A CLASS OF STATIONARY ROTATING STRING COSMOLOGICAL MODELS

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Abstract

We obtain a one parameter class of stationary rotating string cosmological models of which the well-known Gödel Universe is a particular case. By suitably choosing the free parameter function, it is always possible to satisfy the energy conditions. The rotation of the model hinges on the cosmological constant which turns out to be negative.

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They may be one of the sources of density perturbations that are required for formation of large scale structure in the Universe [1,2]. They possess stress-energy and hence couple to gravitational field. Their various features have been considered by some authors [3-5]. Cosmic strings as source of gravitational field in general relativity (GR) was discussed by Letelier [6] and Stachel [7]. Letelier [8] has further constructed string cosmological models for Bianchi I and Kantowski-Sachs spacetimes by introducing the energy-momentum tensor.

$$T_{ik} = \rho u_i u_k - \lambda w_i w_k, \; u_i u^i = 1 = -w_i w^i, \; u_i w^i = 0 \tag{1}$$

as the source term in Einstein's equation

$$R_{ik} - \frac{1}{2} R g_{ik} = -8\pi T_{ik} - \wedge g_{ik} \tag{2}$$

where $\wedge$ is the cosmological constant. T_{ik} represents the energy momentum of a cloud of strings attached with mass particles. The density ρ is made up of particle density ρ_p and the string tensor λ, and is given by

$$\rho = \rho_p + \lambda \tag{3}$$

The energy conditions will require $\rho \geq 0, \; \rho_p \geq 0$ leaving the sign of λ undetermined. λ has however to be positive whenever $\rho_p = 0$. The matter flow and the string fibre directions are respectively specified by the unit timelike u^i and spacelike w^i vectors.

String cosmological models have been studied for Bianchi type spacetimes by several authors [9-13]. It is also shown that cylindrically symmetric non-singular spacetimes also admit physically reasonable string cosmological models [14]. So far stationary rotating string solutions of Einstein's equation. We have obtained a one parameter class of rotating string spacetimes and the free function can be suitably chosen to satisfy the energy conditions. The well-known rotating Gödel Universe follows as a special case of this class. It turns out that the cosmological constant $\wedge$ plays a very simportant role in the sense that it measures rotation as well as particle density ρ_p.

We consider the stationary line-element in the form

$$ds^2 = -dx^2 - \alpha^2 dy^2 - dz^2 + (dt + H dy)^2 \tag{4}$$

where α and H are functions of x alone. We introduce the orthonormal tetrad; $\theta^1 = dx,\ \theta^2 = \alpha dy\ \theta^3 = dz$ and $\theta^4 = dt + H dy$ and in what follows all quantities will be referred to the tetrad frame.

The surviving R_{ab} are given as follows :

$$R_{11} = R_{22} = \alpha''/\alpha - H'^2/2\alpha^2$$

$$R_{44} = -H'^2/2\alpha^2,\ R_{24} = -(1/2\alpha)(H'' - H'\alpha'/\alpha). \tag{5}$$

Substituting this in (2) and using (1), we get

$$R_{24} = 0 \tag{6}$$

$$R_{11} = -4\pi(\rho + \lambda) - \wedge \tag{7}$$

$$R_{44} = 2\wedge = 8\pi(\lambda - \rho) \tag{8}$$

where we have used $u_i = \delta_i^4$ and $w_i = \delta_i^3$ (string is along the z-axis).

From eqns.(5)-(8) we readily obtain

$$H' = m\alpha, \ 8\pi\rho = m^2 - \alpha''/\alpha, \ 8\pi\lambda = m^2/2 - \alpha''/\alpha \tag{9}$$

where m is a constant of integration. Clearly $\rho \geq 0$ is ensured if $\alpha'' \leq 0$ and the particle density

$$\rho_p = \rho - \lambda = m^2/2 \geq 0 \tag{10}$$

whereas the cosmological constant

$$\wedge = -m^2/4 \leq 0. \tag{11}$$

The vorticity of the velocity field, $\Omega = w_{ab}w^{ab}$ turns out to be

$$\Omega = \sqrt{2}m \tag{12}$$

which will vanish only when $\wedge = 0$ and so does the particle density ρ_p.

The metric function α is undetermined and hence we have one parameter class of rotating string spacetimes. In the followng we consider some simple interesting cases.

Solution 1: The simplest case will obviously be $\alpha = x$ leading to

$$\lambda = \rho_p = 2\rho = m^2/16\pi, \ H = mx^2/2, \tag{13}$$

$$ds^2 = -dx^2 - x^2dy^2 - dz^2 + (dt + \frac{1}{2}mx^2dy)^2. \tag{14}$$

Solution 2: Let us put $\lambda = 0$ which will imply $\alpha = e^{mx/\sqrt{2}}$, $H = \sqrt{2}\alpha$. Then we obtain

$$ds^2 = -dx^2 - e^{\sqrt{2}mx}dy^2 - dz^2 + (dt + \sqrt{2}e^{mx/\sqrt{2}}dy)^2 \tag{15}$$

which is the well-known Gödel Universe [15] with $8\pi\rho = m^2/2 = -2\wedge$.

Solution 3: Let us consider the equation of state of the kind $\rho = (1+k)\lambda$ where k is a positive constant. Then we have

$$\alpha'' + \frac{1-k}{2k}m^2\alpha = 0$$

the solution of which depends upon the sign of $\frac{1-k}{2k}$.

Case (i) $a^2 = \frac{1-k}{2k}m^2 > 0$. In that case,

$$\alpha = cosax,\ 8\pi\rho = \frac{m^2(1+3k)}{2k},\ H = -\frac{m}{a}sinax$$

and the metric reads

$$ds^2 = -dx^2 - cos^2axdy^2 - dz^2 + (dt - \frac{m}{a}sinaxdy)^2. \tag{16}$$

Case (ii) $-b^2 = \frac{1-k}{2k}m^2$. We then obtain

$$\alpha = e^{bx},\ 8\pi\rho = \frac{1+k}{2k},\ H = \frac{m}{b}e^{bx}$$

and

$$ds^2 = -dx^2 - e^{2bx}dy^2 - dz^2 + (dt + \frac{m}{b}e^{bx}dy)^2. \tag{17}$$

Solution 4: Let us consider the case of vanishing $\wedge$ which means $\rho_p = \Omega = 0$ and $\rho = \lambda$. This is the case of the Universe filled with the cosmic strings alone. Note that α still remains free to be chosen. We choose $\alpha''/\alpha = -n^2$, n being a constant. Then we get

$$\rho = \lambda = n^2/8\pi$$

and the metric has the simple form

$$ds^2 = -dx^2 - cos^2 nx dy^2 - dz^2 + dt^2. \tag{18}$$

It is interesting to note that ρ is a constant and switching that off leads to flat spacetime.

This metric was earlier obtained by Patel and Vaidya [16] and was interpreted as magnetic Universe with cosmological constant. That is the same spacetime can have two different physical visualisations.

Finally we would like to mention that all the cases considered above satisfy the energy conditions and hence are physically admissible. Further they can always be satisfied by suitably choosing the free function α. It would be interesting to find non-static rotating string models.

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[1] T W S Kibble (1976) J. Phys. **A 9**, 1387.

[2] Ya B Zeldovich (1980) Mon. Not. R. Astron. Soc. **192**, 663.

[3] A Vilenkin (1981) Phys. Rev. **D 24**, 2082.

[4] J R Gott (1985) Astrophys. J. **288**, 422.

[5] D Garfinkle (1985) Phys. Rev. **D32**, 1323.

[6] P S Letelier (1979) Phys. Rev. **D20**, 1294.

[7] J Stachel (1980) Phys. Rev. **D 21**, 2171.

[8] P S Letelier (1983) Phys. Rev. **D 28**, 2414.

[9] K D Krori, T Chaudhuri, C R Mahanta and A Mazumdar (1990) Gen. Rel. Grav. **22**, 123.

[10] A Banerjee, A K Sanyal and S Chakraborty (1990) Pramana - J.Phys. **34**, 1.

[11] R Tikekar and L K Patel (1992) Gen. Rel. Grav. **24**, 397.

[12] R Tikekar and L K Patel (1994) Pramana - J.Phys. **42**, 483.

[13] S D Maharaj, P G L Leach and K S Govinder (1995) - to appear in Pramana.

[14] R Tikekar, L K Patel and N Dadhich (1994) Gen. Rel. Grav. **26**, 647.

[15] K Gödel (1949) Rev. Mod. Phys. **21**, 447.

[16] L K Patel and P C Vaidya (1971) Current Science **40**, 278.