LEPTOPRODUCTION OF HEAVY QUARKS

Pankaj Agrawal[a], Fredrick I. Olness[b,*], Stephan T. Riemersma[b], Wu-Ki Tung[a]
[a]*Michigan State University, East Lansing, Michigan 48824 USA*
[b]*Southern Methodist University, Dallas, Texas 75275 USA*

Abstract

There are presently two approaches to calculating heavy quark production for the deeply inelastic scattering process in current literature. The conventional *fixed-flavor scheme* focuses on the flavor creation mechanism and includes the heavy quark *only* as a final state particle in the hard scattering cross section; this has been computed to next-to-leading order–α_s^2. The more recently formulated *variable-flavor scheme* includes, in addition, the flavor excitation process where the initial state partons of all flavors contribute above their respective threshold, as commonly accepted for calculations of other high energy processes; this was initially carried out to leading order–α_s^1. We first compare and contrast these existing calculations. As expected from physical grounds, the next-to-leading-order *fixed-flavor scheme* calculation yields good results near threshold, while the leading-order *variable-flavor scheme* calculation works well for asymptotic Q^2. The quality of the calculations in the intermediate region is dependent upon the x and Q^2 values chosen. An accurate self-consistent QCD calculation over the entire range can be obtained by extending the *variable-flavor scheme* to next-to-leading-order. Recent work to carry out this calculation is described. Preliminary numerical results of this calculation are also presented for comparison.

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1 Introduction

The production of heavy quarks in photo-, lepto-, and hadro-production processes have become an increasingly important subject of study both theoretically and experimentally. The theory of heavy quark production in perturbative Quantum Chromodynamics (QCD) is subtle because of the additional scales introduced by the quark masses (m_c, m_b, m_t), generically denoted by m_H in the following.[1, 2] Traditional *fixed-flavor scheme* (FFS) calculations treat the heavy quark mass m_H as a *large parameter* for all ranges of the physical momentum scale, generically denoted by[†] Q; hence, it includes only those hard processes initiated by the gluon and the "light quarks" (u, d, s).[3, 4] It is expected to become unreliable at high energies when $Q \gg m_H$, since the presence of powers of $\ln(Q/\mu)$ and $\ln(m_H/\mu)$ in the hard cross section formulas will invalidate the perturbative expansion regardless of how one chooses the renormalization and factorization scale μ. As these ratios can indeed be quite large for charm and bottom quarks at current and future colliders, it is not surprising that results of existing fixed-order calculations have been under much scrutiny. Specifically, both the large size of the next-to-leading-order (NLO) corrections (compared to the leading-order (LO) ones) and the substantial dependence of the results on the (in principle, arbitrary) scale parameter μ are suggestive symptoms of the large logarithm problem.[5] In addition, quantitative comparisons with first experimental results on charm and bottom production have not been entirely satisfactory.[6]

The problem can be stated in another way: in the kinematic regime where $Q \gg m_H$, the heavy quark mass m_H becomes *relatively* "light." It is then expected to behave just like one of the familiar light quarks – it should be more naturally treated like another "parton." This transition from a heavy particle when $Q \lesssim m_H$ to a parton when $Q \gg m_H$ is intuitively obvious; it is implicitly accepted in general considerations of high energy processes both within and beyond the standard model,[7] and explicitly incorporated in all calculations of parton distribution functions (PDF's) inside the hadron[8, 9] – where the number of effective parton flavors increases each time a quark threshold is crossed. However, as mentioned above, the conventional *fixed-flavor scheme* of hard-scattering cross sections of heavy flavor production does not incorporate this effect – a quark with mass m_H is treated as "heavy" for all values of Q, and it is never counted as a parton. Two problems arise from this scheme: (i) the calculation clearly becomes unnatural and unreliable when $Q \gg m_H$; and (ii) the combined use of PDF's calculated in a scheme with scale-dependent number of parton flavors in conjunction with hard cross sections calculated in a different scheme with fixed (scale-independent) number of partons is obviously an inconsistent application of QCD.

The solution to this problem lies in formulating a consistent renormalization and factorization scheme[10] which explicitly implements the above mentioned transition of a "heavy quark" to a "light parton." To distinguish it from the FFS, we shall designate this the *variable flavor scheme* (VFS).[11] In the VFS, there is an intricate interplay between the gluon-boson-fusion (or "flavor creation") and quark-scattering (or "flavor excitation") production mechanisms as the typical physical scale Q varies from the threshold to the asymptotic regions.

In the following, we will review the theoretical issues of heavy quark production relevant for present and future experiments, and present a brief comparison of the existing calculations. We then describe a new three-order calculation for heavy quark production in the VFS which

[†]The generic scale Q can be identified with the familiar variable "Q" in deep inelastic scattering and lepton-pair production (Drell-Yan process), $M_{W,Z}$ in W, Z production, p_t in direct-photon and jet production, ... etc.

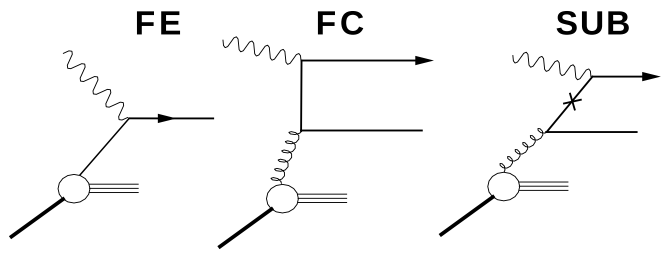

Figure 1: Representative leading order diagrams for flavor-excitation and flavor-creation production mechanisms and the overlap between the two (which must be subtracted for consistency).

is valid from the threshold region to asymptotic energies. This new result will allow us to improve the quantitative QCD theory of heavy flavor lepto-production. The principles described here are also applicable to hadro-production. Not mentioned here are the higher-order QCD corrections to heavy quark production due to large logarithms associated with "small-x" which require resummation of an entirely different type.[12]

2 Variable Flavor Scheme: From Low to High Energy

The intuitive notion that a "heavy quark" with mass becomes *decoupled* from physical processes at a scale $Q \ll m_H$ (thus should not be counted as one of the "partons"), and that the same quark becomes an *active parton* at a much higher scale $Q \gg m_H$, is implemented precisely in terms of the VFS in which the running coupling function $\alpha_s(\mu)$ and the PDF's $f_N^H(\xi,\mu)$ are continuous across the heavy quark threshold $\mu_{th} = m_H$.[cwz,ColTun,lhk2] Decoupling manifests itself in $f_N^H(\xi,\mu) = 0$ for $\mu \le \mu_{th}$, and the effective flavor number n_f does not include H below threshold. Above the threshold $\mu \ge \mu_{th} = m_H$, $f_N^H(\xi,\mu)$ satisfies the PDF evolution equation with the usual $\overline{MS}$ evolution kernel, and the effective flavor number n_f is incremented by one to count H as one of the active partons.[‡]

This scheme naturally includes both the *flavor excitation* and the *flavor creation* production mechanism; the basic idea is illustrated in Fig. 1 which shows one representative leading order diagram of each kind. The quark initiated *flavor excitation* (FE) diagram contributes when the heavy quark PDF $f_N^H(\xi,\mu)$ is non-vanishing. It contains the resummed collinear logarithms to all orders, and represents the dominant physics at large Q^2/m_H^2. The gluon initiated *flavor creation* (FC) diagram captures the correct physics at energy scales of the same order as the quark mass m_H. The *subtraction* (SUB) diagram represents the overlap between the two complementary mechanisms, and serves the dual purpose of removing the double counting and cancelling the collinear singularity contained in FC for large $\log(Q^2/m_H^2)$. It is important to realize that, although the leading FE diagram is formally of $\mathcal{O}(\alpha_s^0)$ and the other two are of $\mathcal{O}(\alpha_s^1)$, all three are, in fact of the *same numerical order* since the heavy-quark distribution function of the FE term is effectively of $\mathcal{O}(\alpha_s^1)$ compared to the gluon distribution of the others. It is in this sense, results based on these diagrams[11] are referred to as "leading-order VFS" calculation.[§]

The main features of the VFS, highlighting the interplay between the FE, FC production mechanisms, and the subtraction term in the various energy ranges, are illustrated in Fig. 2 where the separate contributions to the b-quark production structure function F_2 and the

[‡] As shown in ref. [13], for these intuitively natural properties of $\alpha_s(\mu)$ and $f_N^H(\xi,\mu)$ to hold, the threshold μ_{th} must be chosen at m_H – not at $c\, m_H$ with some arbitrarily chosen c (such as $c = 2$ or 4).

[§] Not included here are the quark-initiated $\mathcal{O}(\alpha_s^1)$ FE contributions which, for the same reason mentioned here, are numerically of the same order as $\mathcal{O}(\alpha_s^2)$ FC terms. They need to be included only for the NLO VFS calculation, as will be discussed in Sec. 4

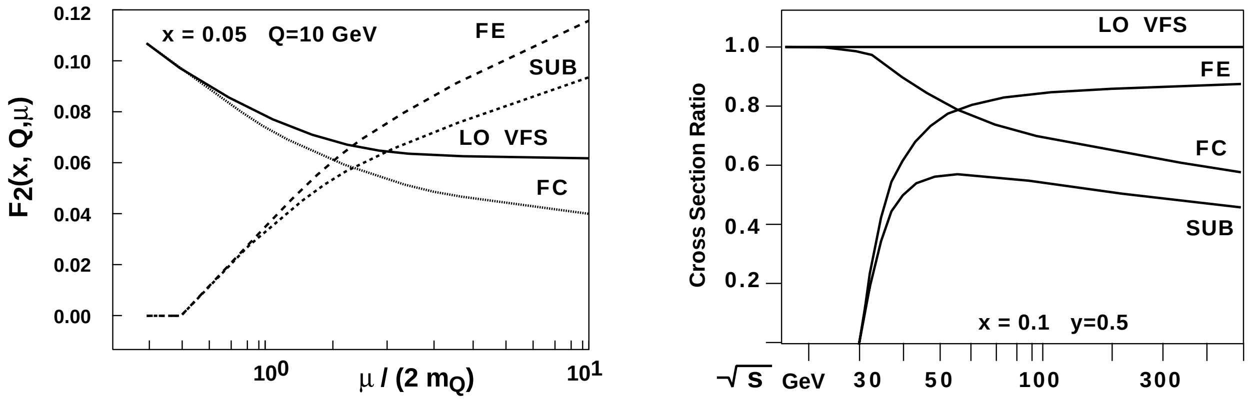

Figure 2: a) Scale dependence of the contributing terms to $F_2(x, Q)$ for bottom production. The factorization scale μ is shown in units of the physical scale $2m_Q$. b) Relative contributions of these terms to the physical cross section for b-production, $d\sigma/dx/dy$ at $\{x, y\} = \{0.1, 0.5\}$ *vs.* CM energy $\sqrt{s}$ in GeV normalized to the full LO result in the *variable flavor scheme* (VFS).

production cross section at a specific $\{x, y\}$ point are shown as a function of the factorization scale μ and the center- of-mass energy respectively.

First, consider Fig. 2a. At a small scale μ, the b-quark is not a constituent of the hadron, therefore the b-quark initiated FE term vanishes. In this region, subtraction vanishes as well so that the total F_2 is given completely by the gluon initiated FC term.

As the μ scale increases, the b-quark evolves as a parton in the hadron, and the b-quark PDF increases rapidly due to splitting of the large number of gluons present. This large μ-dependence by itself appears rather unnatural and may be a cause of serious concern. However, in the threshold region, the collinear subtraction term has precisely the correct form to cancel the unphysical contribution and to remove this artificial scale dependence. This is because in the threshold region ($\mu \simeq m_H$), the heavy quark distribution function is approximately given by the convolution of the gluon distribution with the gluon-quark splitting, $f_H \propto f_g \otimes P_{g \to H}$. This built-in cancellation ensures that the total physical result is actually well-approximated by the FC term (as one would expect just above threshold), and is minimally sensitive to not only the factorization scale, but also the choice of renormalization scheme.

In the limit of very large μ ($\mu \gg m_H$), the large collinear logs in FC are canceled by the subtraction term. The difference is “infrared safe” in the high energy limit, and remains numerically small. Consequently, F_2 is dominated by the FE process, whose contribution becomes dominant as the quark distribution function evolves to an increasingly important size.

The same physics, concerning the two complementary production mechanisms and their relative importance as a function of relevant physics scales, is shown in Fig. 2b for the physical cross section *vs.* energy over the range from fixed-target to HERA. For this comparison, the various terms are normalized to the full result. One can see clearly that $\sigma_{VFS} \simeq \sigma_{FC}$ at low energies, and $\sigma_{VFS} \simeq \sigma_{FE}$ in the high energy limit.[14]

3 Comparison of VFS and FFS

A systematic comparison of the previously available VFS and FFS calculations (LO and NLO respectively) is presented in ref. [15]. We summarize the highlights below. In the NLO FFS calculation,[3] the heavy quark enters into the hard scattering *only* via the FC process. (For example, when considering b production, the number of *light* flavors would be four.) The

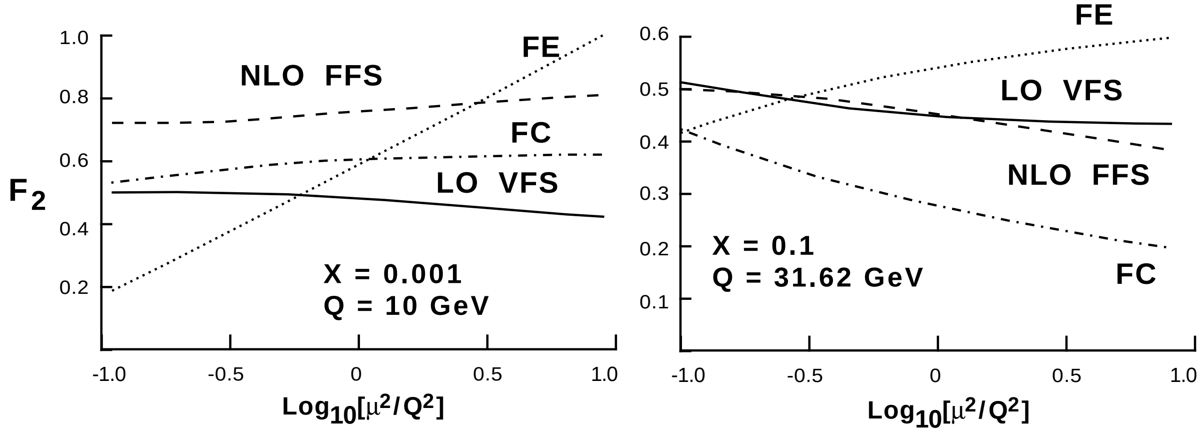

Figure 3: Scale dependence of the various calculations of $F_2(x, Q)$ for charm production on the factorization scale μ.

calculation has been carried out to $\mathcal{O}(\alpha_s^2)$, and this represents the most complete calculation for physical scales Q not too far above m_H. Problems will arise when terms containing "collinear logarithms" $\log^n(Q^2/m_H^2)$ $(n = 1, 2)$ become significant. Fortunately, Q^2/m_H^2 stays moderate even for the highest energy deep inelastic scattering experiments. Thus, as shown in Fig. 3 for a representative situation at HERA energies, the FFS result is quite stable against variation of the (arbitrary) factorization scale–a good indication that it should be reliable. This is in rather sharp contrast to the case of comparable calculations of heavy quark production in hadron colliders[5] where the relevant scale P_T^2 can be very large compared to m_c^2 and m_b^2, leading to well-known unstable results against choice of the scale,[5] (*e.g.*, at the Tevatron with $P_T = 50\,GeV$, $P_T^2/m_c^2 \sim 10^3$).

In the VFS approach, the contribution from the FE process with initial state c- and b-quark distributions represents the resummation of $\alpha_s^n \log^n(Q^2/m_H^2)$ terms *to all orders* in n. The "subtraction term" (Fig. 1) removes these collinear divergences from the FC diagrams and eliminates double counting between the FE and FC terms. The complete treatment of the $\alpha_s^n \log^n(Q^2/m_H^2)$ terms guarantee that the VFS results are reliable at very high energies where $Q^2 \gg m_H^2$. In principle, the VFS approach also reproduces the FFS results when computed to the same order of α_s, (*cf.*, refs. [11, 15]). In practice, since the LO VFS results only include the relevant contributions to $\mathcal{O}(\alpha_s^1)$, these results lack the $\mathcal{O}(\alpha_s^2)$ finite (*i.e.* non-collinear divergent) pieces of the NLO FFS. Hence, the μ-dependence of these results is not obviously better at the energy scale shown. Note, there is a considerable gap between the two calculations in the left-hand-side of the plot; this strongly suggests the need for improvement on existing calculations.

In contrast to the above, both the leading order quark initiated FE process, and the leading order gluon initiated FC process, by themselves, have comparably large scale dependence (*cf.*, Fig. 3) which makes them entirely unreliable in computing structure functions.[15]

4 NLO Calculation in the Variable Flavor Scheme

The LO VFS and NLO FFS calculations include different aspects of higher order corrections to the simple FE and FC heavy quark production mechanisms which become important in different kinematic regions. It is clearly desirable to incorporate both these corrections in one unified treatment which is reliable for all kinematic ranges. The VFS, by its very nature, provides the framework to do so—if we extend the calculation to $\mathcal{O}(\alpha_s^2)$. As mentioned before

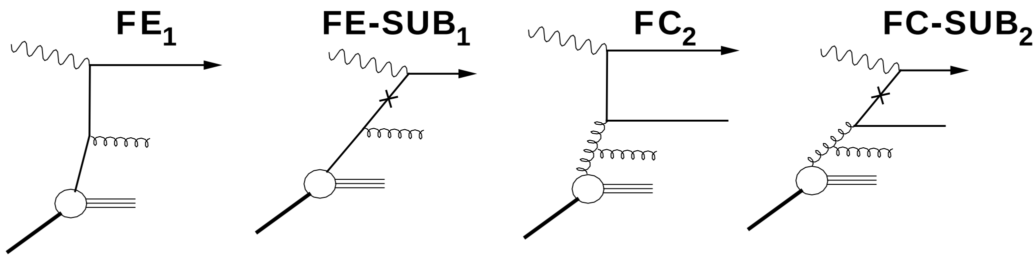

Figure 4: Representative next-to-leading diagrams for FE and FC production mechanisms and their overlaps. All terms are of the same *numerical* order—$\mathcal{O}(\alpha_s^2)$.

(and explained in detail in ref. [11]), this scheme reproduces the FC results of the FFS to the same order near threshold, and it yields the FE results of the naive parton model in the asymptotic region—as it should. The extension of the VFS calculation requires calculating the $\mathcal{O}(\alpha_s^1)$ (heavy) quark-initiated FE and the $\mathcal{O}(\alpha_s^2)$ FC contributions, as well as identifying the overlapping collinear subtraction terms between the two (*cf.* Fig.4). The calculation of the $\mathcal{O}(\alpha_s^1)$ FE results, for general vector boson couplings and arbitrary quark masses, have recently been completed.[16] The $\mathcal{O}(\alpha_s^2)$ FC contribution can be taken from the available results of the NLO FFS calculation.[3] One only needs to identify the appropriate subtraction to remove the $\mathcal{O}(\alpha_s^2)$ collinear-singularity terms which are already resummed into the QCD-evolved FE diagrams.

In the NLO FFS calculation of ref. [3], the collinear singularities for the light degrees of freedom (light quarks and gluons) have, of course, been subtracted using dimensional regularization. All that remains to do is to subtract the collinear singularities associated with the "heavy" quark which manifest themselves as logarithms of m_H. These terms are proportional to the second-order splitting functions P_{gH} and P_{qH}. To ensure the complete removal of double counting and precise matching between the FE contributions and the subtraction contributions near threshold, one must use the 2-loop evolved PDF's in calculating the FE contributions.

Fig. 5 contains preliminary numerical results from this calculation, plotted alongside with those already mentioned from the earlier VFS and FFS calculations. As noted before, the μ-dependence of the LO VFS and NLO FFS calculation compensate each other. The NLO VFS result (for this particular choice of x and Q^2) lies between these complementary calculations. For decreasing μ, the NLO VFS result approaches the NLO FFS, and for increasing μ it approaches the LO VFS. The difference between the NLO VFS result and the NLO FFS is larger for the lighter charm quark than the heavier bottom quark as expected since there is more phase space over which the charm quark PDF can evolve.

These preliminary results are encouraging, and a more complete study is in progress.[16] The result should be a calculation that will provide an important test of perturbative QCD when compared with the results from HERA. Perhaps of more interest is the application of the same principles to heavy quark production in hadron-hadron collisions. As mentioned earlier, because the ratio Q/m_H (where $Q \to p_t$) is much larger there, the conventional FFS calculations are known to be unreliable. The proper implementation of the VFS calculation to the same order there is therefore potentially very useful. For completeness, in the small-x region the complementary small-x resummation may also be important. Theoretical improvement in that front would also be needed.

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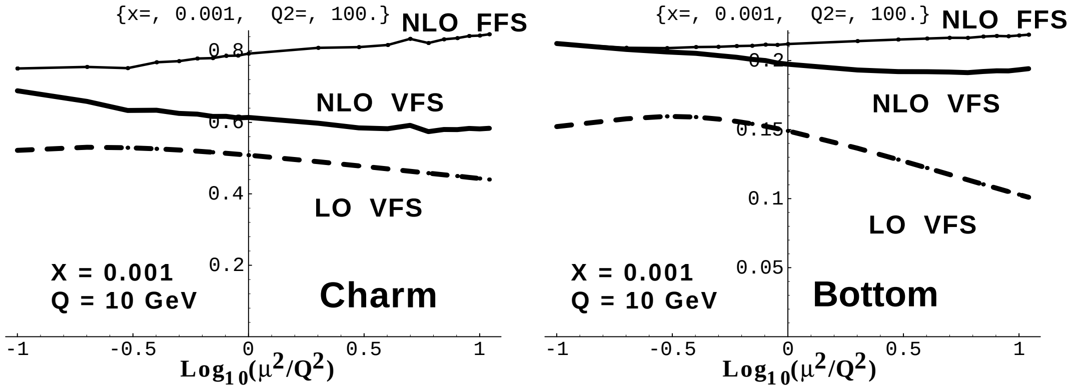

Figure 5: Scale dependence of the $F_2(x,Q)$ for charm and bottom production for $x = 0.001$, $Q = 10\,GeV$, comparing the new NLO variable flavor scheme results with existing ones.

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