\eta_{8} \sin \phi+\eta_{0} \cos \phi, \tag{13}
\end{align*}
$$

where $\phi=-10^{0}$ and $\phi=-19^{0}$ follow from the quadratic mass formula and the two photon decays widths respectively [10]. We employ the following basis [4]

$$
\begin{align*}
\eta & =\frac{1}{\sqrt{2}}(u \bar{u}+d \bar{d}) \sin \theta-(s \bar{s}) \cos \theta \\
\eta^{\prime} & =\frac{1}{\sqrt{2}}(u \bar{u}+d \bar{d}) \cos \theta+(s \bar{s}) \sin \theta \tag{14}
\end{align*}
$$

where $\theta$ is given by

$$
\begin{equation*}
\theta=\theta_{\text {ideal }}-\phi . \tag{15}
\end{equation*}
$$

Performing a similar treatment for $H_{w}^{1}$ and $\tilde{H}_{w}^{1}$, i.e.

$$
\begin{equation*}
<P_{1} P_{2}\left\|\tilde{H}_{w}^{1}\right\| D>\quad=\quad<P_{1} P_{2}\left\|H_{w}^{1}\right\| D> \tag{16}
\end{equation*}
$$

the matrix elements $<P_{1} P_{2}\left|H_{w}^{1}\right| D>$ are obtained from

$$
\begin{align*}
<P_{1} P_{2}\left|H_{w}^{1}\right| D> & =b_{2}\left(P_{a}^{m} P_{m}^{c} P^{b}\right) H_{[b, c]}^{a}+d_{2}\left(P_{a}^{m} P_{m}^{c} P^{b}\right) H_{(b, c)}^{a} \\
& +e_{2}\left(P_{m}^{b} P_{a}^{c} P^{m}\right) H_{(b, c)}^{a}+f_{2}\left(P_{m}^{m} P_{a}^{b} P^{c}\right) H_{(b, c)}^{a} \tag{17}
\end{align*}
$$

Since the C.G. coefficients appearing in the eqs. (11) and (17) are the same, the unknown reduced amplitudes get combined as

$$
\begin{equation*}
b=b_{1}+b_{2}, \quad d=d_{1}+d_{2}, \quad e=e_{1}+e_{2}, \quad f=f_{1}+f_{2}, \tag{18}
\end{equation*}
$$

when the matrix elements are substituted in eq.(6).

There exists a straight correspondence between the terms appearing in (11) and (17) and various quark level processes. The first two terms, involving the coefficients $b^{\prime} s$ and $d^{\prime} s$, represent W -annihilation or W -exchange diagrams. Notice that unlike factorizable W-exchange or W -annihilation diagrams, these diagrams are not suppressed on the basis of the helicity arguments due to the involvement of gluons. The third term, having coefficient $e^{\prime} s$, represents spectator quark like diagram where the uncharmed quark in the parent $D$ meson flows into one of the final state mesons. The last term is like a hair-pin diagram, where $q \bar{q}$ generated in the process hadronizes to one of the final state mesons. Thus obtained nonfactorizable contributions to various $D \rightarrow P P$ decays are given in Table II.

Now we proceed to determine the $\mathrm{SU}(3)$ reduced amplitudes $b, d, e, f$. First, we calculate the factorizable contributions to various decays using $N_{c}=3$, which yields

$$
\begin{equation*}
a_{1}=1.09, \quad a_{2}=-0.09 \tag{19}
\end{equation*}
$$

For the form factors, we use

$$
\begin{equation*}
F_{0}^{D K}(0)=0.76, \quad F_{0}^{D \pi}(0)=0.83 \tag{20}
\end{equation*}
$$

as guided by the semileptonic decays [8,12], and

$$
\begin{array}{cc}
F_{0}^{D \eta}(0)=0.68, & F_{0}^{D \eta^{\prime}}(0)=0.65 \\
F_{0}^{D s \eta}(0)=0.72, & F_{0}^{D s \eta^{\prime}}(0)=0.70 \tag{21}
\end{array}
$$

from the BSW model [1]. Numerical values of the factorizbale amplitudes are given in col (iii) of Table I.
$D \rightarrow \bar{K} \pi$ decays involve elastic final state interactions (FSI) whereas the remaining decays are not affected by them. As a result, the isospin amplitudes
$1 / 2$ and $3 / 2$ appearing in $D \rightarrow \bar{K} \pi$ decays develop different phases;

$$
\begin{gather*}
A\left(D^{0} \rightarrow K^{-} \pi^{+}\right)=\frac{1}{\sqrt{3}}\left[A_{3 / 2} e^{i \delta_{3 / 2}}+\sqrt{2} A_{1 / 2} e^{i \delta_{1 / 2}}\right], \\
A\left(D^{0} \rightarrow \bar{K}^{0} \pi^{0}\right)=\frac{1}{\sqrt{3}}\left[\sqrt{2} A_{3 / 2} e^{i \delta_{3 / 2}}-A_{1 / 2} e^{i \delta_{1 / 2}}\right], \\
A\left(D^{+} \rightarrow \bar{K}^{0} \pi^{+}\right)=\sqrt{3} A_{3 / 2} e^{i \delta_{3 / 2}} . \tag{22}
\end{gather*}
$$

which yield the following phase independent [7,11] expressions:

$$
\begin{gather*}
\left|A\left(D^{0} \rightarrow K^{-} \pi^{+}\right)\right|^{2}+\left|A\left(D^{0} \rightarrow \bar{K}^{0} \pi^{0}\right)\right|^{2}=\left|A_{1 / 2}\right|^{2}+\left|A_{3 / 2}\right|^{2} \\
\left|A\left(D^{+} \rightarrow \bar{K}^{0} \pi^{+}\right)\right|^{2}=3\left|A_{3 / 2}\right|^{2} \tag{23}
\end{gather*}
$$

These relations allow one to work without the phases. Writing the total decay amplitude as sum of factorizable and nonfactorizable parts

$$
\begin{equation*}
A(D \rightarrow \bar{K} \pi)=A^{f}(D \rightarrow \bar{K} \pi)+A^{n f}(D \rightarrow \bar{K} \pi) \tag{24}
\end{equation*}
$$

we obtain

$$
\begin{gather*}
A_{1 / 2}^{n f}=\frac{1}{\sqrt{3}}\left\{\sqrt{2} A^{n f}\left(D^{0} \rightarrow K^{-} \pi^{+}\right)-A^{n f}\left(D^{0} \rightarrow \bar{K}^{0} \pi^{0}\right)\right\}  \tag{25}\\
A_{3 / 2}^{n f}=\frac{1}{\sqrt{3}}\left\{A^{n f}\left(D^{0} \rightarrow K^{-} \pi^{+}\right)+\sqrt{2} A^{n f}\left(D^{0} \rightarrow \bar{K}^{0} \pi^{0}\right)\right\} \\
=\frac{1}{\sqrt{3}}\left\{A^{n f}\left(D^{+} \rightarrow \bar{K}^{0} \pi^{+}\right)\right\} \tag{26}
\end{gather*}
$$

The last relation (26) leads to the following constraint:

$$
\begin{equation*}
\frac{b+d}{e}=\frac{c_{1}+c_{2}}{c_{2}-c_{1}}=-0.424 \pm 0.042 \tag{27}
\end{equation*}
$$

Experimental value $B\left(D^{+} \rightarrow \bar{K}^{0} \pi^{+}\right)=2.74 \pm 0.29 \%$ yields, up to a scale factor $\tilde{G}_{F}$,

$$
\begin{equation*}
e=-0.094 \pm 0.027 G e V^{3} \tag{28}
\end{equation*}
$$

This in turn predicts sum of the branching ratios of $D^{0} \rightarrow \bar{K} \pi$ decay modes,

$$
\begin{equation*}
B\left(D^{0} \rightarrow K^{-} \pi^{+}\right)+B\left(D^{0} \rightarrow \bar{K}^{0} \pi^{0}\right)=6.30 \pm 0.67 \% \quad(6.06 \pm 0.30 \% \text { Expt. }) \tag{29}
\end{equation*}
$$

in good agreement with experiment. Using the experimental value of $B\left(D_{s}^{+} \rightarrow\right.$ $\bar{K}^{o} K^{+}$) $=3.5 \pm 0.7 \%$, we find (in $G e V^{3}$ )

$$
\begin{align*}
& b=+0.080 \pm 0.026  \tag{30}\\
& d=-0.040 \pm 0.026 \tag{31}
\end{align*}
$$

Note that the unknown reduced amplitude $f$ appears only in decays involving $\eta$ and $\eta^{\prime}$ in the final state. We find that experimental values of these decay rates require (in $\mathrm{GeV}^{3}$ ):

$$
\begin{align*}
& f=-0.145 \pm 0.077 \quad \text { for } \quad D^{0} \rightarrow \bar{K}^{0} \eta, \\
& f=-0.115 \pm 0.012 \quad \text { for } \quad D^{0} \rightarrow \bar{K}^{0} \eta^{\prime}, \\
& f=-0.104 \pm 0.163 \quad \text { for } \quad D_{s}^{+} \rightarrow \eta \pi^{+}, \\
& f=-0.081 \pm 0.073 \quad \text { for } \quad D_{s}^{+} \rightarrow \eta^{\prime} \pi^{+} . \tag{32}
\end{align*}
$$

In Tables III, we calculate branching ratios for all the four $\eta, \eta^{\prime}$ emitting decay modes for different choice of $f$, for $\phi=-10^{\circ}$ and $-19^{\circ}$. It is clear that for $f=$ -0.12 and $\phi=-10^{\circ}$, all the branching ratios match well with experiment. For the sake of comparison with factorizable terms, nonfactorizable contributions to various modes for $f=-0.12$ are given in column (iii) of the Table II. Color-suppressed decays obviously require large nonfactorizable contributions.

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Table I
Spectator-quark decay amplitudes $\left(\times \tilde{G}_{F} G e V^{3}\right)$

| Process | Amplitude | $\phi=-10^{0}$ | $\phi=-19^{0}$ |
| :---: | :---: | :---: | :---: |
| $D^{+} \rightarrow \bar{K}^{0} \pi^{+}$ | $a_{1} f_{\pi}\left(m_{D}^{2}-m_{K}^{2}\right) F_{0}^{D K}\left(m_{\pi}^{2}\right)$ |  |  |
|  | $+a_{2} f_{K}\left(m_{D}^{2}-m_{\pi}^{2}\right) F_{0}^{D \pi}\left(m_{K}^{2}\right)$ | +0.311 | +0.311 |
| $D^{0} \rightarrow K^{-} \pi^{+}$ | $a_{1} f_{\pi}\left(m_{D}^{2}-m_{K}^{2}\right) F_{0}^{D K}\left(m_{\pi}^{2}\right)$ | +0.354 | +0.354 |
| $D^{0} \rightarrow \bar{K}^{0} \pi^{0}$ | $\frac{1}{\sqrt{2}} a_{2} f_{K}\left(m_{D}^{2}-m_{\pi}^{2}\right) F_{0}^{D \pi}\left(m_{K}^{2}\right)$ | -0.030 | -0.030 |
| $D^{0} \rightarrow \bar{K}^{0} \eta$ | $\frac{1}{\sqrt{2}} a_{2} \sin \theta f_{K}\left(m_{D}^{2}-m_{\eta}^{2}\right) F_{0}^{D \eta}\left(m_{K}^{2}\right)$ | -0.016 | -0.019 |
| $D^{0} \rightarrow \bar{K}^{0} \eta^{\prime}$ | $\frac{1}{\sqrt{2}} a_{2} \cos \theta f_{K}\left(m_{D}^{2}-m_{\eta^{\prime}}^{2}\right) F_{0}^{D \eta^{\prime}}\left(m_{K}^{2}\right)$ | -0.013 | -0.010 |
|  |  |  |  |
| $D_{s}^{+} \rightarrow \bar{K}^{0} K^{+}$ | $a_{2} f_{K}\left(m_{D_{s}}^{2}-m_{K}^{2}\right) F_{0}^{D s K}\left(m_{K}^{2}\right)$ | -0.035 | -0.035 |
| $D_{s}^{+} \rightarrow \pi^{0} \pi^{+}$ | 0 | 0 | 0 |
| $D_{s}^{+} \rightarrow \eta \pi^{+}$ | $-a_{1} \cos \theta f_{\pi}\left(m_{D_{s}}^{2}-m_{\eta}^{2}\right) F_{0}^{D s \eta}\left(m_{\pi}^{2}\right)$ | -0.261 | -0.216 |
| $D_{s}^{+} \rightarrow \eta^{\prime} \pi^{+}$ | $a_{1} \sin \theta f_{\pi}\left(m_{D_{s}}^{2}-m_{\eta^{\prime}}^{2}\right) F_{0}^{D_{s} \eta^{\prime}}\left(m_{\pi}^{2}\right)$ | +0.213 | +0.243 |

Table II
Nonfactorizable contributions to $D \rightarrow P P$ decays $\left(\times \tilde{G}_{F} G e V^{3}\right)$

| Process | Amplitude | $\phi=-10^{0}$ | $\phi=-19^{0}$ |
| :---: | :---: | :---: | :---: |
| $D^{+} \rightarrow \bar{K}^{0} \pi^{+}$ | $2\left(c_{1}+c_{2}\right) e$ | -0.141 | -0.141 |
|  |  |  |  |
| $D^{0} \rightarrow K^{-} \pi^{+}$ | $c_{2}(b+d+e)$ | +0.028 | +0.028 |
| $D^{0} \rightarrow \bar{K}^{0} \pi^{0}$ | $\frac{1}{\sqrt{2}} c_{1}(-b-d+e)$ | -0.119 | -0.119 |
| $D^{0} \rightarrow \bar{K}^{0} \eta$ | $c_{1}\left[\frac{\sin \theta}{\sqrt{2}}(b+d+e+2 f)-\cos \theta(b+d+f)\right]$ | -0.115 | -0.154 |
| $D^{0} \rightarrow \bar{K}^{0} \eta^{\prime}$ | $c_{1}\left[\frac{\operatorname{cose} \theta}{\sqrt{2}}(b+d+e+2 f)+\sin \theta(b+d+f)\right]$ | -0.256 | -0.235 |
|  |  |  |  |
| $D_{s}^{+} \rightarrow \bar{K}^{0} K^{+}$ | $c_{1}(-b+d+e)$ | -0.268 | -0.268 |
| $D_{s}^{+} \rightarrow \pi^{0} \pi^{+}$ | 0 | 0 | 0 |
| $D_{s}^{+} \rightarrow \eta \pi^{+}$ | $c_{2}[\sqrt{2} \sin \theta(-b+d+f)-\cos \theta(e+f)]$ | +0.046 | +0.076 |
| $D_{s}^{+} \rightarrow \eta^{\prime} \pi^{+}$ | $c_{2}[\sqrt{2} \cos \theta(-b+d+f)+\sin \theta(e+f)]$ | +0.199 | +0.189 |

## Table III

Branching (\%) of $\eta / \eta^{\prime}$ emitting decays including nonfactorization terms

| Decay | $\phi=-10^{\circ}$ |  |  |  | $\phi=-19^{\circ}$ |  |  |  | Expt. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $f=-0.10$, | -0.12, | -0.14 | $f=-0.10$, | -0.12, | -0.14 |  |  |  |
|  |  |  |  |  | 0.86 | 1.02 | 1.19 |  |  |
| $D^{0} \rightarrow \eta \bar{K}^{0}$ | 0.53 | 0.59 | 0.66 | 1.04 | 1.51 | 2.06 | $0.68 \pm 0.11$ |  |  |
| $D^{0} \rightarrow \eta^{\prime} \bar{K}^{0}$ | 1.28 | 1.81 | 2.43 |  |  |  | $1.66 \pm 0.29$ |  |  |
|  |  |  |  | 0.86 | 0.80 | 0.73 | $1.9 \pm 0.4$ |  |  |
| $D_{s}^{+} \rightarrow \eta \pi^{+}$ | 1.93 | 1.87 | 1.82 | 5.73 | 6.22 | 6.72 | $4.7 \pm 1.4$ |  |  |
| $D_{s}^{+} \rightarrow \eta^{\prime} \pi^{+}$ | 5.17 | 5.64 | 6.13 |  |  |  |  |  |  |

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