Bulk properties of nuclear matter in the relativistic Hartree approximation with cut-off regularization

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**Abstract**
    A method of cut-off regularization is proposed to evaluate vacuum corrections in nuclear matter in the framework of the Hartree approximation. Bulk properties of nuclear matter calculated by this method are a good agreement with results analyzed by empirical values. The vacuum effect is quantitatively evaluated through a cut-off parameter and its role for saturation property and compressional properties is clarified.

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In a relativistic approach for the study of nuclear matter vacuum effects are very important corrections which can not be taken into account in any non-relativistic formalism. About twenty years ago, using a simple $\sigma$-$\omega$ model (Walecka model), the mean field theory (MFT) clarified the saturation mechanism of nuclear matter relativistically[1] and in succession vacuum effects to the saturation property were evaluated by a method of dimensional regularization in the framework of the Hartree approximation (RHA)[2]. It was shown that vacuum corrections made an effective nucleon mass larger and the incompressibility of nuclear matter smaller. A small incompressibility is desirable for experimental findings of today[3,4]. Recently it has been reported that there is a strong correlation between an increase of effective mass and a decrease in incompressibility[5].

Nearly ten years after a proposal of the Walecka model, corrections due to vacuum polarization in quantum fluctuation around the mean field (the Hartree field) were evaluated[6]. The results were undesirable. Corrections were larger by far than the magnitude of mean field and furthermore brought forth instable ghost poles (the Landou ghost) in meson propagators which made nuclear matter unstable. One of ad hoc but powerful recipes to escape from these disasters was to introduce some form factors at each vertex[7]. There was another idea that the form factors should be derived from vertex corrections[8]. These recipes are grounded on existence of internal structure of hadron. We are afraid that vacuum corrections evaluated by the method of dimensional regularization may be overestimated because the size of hadron is not considered in such a regularization as the one used well in the elementary particle physics.

When we use a conventional form factor in a calculation of polarization insertion of vector meson, however, we need a safety device to assure the baryon current conservation. In this report, then, we show another recipe to estimate vacuum corrections in the framework of Hartree approximation in simple Walecka model. This is the first request to a method of cut-off regularization stated in the following discussion.

A nucleon propagator $G(k)$ in the relativistic Hartree approximation has the following standard form

$$G(k) = G_F(k) + G_D(k)$$

$$= \frac{-1}{i\gamma_\mu k_\mu^* + M^* - i\epsilon} + \left( -i\gamma_\mu k_\mu^* + M^* \right) \frac{i\pi}{E_k^*} \theta(k_F - |\vec{k}|) \delta(k_0^* - E_k^*), \qquad (1)$$

$$E_k^* = \sqrt{\vec{k}^2 + M^{*2}}, \qquad (2)$$

$$M^* = M + \Sigma_S, \qquad (3)$$

$$k_\mu^* = (\vec{k}, k_4 + \Sigma_4) = (\vec{k}, ik_0 + i\Sigma_0), \qquad (4)$$

where the subscripts "$F$" and "$D$" of $G(k)$ denote the Feynman part and the density part, respectively, $M$ and $k_F$ denote the physical nucleon mass and Fermi

momentum, respectively. In the simple Walecka model the nucleon self-energies are given as follows,

$$\Sigma_S = \Sigma_{SD} + \Sigma_{SF} = i\lambda \left(\frac{g_s}{m_s}\right)^2 \int \frac{d^4q}{(2\pi)^4} \text{tr}\{G_D(q) + G_F(q)\}, \qquad (5)$$

$$\Sigma_0 = i\lambda \left(\frac{g_v}{m_v}\right)^2 \int \frac{d^4q}{(2\pi)^4} \text{tr}\{\gamma_0 G(q)\} = -\left(\frac{g_v}{m_v}\right)^2 \rho_B, \qquad (6)$$

where $\rho_B$ denotes the baryon density and $m_s$, $m_v$, $g_s$ and $g_v$ denote the $\sigma$-meson mass, the $\omega$-meson mass, the $\sigma$-nucleon coupling and the $\omega$-nucleon coupling, respectively, and $\lambda$ denotes the degeneracy, $\lambda = 2$ for nuclear matter and $\lambda = 1$ for neutron matter. The Feynman part of self-energy $\Sigma_{SF}$ is a divergent integral in the 4-dimensional momentum space while $\Sigma_{SD}$ and $\Sigma_0$ have finite values. In the Hartree approximation there appears another divergent integral in the Feynman part of baryon energy density defined as follows,

$$\varepsilon_B = \varepsilon_{BD} + \varepsilon_{BF} = \frac{\lambda}{\pi^2} \int_0^{k_F} k^2 \sqrt{k^2 + M^{*2}} dk - \lambda \int \frac{d^4k}{(2\pi)^4} \text{tr}[\gamma_4 G_F(k)]\gamma_4. \quad (7)$$

We discuss how we introduce a cut-off parameter into the two divergent integrals in the formalism of the Hartree approximation. We start discussion with the nucleon effective mass $M^*$. The effective mass is given by using the scalar self-energy with a cut-off parameter $\Lambda$,

$$M^* = M_0 + \Sigma_{SD}(M^*, \rho_B) + \Sigma_{SF}(M^*, \Lambda)$$

$$= M + \Sigma_{SD}(M^*, \rho_B) + \Sigma_{SF}(M^*, \Lambda) - \Sigma_{SF}(M, \Lambda), \qquad (8)$$

$$M_0 = M - \Sigma_{SF}(M, \Lambda), \qquad (9)$$

where $M_0$ denotes the bare mass of nucleon. The last term in eq.(8) is the self-energy at zero density and is introduced for the effective mass $M^*$ to be the physical mass $M$ at zero density. We require the bare nucleon mass $M_0$ to have cut-off dependence so that the physical nucleon mass $M$ dose not depend on any cut-off parameter $\Lambda$. We note that the bare nucleon mass $M_0(\Lambda)$ becomes the physical mass as $\Lambda \to 0$.

To make progress we write the energy density as follows,

$$\varepsilon = \frac{1}{2}\pi^2 \alpha_V \rho_B^2 + \frac{1}{2\pi^2} \frac{1}{\alpha_S} \left(M^* - M\right)^2 + \varepsilon_B, \qquad (10)$$

where $\alpha_S$ and $\alpha_V$ are defined,

$$\alpha_S = \left(\frac{g_s}{\pi m_s}\right)^2, \qquad \alpha_V = \left(\frac{g_v}{\pi m_v}\right)^2. \qquad (11)$$

As the second request we require Hugenholtz-von Hove theorem[9],

$$(\varepsilon + P)/\rho_B = E_N(k_F) \equiv E_F, \qquad (12)$$

3

where $P$ denotes pressure and $E_N(k_F)$ denotes a nucleon energy at Fermi momentum. The condition to satisfy this identity is given by

$$\frac{\partial \varepsilon}{\partial M^*} = 0 \iff M^* = M - \pi^2 \alpha_S \frac{\partial \varepsilon_B}{\partial M^*} \iff \Sigma_S = -\pi^2 \alpha_S \frac{\partial \varepsilon_B}{\partial M^*}. \qquad (13)$$

The relation Eq.(13) between $\Sigma_S$ and $\varepsilon_B$ is exactly satisfied both in the density dependent part and in the Feynman part, respectively, in RHA[2]. In the method of cut-off regularization, however, the same relation is not satisfied in Feynman part although it is satisfied in the density part. Then we require that $\Sigma_{SF}$ is combined with $\varepsilon_{BF}$ by Eq. (13) and obtain the expression for $\varepsilon_{BF}$ as follows,

$$\varepsilon_{BF} = -\frac{1}{\pi^2 \alpha_S} \int_M^{M^*} \Big[ \Sigma_{SF}(x, \Lambda) - \Sigma_{SF}(M, \Lambda) \Big] dx$$

$$= \frac{\lambda}{4} \frac{1}{4\pi^2} \Big[ -\Lambda^4 \ln \big\{ \frac{\Lambda^2 + M^{*2}}{\Lambda^2 + M^2} \big\} - \Lambda^2 \big( M^{*2} - M^2 \big) + M^{*4} \ln \big\{ \frac{\Lambda^2 + M^{*2}}{M^{*2}} \big\}$$

$$- M^4 \ln \big\{ \frac{\Lambda^2 + M^2}{M^2} \big\} + 4 \big( M^* - M \big) M \Big\{ \Lambda^2 - M^2 \ln \big\{ \frac{\Lambda^2 + M^2}{M^2} \big\} \Big\} \Big],$$

$$\qquad (14)$$

where

$$\Sigma_{SF}(M^*, \Lambda) = \frac{\lambda}{4} \alpha_S M^* \Big[ \Lambda^2 - M^{*2} \ln \big\{ \frac{\Lambda^2 + M^{*2}}{M^{*2}} \big\} \Big], \qquad (15)$$

and the lower limit $M$ in the integral is chosen so that $\varepsilon_{BF}$ vanishes at zero density. The scalar self-energy and the baryon energy density in the density dependent part are given by

$$\Sigma_{SD} = -\frac{\lambda}{2} \alpha_S M^* \Big[ k_F E_F^* - M^{*2} \ln \big\{ \frac{k_F + E_F^*}{M^*} \big\} \Big], \qquad (16)$$

$$\varepsilon_{BD} = \frac{\lambda}{8\pi^2} \Big[ 2 k_F E_F^{*3} - M^{*2} k_F E_F^* - M^{*4} \ln \big\{ \frac{k_F + E_F^*}{M^*} \big\} \Big], \qquad (17)$$

respectively, where $E_F^* = \sqrt{k_F^2 + M^{*2}}$ .

We make use of the saturation condition to determine the cut-off parameter at the normal density because the cut-off parameter is independent of density. From Hugenholtz-von Hove theorem the saturation condition is expressed by

$$P = \rho_B \big[ E_F^* + \pi^2 \alpha_V \rho_B - e \big] = 0, \qquad (18)$$

$$e = M - 15.75 \quad [MeV], \qquad (19)$$

at the normal density. Then we have a following relation between $M^*$ and $\alpha_V$ at the normal density.

$$E_F^* + \pi^2 \alpha_V \rho_B = M - 15.75. \qquad (20)$$

4

On the other hand, the coupling strength $\alpha_S$ is given by

$$\alpha_S = \frac{M^* - M}{\bar{\Sigma}_s},\qquad(21)$$

where $\bar{\Sigma}_s$ is defined as $\Sigma_s = \alpha_S \cdot \bar{\Sigma}_s$. Putting these $\alpha_S$ and $\alpha_V$ into energy density equation, Eq. (10), we have a kind of relation between the effective mass $M^*$ and the cut-off parameter $\Lambda$ which satisfies the saturation condition at the normal density.

In Fig. 1, we show the saturation curves for several sets of $M^*$ and $\Lambda$. The other bulk properties of nuclear matter can be calculated for each set and are summarized in Table.

Fig. 1 and Table

First of all we discuss the magnitude of cut-off parameter $\Lambda$ which looks very small at first glance. We try to calculate $\Sigma_{SF}$ again by introducing the following conventional monopole type of form factor into vertex,

$$F_{NN\sigma}(p,q) = \frac{\Lambda_N^2 - M^2}{p^2 + \Lambda_N^2} \cdot \frac{\Lambda_N^2 - M^2}{(p+q)^2 + \Lambda_N^2} \cdot \frac{\Lambda_\sigma^2}{q^2 + \Lambda_\sigma^2},\qquad(22)$$

Fig. 2

We note that we use this form factor for $\Sigma_{SF}(M,\Lambda)$ at zero density while we modify it for $\Sigma_{SF}(M^*,\Lambda)$ at finite density by replacing $M$ with $M^*$ in Eq.(22). We make a comparison between the new result and $\Sigma_{SF}(M^*,\Lambda)$, and obtain the following relation between two kind of cut-off parameter,

$$\Lambda_N^2 = M^2 + \Lambda^2, \qquad \left(\Lambda_N^{*\,2} = M^{*2} + \Lambda^2\right),\qquad(23)$$

where another cut-off parameter $\Lambda_\sigma$ dose not participate in Hartree calculation because $q^2 = 0$. When we take about a half of nucleon mass as the cut-off parameter $\Lambda$, for example, we have familiar values for the cut-off $\Lambda_N$ of the monopole type of form factor.

Next, we are very interested in the saturation curves in Fig. 1. The saturation curves are softer as an increase of $\Lambda$. Starting from the most stiff curve (MFT) obtained without vacuum effect ($\Lambda = 0$), there exist the curves with smaller values of incompressibility less than 200 MeV. We can understand the saturation mechanism as a stable balance of three force, i.e., the attractive $\sigma$-meson, the repulsive $\omega$-meson and another repulsive vacuum effect. Contributions of meson to energy density depend strongly on the baryon density whereas contributions of vacuum effect depend weakly on the baryon density. So, since the energy density is fixed at the normal density, if the vacuum contribution to the energy density is larger, the $\omega$-meson contribution is smaller. Thus, the increase of $\Lambda$ dulls the density dependence of energy density in the neighbourhood of the normal density. This is the reason that the incompressibility $K$ becomes small if $\Lambda$ increase, as shown in Table.

Also we make a remark on the relation between $M^*$ and $\Lambda$. The effective mass depends on the attractive $\sigma$-nucleon coupling $g_s$ and the repulsive vacuum parameter $\Lambda$. The former makes $M^*$ small and the latter dose $M^*$ larger. The effective mass continues to increase if $0 < \Lambda/M < 0.4$, reaches to the maximum at $\Lambda/M \sim 0.4$ and decreases if $0.5 < \Lambda/M < 0.56$. Then, the vacuum effect for the effective mass is largest at $\Lambda/M \sim 0.4$. It is reasonable that the repulsive $\omega$-nucleon coupling strength $g_v$ shows a minimum value at $\Lambda/M \sim 0.4$ ( the maximum effect of vacuum ). It can be observed through the effective mass that the parameter $\Lambda$ and $g_v$ are complement each other as the two repulsive effects.

The effective mass $M^*$ decreases as the incompressibility $K$ decreases less than 400 MeV. The small $M^*$ is rather desirable since the empirical spin-orbit splitting in light nuclei supports $M^* = 0.6M$ [10]. The skewness $K'$ ( the third order derivative of saturation curve at the normal density in Ref. [5]) can also be calculated. Using this $K'$, the Coulomb coefficient $K_c$ is given by

$$K_c = -\frac{3q_e^2}{5r_0}\Big(\frac{9K'}{K} + 8\Big), \qquad r_0 = \Big(\frac{3}{4\pi\rho_0}\Big)^{1/3}, \tag{24}$$

based on the scaling model[11], where $g_e$ denotes the proton electric charge. $K_c$ is the coefficient of leptdermous expansion[11],

$$K(A,Z) = K + K_{sf}A^{-1/3} + K_{vs}I^2 + K_cZ^2A^{-4/3} + \cdots, \qquad I = 1 - 2\frac{Z}{A}, \tag{25}$$

where $K_{sf}$ and $K_{vs}$ are the surface-term coefficient and the volume-symmetry-term coefficient, respectively. These coefficients are determined from the giant monopole resonance(GMR) data of many nuclei. In fig. 3, we show the $K - K_c$ relation together with results analyzed by empirical values in Table 3 of [3] and in Table IV of [4]. We have a fine agreement with results analyzed empirically if 200 MeV $< K <$ 350 MeV.

Fig. 3

The symmetry energy $a_4$ in Table includes the $\rho$-meson contribution which depends on the square of the ratio of coupling strength to mass of $\rho$-meson and makes $a_4$ increase to about 30 MeV, i.e., without this contribution $a_4$ becomes 20.6 MeV at $K = 300$ MeV, in Table.

The $K - K_{vs}$ relation is shown in Fig. 4. The quantity $K_{vs}$ is given by

$$K_{vs} = K_{sym} - L\Big(9\frac{K'}{K} + 6\Big), \tag{26}$$

in the scaling-model, where $L$ and $K_{sym}$ are the first and second order derivative of asymmetry energy, respectively ( see the detailed definitions in Ref. [5]). Also we have another fine agreement with empirical values[3,4] if 250 MeV $< K <$ 400 MeV.

In summary, we proposed the method of cut-off regularization to evaluate the vacuum corrections in nuclear matter in the framework of the Hartree

6

approximation. We found that this method, RHAC( Relativistic Hartree Approximation with Cut-off regularization), can prepare the values from 200 MeV to 546 MeV for nuclear incompressibility in spite of a few adjustable parameters. So we note that the RHAC method is a very useful phenomenological one under the present situation that there is much uncertainty in the experimental determination of compressional properties.

We made quantitative analysis of the vacuum correction by the cut-off parameter. The results are summarized as follows.

(1) An increase of $\Lambda$ means an increase of vacuum correction. The parameter $\Lambda$ can be connected with the cut-off parameter of the conventional monopole type of form factor.

(2) The vacuum correction gives the repulsive effect both to the effective nucleon mass and to the baryon energy density. The saturation property is yielded by the interplay among the attractive $\sigma$-meson, the repulsive $\omega$-meson and the repulsive vacuum effect. The repulsive vacuum effect makes the nucleon incompressibility small because of its weak dependence on the baryon density.

(3) The calculated asymmetry energies in Table agree well with the empirical values if $C_\rho^2 = (g_\rho M/m_\rho)^2 = 54.71$ is used as the $\rho$-meson coupling strength[5].

(4) The calculated curve on the $K - K_c$ plane is a fine agreement with the empirical candidates in region 200 MeV $< K < 350$ MeV and also the curve on the $K - K_{vs}$ plane is a good agreement with the empirical candidates in region 250 MeV $< K < 400$ MeV. Therefore, to account for $K_c, K_{vs}$ and $a_4$ simultaneously, the RHAC method is valid if 250 MeV $< K < 350$ MeV.

## Acknowledgment

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**Table and Figure Captions**

Table    The numerical results for parameter sets of $(M^*, \Lambda)$. $K$, $K'$, $K_c$, $a_4$, $K_{vs}$, $K_{sym}$ and $L$ are shown in MeV. The $a_4$, $K_{vs}$, $K_{sym}$ and $L$ depend on value of $C_\rho = g_\rho M/m_\rho$ where $C_\rho^2 = 54.71[5]$.

Fig.1    The $k_F$-dependence of binding energy. All curves take a minimum value( -15.75 MeV ) at the normal Fermi momentum ( 1.35 fm$^{-1}$ ). The bold solid line, the bold dash-dotted line, the bold dashed line, the bold dotted line, the dash-dotted line, the dashed line, the dotted line and the solid line are the results for $K = 546, 500, 450, 400, 350, 300, 250$ and $200$ MeV in Table , respectively. The bold solid line is the result of MFT without the vacuum effect.

Fig. 2    Diagrammatic representation of the form factor at $NN\sigma$ vertex. The solid ( wavy ) lines denote nucleon ( $\sigma$-meson ) propagators. The form factor is shown as the circle at vertex.

Fig.3    The $K - K_c$ relation. The crosses with error bars are results in ref. [3] and the solid squares are the data from the table IV in ref. [4].

Fig.4    The $K - K_{vs}$ relation. The crosses with error bars are results in ref. [3] and the solid squares are the data from the table IV in ref. [4].

| $\Lambda/M$ | $K$ | $K'$ | $K_c$ | $a_4$ | $K_{vs}$ | $K_{sym}$ | $L$ | $M^*/M$ | $g_s$ | $g_v$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0000 | 546 | 226.39 | -8.971 | 30.2 | -890.4 | 86.3 | 100.36 | 0.5470 | 10.438 | 12.899 |
| 0.2827 | 500 | 160.77 | -8.330 | 29.5 | -775.3 | 72.0 | 95.27 | 0.5722 | 10.447 | 12.498 |
| 0.3884 | 450 | 101.96 | -7.676 | 29.0 | -671.5 | 65.6 | 91.69 | 0.5924 | 10.779 | 12.165 |
| 0.4892 | 400 | 54.71 | -7.058 | 29.0 | -587.6 | 79.8 | 92.29 | 0.5919 | 11.903 | 12.173 |
| 0.5407 | 350 | 0.05 | -6.118 | 29.7 | -463.3 | 127.9 | 98.50 | 0.5650 | 13.467 | 12.615 |
| 0.5559 | 300 | -78.33 | -4.320 | 30.5 | -197.4 | 186.0 | 105.02 | 0.5398 | 14.535 | 13.011 |
| 0.5609 | 250 | -177.10 | -1.242 | 31.1 | 288.0 | 246.4 | 110.74 | 0.5202 | 15.262 | 13.308 |
| 0.5624 | 200 | -293.20 | 3.972 | 31.6 | 1142.4 | 308.9 | 115.86 | 0.5044 | 15.801 | 13.542 |

Table