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The overlap is not a waveguide.

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Abstract

Golterman and Shamir falsely claim that a waveguide model modified by adding many charged bosonic spinors, in the limit of an infinite number of matter fields, becomes identical to the overlap if in the target theory every fermion appears in four copies. Their modified model would give wrong results even in the vectorial four flavor massless Schwinger model, while a dynamical simulation of this model with the overlap works correctly. In this note we pinpoint the error in the derivation of Golterman and Shamir.

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In a recent letter [1] the overlap method of defining chiral gauge theories was criticized on the grounds that it is similar to a Yukawa model known as the “waveguide”. In the seventh paragraph of [1] the main result is identified as an exact equivalence between the overlap of ref. [2] and a “modified waveguide model” with Higgs hopping parameter κ set to 0 and Yukawa coupling y set to 1. The equivalence is allegedly shown for a situation where there are $4n$ identical chiral families. The proof of the equivalence concludes with the last equation in [1] (eq. (42)) which is claimed to have been rigorously established.

The modified waveguide differs in a major way from the original waveguide. The discussion of the properties of the modified waveguide is highly speculative. However, we don’t need to engage in imprecise arguments here since the main result of ref. [1] is false. The error made by Golterman and Shamir was specified as a possible pitfall in our paper [3] (last paragraph in section 6 there), also listed in reference [3] of [1].

The modified waveguide, when analyzed using the second quantized transfer matrices introduced in [2], $T_{\pm}(U)$, depending parametrically on the background gauge field U , leads after the integration of all matter fields to:

$$\left[\frac{\text{Tr} [T_-^L(1)T_-^L(U)T_+^L(U)T_+^L(1)]}{\sqrt{\text{Tr} [T_-^{2L}(1)T_-^{2L}(U)]}\sqrt{\text{Tr} [T_+^{2L}(1)T_+^{2L}(U)]}} \right]^{4n}. \quad (*)$$

L is a large integer. The original waveguide model gives an effective action equal to the logarithm of the numerator of the above expression. It is claimed in ref. [1] that as $L \rightarrow \infty$ (*) converges to the overlap. This is incorrect in general because the ground states of $T_-(U)$ and $T_+(U)$ will be orthogonal to each other for large chunks of gauge field space. Such gauge field backgrounds will be generated with finite probability in a Monte Carlo simulation using (*). In particular this happens when the gauge field background is close to a continuum connection on a principal bundle carrying nonzero instanton number. The overlap gives a vanishing result reflecting the known zero modes while (*) would typically approach some finite limit, the traces being dominated by a combination of second quantized fermionic ground and excited states.

Let us explain this in some more detail: The transfer matrices are exponents of bilinears in fermion creation/annihilation operators that conserve a total fermion number. When $L \rightarrow \infty$ the three traces in (*) will be dominated by specific states of definite fermion number. (Typically, states with the same fermion number will have nonzero inner products and exact accidental degeneracies will not happen.) Suppose the background has lattice topological charge 1. The single way (*) could vanish in the limit $L \rightarrow \infty$ is when the fermion numbers dominating the two traces in the denominator of (*) are different. However, deforming the gauge background towards a configuration of zero topological charge

and where we have near degeneracy in $T_+(U)$ (the single matrix that is substantially sensitive to the topology of the background [3]) we see that there will be situations where the fermion numbers of the dominating states in the denominator will be the same and (*) will have a finite limit. The regions in gauge field space where the overlap is strictly zero and (*) is non-zero are of codimension zero and there is no obvious mechanism to suppress them with probability one.

The claimed equivalence could not have been correct *a priori* since for any finite L the modified waveguide has exact global symmetries that are known to be violated in the continuum. This observation holds also in the context of a vector-like theory, showing that the failure of the modified waveguide is independent of the chiral nature of the fermionic content of the target theory. More precisely, had we replaced the fermion subroutines implementing the overlap in our Monte Carlo simulation of the four flavor massless Schwinger model by subroutines implementing the modified waveguide, we would have obtained on a torus $\langle \prod_{f=1}^4 \bar{\psi}_f \psi_f \rangle = 0$. Numerical results obtained with the overlap yield a nonvanishing $\langle \prod_{f=1}^4 \bar{\psi}_f \psi_f \rangle$ in quantitative agreement with continuum [4].

The unwanted preservation of global symmetries is a common feature of the majority of the attempts to regulate nonperturbatively gauge theories with massless fermions using Yukawa models. For this reason, an understanding of the detailed dynamics of these models (in particular for $F_{\mu\nu} \equiv 0$, the focus of most recent research) is likely to be irrelevant to the problem of putting *chiral* gauge theories on the lattice. Of course, this opinion is open to debate.

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