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A Mechanism for Instanton induced Chiral Symmetry Breaking in QCD

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Abstract

We propose a mechanism for instanton induced chiral symmetry breaking in QCD with fundamental scalars. The model Lagragian that we use has the same symmetry properties as QCD. The scalar fields develop vacuum expectation values at a non-trivial minimum and generate masses for thhe light quarks. The minimization condition is also used to break the $SU(N_f)$ flavour symmetry in order to make the s quark heavier than the two lighter ones. Thus a vacuum of the theory that is not chirally invariant is obtained.

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I. INTRODUCTION

In the gauge theory of strong interactions viz. QCD, the mechanism of chiral symmetry breaking is an important issue. The current quark masses for u, d and s quarks are known not to be zero. In the absence of explicit mass terms for the quarks, the Lagrangian for QCD has exact chiral symmetry $SU(N_f)_L \times SU(N_f)_R$, in addition to $U(1)_V$ (baryon number conservation) and $U(1)_A$ which, as is well known, is violated by quantum effects. Apart from the issue of the non-trivial nature of the QCD vacuum with the quark condensate $< 0|\bar{q}q|0 > \neq 0$, the nature of the chiral phase transition in QCD continues to be of interest. This has been studied in various ways, by assuming an effective theory with the desired properties. In particular, we would like to mention here the work of Pisarski and Wiilczek [?] and the related earlier work of Raby [?]. While the former authors studied the restoration of chiral symmetry at finite temperature, the latter study concentrated more on phenomenological aspects of the η decay width and related issues. However, the usual practise followed by these authors to break the symmetry in the first place is to add a term $c(detM + detM^{\dagger})$, where M is a colour singlet complex $N_f \times N_f$ matrix, which is also used to give mass to the η' meson.

The effect of instantons in QCD (without fundamental scalars) to produce chiral symmetry breaking is by now an old isuue and was studied amongst others, by Callan, Dashen and Gross [?], by 't Hooft [?] in the context of the $U(1)$ problem and by Shuryak [?]. However, the mechanism of dynamical chiral symmetry breaking continues to be an important problem in QCD.

In this paper, we re-examine the issue in the context of the effect of instantons in QCD with fundamental scalars described by a color singlet $N_f \times N_f$ complex matrix, allowing these scalars to have only a kinetic energy term and Yukawa coupling with the quark fields. The idea of treating scalars as elementary fields interacting with quarks has been earlier considered by D'yakonov and Eldes [?], Dhar, Shankar and Wadia [?] and others in the Nambu-Jona Lasinio model motivated effective QCD Lagrangian. However our model differs from these in the sense the we evaluate the effective potential for M by integrating out the quark fields in the instanton background at the one-loop level. In that sense we use a fundamental rather than an effective Lagrangian. A similar approach to explain small fermion masses in QCD has been considered by Huang and Viswanathan [?] with M as a color singlet complex *single component* scalar field. We show that with M as a $N_f \times N_f$ complex matrix, the effective potential induced by the instanton one-loop effects leads to chiral symmetry breaking by producing a $(detM + detM^\dagger)$ from the zero modes of the Dirac operator. However before we go on to describe the work in the rest of the paper, it is appropriate at this point to motivate the use of our Lagrangian which we shall be using subsequently.

Though QCD is and has been an acceptable theory of strong interactions, its low energy behavior still eludes explicit calculations, where effective theories like the Gell-Mann Levy linear σ model are used to give a field theoretical description of the strong interactions. The linear sigma model is not asymptotically free, and therefore, inspite of being perturbatively renormalizable, is usually considered a low energy effective theory. Recently however, the chiral sigma model with quarks substituted for nucleons [?] has found support as a reasonable description of the nucleon [?], strong interaction properties at finite temperature and baryon density [?,?,?] even at scales well above chiral restoration, as well as weak interaction properties [?]. At first sight this is unexpected from an effective theory. However it has very recently been shown that the chiral linear sigma model with quarks, when coupled to gluons can be asymptotically free [?]. Lattice studies [?,?] indicate that the chiral sigma model with quarks reproduce QCD lattice results rather well at finite temperature with the pions and sigma being elementary for all $T > T_\chi$ except not as Goldstone bosons. However gluons were neglected in these references. In this paper, we consider QCD with the addition of a color singlet scalar multiplet as fundamental fields. We will allow for $SU(N_f) \times SU(N_f)$ flavor symmetry which entails taking $\sigma = \sigma^a \lambda^a, \pi = \pi^a \lambda^a$, collectively denoted by a complex $N_f \times N_f$ matrix M as stated earlier (λ^a are the usual Gell-Mann $SU(3)$ matrices for $N_f = 3$). We use this as a candidate theory for strong interactions with quarks, gluons and mesons as elementary excitations. We assume that we are in a phase of the theory (dictated by initial conditions on the QCD and Yukawa coupling and the renormalization scale μ) where not only the quarks but also the scalar and pseudoscalar mesons contained in M are elemenatary.

We describe the model in Section II and evaluate the quantum one-loop effective action with instanton as the classical background. In Section III the effective potential is obtained in the dilute gas approximation for the instantons. The potential is examined for a non-trivial minimum in Section IV and the quark mass generation is studied. Improvements following [?] and [?] are also examined. The results are summarized in Sec. V.

II. THE MODEL

The proposed Lagrangian consists of the usual Lagrangian of QCD and an $(N_f \times N_f)$ color singlet complex matrix M whose matrix elements consist of elementary scalars and pseudoscalars, interacting with quarks through the Yukawa coupling, viz.

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu a} + \bar{\psi} i \not{D} \psi + i\theta \bar{F} F \\ & + g_y (\bar{\psi}_L M \psi_R + \bar{\psi}_R M^\dagger \psi_L) + \frac{1}{2} \partial_\mu M \partial^\mu M^\dagger, \end{aligned} \tag{1}$$

where $\not{D} = \not{\partial} - ig\not{A}$, g_y is the Yukawa coupling and we have added a kinetic energy term for M. A few remarks about (??) are in order. This Lagrangian possesses global $SU(N_f)_L \times SU(N_f)_R$ chiral invariance and of course, $SU(3)_c$ local color invariance. In addition it has $U(1)_V$ (baryon number) and $U(1)_A$ invariances, the latter being broken by quantum corrections, leaving only a discrete $Z(N_f)_A$ symmetry. The raison d'etre for using this Lagrangian has already been explained in the Introduction. The topological term $i\theta \bar{F} F$ has been added although we know that experimental limits on the neutron dipole moment sets a limit on $\theta < 10^{-9}$.

The partition function for (??) is

$$Z = \int \mathcal{D}\bar{\psi} \mathcal{D}\psi \mathcal{D}A_\mu \mathcal{D}M \mathcal{D}M^\dagger e^{i \int d^4x \mathcal{L}[\psi, \bar{\psi}, A_\mu, M, M^\dagger]} \tag{2}$$

We evaluate the partition function, first by going over to Euclidean space and choosing instanton background for the gluon field. This consists in writing $A_\mu^a = \bar{A}_\mu^a(instanton) + a_\mu^a$. The integration over ψ and $\bar{\psi}$ is done first, giving the result

$$Z = \int \mathcal{D}A_\mu \mathcal{D}M\mathcal{D}M^\dagger e^{-\int d^4x[-\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a}+i\theta\tilde{F}F]}$$
$$det(i\bar{\not{D}} + ReM + i\gamma_5 ImM), \tag{3}$$

where $\bar{\not{D}} = \not{\partial} - ig\bar{\not{A}}$(instanton)and the determinant above is over Lorentz and flavour indices. The functional integration over A_μ^a can be split over instanton locations $\bar{A}_\mu^a$(instanton)and and an integral over the fluctuations a_μ^a around the instantons. We make the usual dilute gas approximation for the instantons. The Gaussian integration over a_μ^a givs a contribution [?] which we denote by K and the integration over $\bar{A}_\mu^a$ (instanton) is replaced by a summation over instanton winding numbers, apart from an overall Jacobian for the change of measure.The determinant $det(i\not{D} + ReM + i\gamma_5 ImM)$ gives a contribution $detM$ and $detM^\dagger$ for each zero mode of the $\not{D}$ operator ($\gamma_5\psi_0 = \pm\psi_0$) and $det'(-\not{D}^2 + MM^\dagger)$ from the non-zero modes. The result of all this is

$$Z = \int \mathcal{D}M\mathcal{D}M^\dagger \sum_{n_+} \frac{1}{n_+!} e^{in_+\theta} K^{n_+} (detM)^{n_+}$$
$$\sum_{n_-} \frac{1}{n_-!} e^{in_-\theta} K^{n_-} (detM)^{n_-}$$
$$det^{'}(-\not{D}^2 + MM^\dagger) e^{-\int d^4x \partial_\mu M \partial^\mu M^\dagger} \tag{4}$$

where we have taken into account the exchange symmetry of the instantons. The summations over n_+ and n_- can be carried out and the effective action Γ defined through $Z = \int \mathcal{D}M\mathcal{D}M^\dagger e^{-\int \Gamma}$, can be written down to be

$$\Gamma = \frac{1}{2}\partial_\mu M \partial M^\dagger + K detM + K detM^\dagger$$
$$+ ln det(-\not{D}^2 + MM^\dagger) \tag{5}$$

where we have taken, without loss of generality $\theta = 0$.

III. ONE LOOP EFFECTIVE POTENTIAL

The result of the previous section shows that with the model proposed, the effect of the quantum one loop calculations around an instanton background is to produce an effective action given in (??). Before continuing with our study, it will be instructive to compare our effective action (??) with that of Raby [?] and Pisarski and Wilczek [?]. The point we wish to make is that the term $(detM + detM^\dagger)$ that they add for phenomenological reasons is derived in our paper as a contribution arising out of the zero modes of the Dirac operator in the instanton background. The importance of this term, as pointed out in these references, is to break $U(N_f) \times U(N_f) \rightarrow SU(N_f) \times SU(N_f) \times U(1)_V$ which is the quantum symmetry of QCD. To be precise, the Lagrangian (??) has $G = SU(3)_c \times SU(N_f) \times SU(N_f) \times U(1)_V \times U(1)_A$ symmetries with the generators of G acting on ψ. By integrating out the fermion field, we obtain an effective action (??) for M which has the same symmetry G with the generators now acting on M. This term makes the η' massive as the broken $Z_A(N_f)$ symmetry is discrete. In order to appreciate the role of this term in our model, we see from (??) and (??) that if $\theta \neq 0$, then we would have gotten $K cos\theta(detM + detM^\dagger) + K i sin\theta(detM - detM^\dagger)$. The second term violates CP and hence we have put $\theta = 0$ consistent with the data on the electric dipole moment of the neutron.

The other important term is expected to come from the contribution of the non-zero eigenvalue sector and as shown, contributes $lndet'(-\bar{\not{D}}^2 + MM^\dagger)$. This does not break $U(1)_A$ and will be a function of $MM^\dagger$. The exact calculation of this piece if of course difficult. Following 't Hooft [?] we see that the regularized product of the non-vanishing eigenvalues is proportional to the one with $\bar{A}_\mu = 0$, the proportionality constant being dependent on the instanton quantum numbers (see, for example, Eq. 6.15 of [?]). Therefore, we write this term as

$$A\ lndet'(-k^2 + MM^\dagger) \tag{6}$$

where A contains the effect of instantons taken to be compact in each small volume ΔV of space-time and the ghost determinant arising from gauge fixing. We further consider a basis in which the complex matrix M is diagonal with elements $\lambda_i, (i = 1, \ldots, N_f)$. This is done by considering the term $(\bar{\psi}_L M \psi_R + \bar{\psi}_R M^\dagger \psi_L)$ in the Lagrangian. We diagonalize M,the complex $N_f \times N_f$ matrix, by independent left and right unitary transformations viz. $U_L^\dagger M U_R = \Lambda$ where $\Lambda_{ij} = \lambda_i \delta_{ij}$. This transformation, of course, introduces a mixing of flavours in ψ; nevertheless, the strong interaction part $\bar{\psi} \not{A} \psi (= \bar{\psi}_L \not{A} \psi_L + \bar{\psi}_R \not{A} \psi_R)$ is unaffected by this mixing. In general these λ_i's are complex. Using ϵ-regularizing scheme (see, for example, [?]) we can then write

$$A\ lndet'(-k^2 + MM^\dagger) \simeq A \sum_{i=1}^{N_f} |\lambda_i|^4 ln(\frac{|\lambda_i|^2}{\mu^2}) \tag{7}$$

where μ is a regularization scale.

Combining all this, the effective potential for M becomes

$$V_{eff} = K \prod_{i=1}^{N_f} \lambda_i + K \prod_{i=1}^{N_f} \lambda_i^* + A \sum_{i=1}^{N_f} |\lambda_i|^4 ln(\frac{|\lambda_i|^2}{\mu^2}) \tag{8}$$

Note that the first two terms are a consequence of the topologically non-trivial instanton background for gluons that we have chosen. Such terms will not be present in a usual topological trivial configuration like, for instance, the Saviddy background. This will prove to be crucial for our purposes.

IV. MECHANISM OF CHIRAL SYMMETRY BREAKING

The minimum of the effective potential (**??**) will provide us with the vacuum expectation values for the diagonal elements of M. To simplify the algebra, we will henceforth restrict ourselves to $N_f = 3$. We are interested in the minimum of the effective potential (**??**) so that the vacuum expectation values $< M >$ can be identified with $\lambda_i, (i = 1, 2, 3)$. The minimisation with respect to $\lambda_1, \lambda_2, \lambda_3$ yields

$$K\lambda_2\lambda_3 = -A\lambda_1^*|\lambda_1|^2(1 + 2ln(\frac{|\lambda_1|^2}{\mu^2}))$$
$$K\lambda_3\lambda_1 = -A\lambda_2^*|\lambda_2|^2(1 + 2ln(\frac{|\lambda_2|^2}{\mu^2}))$$

$$K\lambda_1\lambda_2 = -A\lambda_3^*|\lambda_3|^2(1+2ln(\frac{|\lambda_3|^2}{\mu^2})) \tag{9}$$

from which, it immediately follows

$$\begin{aligned} |\lambda_1|^4(1+2ln(\frac{|\lambda_1|^2}{\mu^2})) &= |\lambda_2|^4(1+2ln(\frac{|\lambda_2|^2}{\mu^2})) \\ &= |\lambda_3|^4(1+2ln(\frac{|\lambda_3|^2}{\mu^2})) \end{aligned} \tag{10}$$

At this stage, the λ_i's are constants representing vacuum expectation values $< M >$. In general, the λ_i's are complex. Since however these are now space-time independent constants, their phases can be absorbed in U_L or U_R. With this, the λ_i's can be treated as real.

The trivial solution to (??) is of course $\lambda_1 = \lambda_2 = \lambda_3 = 0$ which corresponds to $V_{eff} = 0$. However, a non-trivial solution to (??) is given by

$$\lambda_1 = \lambda_2 = \lambda_3 = \lambda \neq 0 \tag{11}$$

where λ is given by

$$\lambda = \mu e^{-(1+\frac{K}{A\lambda})/4} \tag{12}$$

The value of the effective potential for this solution is

$$V_{eff}(\lambda_1 = \lambda_2 = \lambda_3 = \lambda) = -2A\lambda^4(1+ln(\frac{\lambda}{\mu})) \tag{13}$$

We require this to be lower than $V_{eff} = 0$ (corresponding to the trivial solution $\lambda_1 = \lambda_2 = \lambda_3 = 0$) which is possible if

$$\mu < e\lambda \tag{14}$$

This sets the energy scale μ for which chiral symmetry breaking is possible. The symmetric solution (??)spontaneously breaks $SU(3)_L \times SU(3)_R$ to $SU(3)$ symmetry. We stress again that the $(detM + detM^{\dagger})$ coming from the instanton zero modes is crucial for this symmetry breaking. It is also clear that this breaking gives rise (in the 3 flavor case) to masses for the quarks

$$m_u = m_d = m_s = g_y \lambda. \tag{15}$$

Thus as conjectured in early works (see, for example, [?]), this model provides us with an explicit realization of chiral symmetry through instantons, due to the presence of fundamental scalars. A numerical value of μ cannot be obtained since λ is not specified in terms of known masses. However an estimate can be made if the Yukawa coupling in (??) is taken to be approximately around unity. Then λ can be identified with the quark masses as given in the equation above. Taking $m_s \sim 150$ MeV, we get $\mu < 300$ MeV, which sets the scale in this model for chiral symmetry breaking.

The minimization condition also provides us with a more phenomenologically realistic scenario where all the quark masses are not equal. For this, we will assume that $\lambda_1 = \lambda_2 = \lambda \neq \lambda_3$. In particular we look for a solution for which $\lambda_3 = a\lambda$, such that

$$m_u = m_d \neq m_s \tag{16}$$

Substituting in the minimization condition (??), the result $\lambda_3 = a\lambda$ we get the following equation

$$\frac{a^4}{1-a^4} ln a^2 = \frac{1}{2} + ln(\frac{\lambda^2}{\mu^2}). \tag{17}$$

This equation is numerically solved for many values of a (see Appendix). In particular we choose a value of $a = 20$ as the ratio of the s current quark mass to the u (or d) current quark mass. This immediately gives us a value of $\mu \simeq 26\lambda$ which for $g_y \sim 1$ gives $\mu \simeq 210$MeV, consistent with our previous estimate $\mu < 300$ MeV. We have broken the degeneracy between the quark masses between the s quark and the two lighter quarks, thereby breaking the $SU(3)$ flavour symmetry to isospin ($SU(2)$) and strangeness ($U(1)$). This kind of sequential symmetry breaking was envisaged long ago by Gell-Mann [?] by introducing $(\bar{\psi}_L M \psi_R + \bar{\psi}_R M^\dagger \psi_L)$ as an ideal pattern of couplings to "mesons". He added two types of terms; one is $u_0 \sim -m\bar{\psi}\psi$ which corresponds to, in our case $\lambda_1 = \lambda_2 = \lambda_3 = \lambda$ (Eq. ??) and a term $u_8 \sim m_{02}(\bar{N}N - 2\bar{\lambda}\lambda)$ (in the notation of [?]) which corresponds to $\lambda_1 = \lambda_2 = \lambda; \lambda_3 \neq \lambda$, corresponding, in our case to (??). In this way, the effect of the u_0 and u_8 terms of Gell-Mann, which have been introduced later by many other authors (see, for example, [?]), can be reproduced from the solution (??) of the minimum of the effective potential in our model. With the addition of these terms, the study of the effective action (??) proceeds along the lines elucidated by Raby, by expanding $< M >$ around $< M >_0$.

V. CONCLUSION

A mechanism for producing chiral symmetry breaking in QCD with scalars, is demonstrated. The model Lagrangian has the same symmetry properties as QCD. The effective potential for the "field" M is obtained in an instanton background and is shown to have a non trivial minimum. This generates masses for the quarks due to the breakdown of chiral symmetry. The connection between this study and earlier work *viz.* that of Pisarski and Wilczek and of Raby is pointed out. The method of Raby of introducing tadpole terms to break the $SU(3)$ flavor symmetry is realized here by the minimization condition with $m_u = m_d \neq m_s$. Thus a vacuum of the theory that is not chirally invariant (as dictated by QCD sum rules and other studies) has been obtained.

VI. APPENDIX

To see that equation (??) can accomodate many solutions where the λ's are not degenerate, we make the change of variable $x = \frac{\lambda_1^4}{\mu^4}$ and $y = \frac{\lambda_3^4}{\mu^4}$. Then the minimization condition (??) becomes

$$ln(e^x x^x) = ln(e^y y^y) \qquad (18)$$

Now taking y to be a multiple of x, say $y = bx$ (where $b = a^4$ as defined earlier in the text), the above equation reduces to

$$e^{x(1-b)} x^{x(1-b)} b^{-bx} = 1 \tag{19}$$

The easiest way to solve this equation is graphically by plotting the function $f(x) = e^{x(1-b)} x^{x(1-b)} b^{-bx}$ as a function of x for different values of the parameter b and checking where it cuts the $f(x) = 1$ line. This is shown graphically in the figure. Notice that we have to choose very large values of $b = a^4$ since $a = 20$ from phenomenological considerations as mentioned before. Note also that the $f(x) = 1$ line also coincides with the $b = 1$ case when $f(x) = 1$ for all values of x. We have used different values of b to indicate that they all cut the $f(x) = 1$ line at some point corresponding to some value of x which, in turn, will provide us with the λ to μ ratio, as indicated in section IV.

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Figure caption

Plot of $f(x) = e^{x(1-b)}x^{x(1-b)}b^{-bx}$ as a function of x for different b's. The horizontal line is the $f(x) = 1$ line (as also the $b = 1$ line for all x).