Strong Constraints on the Parameter Space of the MSSM from Charge and Color Breaking Minima [*]

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Abstract

A complete analysis of all the potentially dangerous directions in the field-space of the minimal supersymmetric standard model is carried out. They are of two types, the ones associated with the existence of charge and color breaking minima in the potential deeper than the realistic minimum and the directions in the field-space along which the potential becomes unbounded from below. The corresponding new constraints on the parameter space are given in an analytic form, representing a set of necessary and sufficient conditions to avoid dangerous directions. They are very strong and, in fact, there are extensive regions in the parameter space that become forbidden. This produces important bounds, not only on the value of A, but also on the values of B and $M_{1/2}$. Finally, the crucial issue of the one-loop corrections to the scalar potential has been taken into account in a proper way.

1 Introduction

As is well known, the presence of scalar fields with color and electric charge in supersymmetric (SUSY) theories induces the possible existence of dangerous charge and color breaking (CCB) minima, which would make the standard vacuum unstable. This is not necessarily a shortcoming since many SUSY models can be discarded on these grounds, thus improving the predictive power of the theory.

This fact has been known since the early 80's [1, 2]. Since then, several interesting papers have appeared in the subject [3, 4, 5, 6]. However, a complete study of this crucial issue is still lacking. This is mainly due to two reasons. First, the enormous complexity of the scalar potential, V, in a SUSY theory, which has motivated that only analyses examining particular directions in the field–space have been performed. Second, as we will see, the radiative corrections to V have not been normally included in a proper way.

Concerning the first point, and to introduce some notation, let us write the tree-level scalar potential, V_o, in the minimal supersymmetric standard model (MSSM):

$$V_o = V_F + V_D + V_{\text{soft}} \ , \tag{1}$$

with

$$V_F = \sum_\alpha \left| \frac{\partial W}{\partial \phi_\alpha} \right|^2 \ , \tag{2a}$$

$$V_D = \frac{1}{2} \sum_a g_a^2 \left(\sum_\alpha \phi_\alpha^\dagger T^a \phi_\alpha \right)^2 \ , \tag{2b}$$

$$
\begin{aligned}
V_{\text{soft}} \ = \ & \sum_\alpha m_{\phi_\alpha}^2 |\phi_\alpha|^2 \ + \sum_{i \equiv generations} \left\{ A_{u_i} \lambda_{u_i} Q_i H_2 u_i + A_{d_i} \lambda_{d_i} Q_i H_1 d_i \right. \\
& + \ \left. A_{e_i} \lambda_{e_i} L_i H_1 e_i + \text{h.c.} \right\} + (B\mu H_1 H_2 + \text{h.c.}) \ ,
\end{aligned} \tag{2c}
$$

where W is the MSSM superpotential

$$W = \sum_{i \equiv generations} \left\{ \lambda_{u_i} Q_i H_2 u_i + \lambda_{d_i} Q_i H_1 d_i + \lambda_{e_i} L_i H_1 e_i \right\} + \mu H_1 H_2 \ , \tag{3}$$

ϕ_α runs over all the scalar components of the chiral superfields and a,i are gauge group and generation indices respectively. Q_i (L_i) are the scalar partners of the quark (lepton) $SU(2)_L$ doublets and u_i, d_i (e_i) are the scalar partners of the quark (lepton) $SU(2)_L$ singlets. In our notation $Q_i \equiv (u_L, \ d_L)_i$, $L_i \equiv (\nu_L, \ e_L)_i$, $u_i \equiv u_{R_i}$, $d_i \equiv d_{R_i}$, $e_i \equiv e_{R_i}$. Finally, $H_{1,2}$ are the two SUSY Higgs doublets. The previous potential is extremely involved since it has a large number of independent fields. Furthermore, even assuming universality of the soft breaking terms at the unification scale, M_X, it contains a large number of independent parameters: m, M, A, B, μ, i.e. the universal scalar and gaugino masses, the universal coefficients of the trilinear and bilinear scalar terms, and the Higgs mixing mass, respectively. In addition, there are the gauge (g) and Yukawa (λ) couplings which are constrained by the experimental data. Notice that M does not appear explicitly in V_o, but it does through the renormalization group equations (RGEs) of all the remaining parameters.

As mentioned above, the complexity of V has made that only particular directions in the field-space have been explored. It will be useful for us to remind here two of them. First, there is the "traditional" bound, first studied by Frere et al. and subsequently by others [1, 2]. These authors considered just the three fields present in a particular trilinear scalar coupling, e.g. $\lambda_u A_u Q_u H_2 u$, assuming equal vacuum expectation values (VEVs) for them:

$$|Q_u| = |H_2| = |u| \quad , \tag{4}$$

where only the u_L-component of Q_u takes a VEV in order to cancel the D–terms. The phases of the three fields are taken in such way that the trilinear scalar term in the potential has negative sign. Then, they showed that a very deep CCB minimum appears *unless* the famous constraint

$$|A_u|^2 \leq 3 \left(m_{Q_u}^2 + m_u^2 + m_2^2 \right) \tag{5}$$

is satisfied. In the previous equation $m_{Q_u}^2, m_u^2, m_2^2$ are the mass parameters of Q_u, u, H_2. Notice from eq.(1) that m_2^2 is the sum of the H_2 squared soft mass, $m_{H_2}^2$, plus μ^2. Similar constraints for the other trilinear terms can straightforwardly be written. These "traditional" bounds have extensively been used in the literature. The second example is due to Komatsu [5], who realized that the potential of eq.(1) along the direction

$$\begin{aligned} |L_i|^2 \quad &= |H_2|^2 + |Q_j|^2 \\ Q_j d_j \quad &= -\frac{\mu}{\lambda_{d_j}} H_2 \\ |Q_j|^2 \quad &= |d_j|^2 \quad , \end{aligned} \tag{6}$$

with L_i and Q_j VEVs taken along ν_L and d_L respectively, is unbounded from below (UFB) *unless* the constraint

$$m_2^2 - \mu^2 + m_{L_i}^2 \geq 0 \tag{7}$$

is satisfied. Komatsu claimed that for $M_{\text{top}} = 100$ GeV this constraint is extremely strong. To see this, notice that at the M_Z scale $m_2^2 - \mu^2$ is normally negative and of the same order as $m_{L_i}^2$.

Let us go now to the issue of the radiative corrections. Usually, the scalar potential is considered at tree-level, improved by one-loop RGEs, so that all the parameters appearing in it (see eq.(1)) are running with the renormalization scale, Q. Then it is demanded that the previous CCB constraints, i.e. eqs.(5), (7) and others, are satisfied at any scale between M_X and M_Z. However, as was clarified by Gamberini et al. [6], this is not correct. V_o is strongly Q–dependent and the one-loop radiative corrections to it, namely

$$\Delta V_1 = \sum_\alpha \frac{n_\alpha}{64\pi^2} M_\alpha^4 \left[\log \frac{M_\alpha^2}{Q^2} - \frac{3}{2} \right] \quad , \tag{8}$$

are crucial to make the potential stable against variations of the Q scale. In eq.(8) $M_\alpha^2(Q)$ are the improved tree-level (field–dependent) squared mass eigenstates and $n_\alpha = (-1)^{2s_\alpha}(2s_\alpha + 1)$, where s_α is the spin of the corresponding particle. Clearly, the complete one-loop potential $V_1 = V_o + \Delta V_1$ has a structure that is even far more involved than V_o (notice that ΔV_1 is a complicated function of all the scalar fields). This

makes in practice the minimization of the complete V_1 an impossible task. However, in the region of Q where ΔV_1 is small, the predictions of V_o and V_1 essentially coincide. This occurs for a value of Q of the order of the most significant M_α mass appearing in (8), which in turn depends on what is the direction in the field-space that is being analyzed. Moreover, this corresponds to the region of maximal Q–invariance of V_1 [6, 7]. Therefore, one can still work just with V_o, but with the appropriate choice of Q. In this way it was shown in ref.[6] that the apparently very strong constraint (7) was in fact extremely weak. It should be mentioned however that the analysis was performed assuming $M_{\text{top}} = M_W$. As we will see in sect.3 and sect.6, once the constraint (7) is improved and the top quark mass is set at its current value, the corresponding bound is really very restrictive.

To summarize the situation, due to the complexity of the SUSY scalar potential, only particular directions in the field-space have been considered, thus obtaining necessary but not sufficient conditions to avoid dangerous CCB minima. Furthermore, the usual lack of an optimum scale to evaluate the constraints implies that their restrictive power has been normally overestimated. E.g., eq.(5)-type constraints when (incorrectly) analyzed at M_X are very strong. The aim of this paper is to improve, and hopefully fix, this situation.

In sect.2 we review the realistic minimum that corresponds to the standard vacuum. In particular, we derive the correct scale at which the minimization of the potential has to be evaluated and summarize all the theoretical and experimental constraints that the realistic minimum must satisfy. In sect.3 we carry out a complete analysis of all the potentially dangerous directions in the field-space along which the potential can become unbounded from below, obtaining the corresponding constraints on the parameter space. The possibility of spontaneous lepton number breaking is also discussed since one of those directions involves the sneutrino. In sect.4 we perform a complete analysis of all the constraints arising from the existence of charge and color breaking minima in the potential deeper than the realistic minimum. Let us remark that the bounds obtained in this section, as well as in sect.3, are completely general and are expressed in an analytical way. Hence, they represent necessary and sufficient conditions on the parameters of the MSSM, which can also be applied to the non-universal case. The correct choice of the scale to evaluate the constraints is also discussed. The reader not interested in the precise details of the calculation of the constraints may jump over the two previous sections and go directly to sect.5, where we summarize all the previous results. In sect.6 we analize numerically how the previously found constraints restrict the whole parameter space of the MSSM. Although the "traditional" bounds evaluated at the correct scale turn out to be very weak, we will show that the new charge and color breaking constraints found here are much more important and, in fact, there are extensive regions in the parameter space which are forbidden. The unbounded from below-like constraints turn out to be even stronger. All together produces important bounds not only on the value of A, but also on the values of B and M. The conclusions are left for sect.7. Finally, the Appendix is devoted to the proof of some relevant general properties concerning CCB minima which are used throughout the paper.

2 The realistic minimum

The neutral part of the Higgs potential in the MSSM is

$$
\begin{aligned}
V_{\text{Higgs}} &= m_1^2|H_1|^2 + m_2^2|H_2|^2 - 2|m_3^2||H_1||H_2| + \frac{1}{8}(g'^2 + g_2^2)(|H_2|^2 - |H_1|^2)^2 \\
&+ \Delta V_1 \ ,
\end{aligned} \tag{9}
$$

where $m_1^2 \equiv m_{H_1}^2 + \mu^2$, $m_2^2 \equiv m_{H_2}^2 + \mu^2$, $m_3^2 \equiv -\mu B$, $g_3 = g_2 = g_1 = \sqrt{\frac{5}{3}}g'$ at M_X, and ΔV_1 is given in eq.(8). It should develope a minimum at $|H_1| = v_1$, $|H_2| = v_2$, such that $v_1^2 + v_2^2 = 2M_W^2/g_2^2$. This is the realistic minimum that corresponds to the standard vacuum. In this way the requirement of correct electroweak breaking fixes one of the five independent parameters of the MSSM (i.e. m, M, A, B, μ), say μ. Actually, for some choices of the four remaining parameters (m, M, A, B), there is no value of μ capable of producing the correct electroweak breaking. Therefore, this requirement restricts the parameter space further, as is illustrated in Fig.1 (central darked region) with a representative example. The value of the potential at the realistic minimum is

$$
V_{\text{real min}} = -\frac{1}{8}\left(g'^2 + g_2^2\right)(v_2^2 - v_1^2)^2 = -\frac{\left\{\left[\,(m_1^2 + m_2^2)^2 - 4|m_3|^4\,\right]^{1/2} - m_1^2 + m_2^2\,\right\}^2}{2\left(g'^2 + g_2^2\right)} \ . \tag{10}
$$

Note that this is the result obtained by minimizing just the tree-level part of (9). As explained in sect.1 this is correct if the minimization is performed at some sensible scale around which V_{Higgs} is Q-invariant. We have chosen for this the scale $Q = M_S$, where the predictions for $v_{1,2}$ from V_{Higgs} with and without radiative corrections coincide[1]. More precisely, the requirement $\left.\frac{\partial \Delta V_1}{\partial H_2}\right|_{Q=M_S} = 0$ gives

$$
M_S = e^{-1/2}\prod_\alpha M_\alpha^{\frac{d_\alpha M_\alpha^2}{\sum_\beta d_\beta M_\beta^2}} \tag{11}
$$

$$
d_\beta = n_\beta \frac{\partial M_\beta^2}{\partial H_2} \ . \tag{12}
$$

Note that M_S is a certain average of typical SUSY masses.

In all the previous calculation, one has to run the parameters through their respective RGEs, which depend on the value of the gauge and Yukawa couplings. The boundary conditions for these are determined by the experimental values of $\alpha_1(M_Z)$, $\alpha_2(M_Z)$, $\alpha_3(M_Z)$ and the quark masses. In particular, we take $M_{\text{top}}^{\text{phys}} = 174$ GeV as the physical (pole) top mass, which is related to the running top mass through a standard expression [8]. Actually, not for all the parameter space it is possible to choose the boundary condition of λ_{top} so that the experimental mass is reproduced because the RG infrared fixed point of λ_{top} puts an upper bound on M_{top}, namely $M_{\text{top}} \lesssim 197\sin\beta$ GeV [9], where $\tan\beta = v_2/v_1$. The corresponding restriction in the parameter space is certainly substantial as is illustrated in Fig.1 (upper and lower darked regions). Let us also mention that whenever $\tan\beta$ is not large ($\lesssim 10$), it is a good approximation

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[1]Strictly, this can only be demanded for one of the two Higgs VEVs, say v_2, but then it also occurs for v_1 with high accuracy.

to neglect the effect of the bottom and tau Yukawa couplings in the set of RGEs. We have adopted this simplification throughout the paper.

To be considered as realistic, the previous minimum has to satisfy a number of further constraints. First of all, V_{Higgs} should not be unbounded from below. Working just with the tree-level part of (9), this leads to the well-known condition

$$m_1^2 + m_2^2 \geq 2|m_3^2| \ . \tag{13}$$

Actually, (13) is automatically satisfied at $Q = M_S$, but this is not necessarily true for $Q > M_S$. If it is not, then for large VEVs of the Higgs fields ($H_{1,2} \sim Q > M_S$), the potential becomes much deeper than the realistic minimum. Hence, we must impose (13) at any $Q > M_S$ and, in particular, at $Q = M_X$. Very often the additional condition

$$m_1^2 m_2^2 - |m_3|^4 < 0 \ , \tag{14}$$

is demanded at the M_S scale to ensure that the $H_1 = H_2 = 0$ (non-electroweak-breaking) point is unstable. However, it can be checked that (14) is automatically satisfied once a realistic minimum has been found.

Second, we must be sure that the realistic minimum of the (neutral) Higgs potential is really a minimum in the whole field-space. This simply implies that all the scalar squared mass eigenvalues (charged Higgses, squarks and sleptons) must be positive. This is guaranteed for the charged Higgs fields since in the MSSM the minimum of the Higgs potential always lies at

$$H_2^+ = H_1^- = 0 \ , \tag{15}$$

but not for the rest of the sparticles. Actually, we have verified that the charged Higgs fields do not play any significant role not only for the realistic minimum, but also for any CCB direction. So, we have assumed (15) throughout the paper. Finally, we must go further and demand that all the not yet observed particles, i.e. gluino (g), charginos ($\chi^{\pm}$), neutralinos (χ^o), Higgses, squarks (q) and sleptons (l), have masses compatible with the experimental bounds. Conservatively enough, we have imposed

$$
\begin{aligned}
M_g &\geq 120 \text{ GeV} \ , \quad M_{\chi^{\pm}} \geq 45 \text{ GeV} \\
M_{\chi^o} &\geq 18 \text{ GeV} \ , \quad M_q \geq 100 \text{ GeV} \\
M_t &\geq 45 \text{ GeV} \ , \quad M_l \geq 45 \text{ GeV} \ ,
\end{aligned}
\tag{16}
$$

in an obvious notation. The effect of strengthening these bounds can be trivially incorporated to the results of the paper.

3 Improved UFB constraints

These constraints arise from directions in the field-space along which the (tree-level) potential can become unbounded from below (UFB). It is interesting to note that usually this is only true at tree-level since radiative corrections eventually raise the potential for large enough values of the fields. This is the case of UFB-2,3 directions studied below. We have already mentioned the UFB direction of eq.(6) [5], and the one in the Higgs part of the potential involving only the Higgs fields (see eq.(13)). However, as we are about to see, it is possible to do a complete clasification of *all* the potentially dangerous UFB directions and constraints in the MSSM. We will also consider the radiative corrections in a proper way by making an suitable choice of the renormalization scale (for more details see subsect.4.5).

3.1 General properties

1 It is easy to check that trilinear scalar terms cannot play a significant role along an UFB direction since for large enough values of the fields the corresponding quartic (and positive) F–terms become unavoidably larger.

2 Since all the physical masses must be positive at $Q = M_S$, the only negative terms in the (tree-level) potential that can play a relevant role along an UFB direction are[2]

$$m_2^2|H_2|^2 \ , \quad -2|m_3^2||H_1||H_2| \quad . \tag{17}$$

Therefore, any UFB direction must involve, H_2 and, perhaps, H_1. Furthermore, since the previous terms are cuadratic, all the quartic (positive) terms coming from F– and D–terms must be vanishing or kept under control along an UFB direction. This means that, in any case, besides H_2 some additional field(s) are required.

3.2 UFB constraints

Using the previous general properties we can completely clasify the possible UFB directions in the MSSM:

UFB-1

The first possibility is to play just with H_1 and H_2. Then, the relevant terms of the potential are those written in eq.(9). Obviously, the only possible UFB direction corresponds to choose $H_1 = H_2$ (up to $O(m_i)$ differences which are negligible for large enough values of the fields), so that the quartic D–term is cancelled. Thus, the (tree-level) potential along the UFB-1 direction is

$$V_{\mathrm{UFB}-1} = (m_1^2 + m_2^2 - 2|m_3^2|)|H_2|^2 \ . \tag{18}$$

The constraint to be imposed is that, for any value of $|H_2| < M_X$,

$$V_{\mathrm{UFB}-1}(Q = \hat{Q}) > V_{\mathrm{real\ min}}(Q = M_S) \ , \tag{19}$$

where $V_{\mathrm{real\ min}}$ is the value of the realistic minimum, given by eq.(10), and $V_{\mathrm{UFB}-1}$ is evaluated at an appropriate scale $\hat{Q}$. (Recall that since we are dealing with the tree-level part of the Higgs potential, this has to be computed at a correct renormalization scale.) More precisely $\hat{Q}$ must be of the same order as the most significant mass along this UFB-1 direction, which is $\hat{Q} \sim \mathrm{Max}(g_2|H_2|, \lambda_{top}|H_2|, M_S)$.

However, in this case, as already discussed in sect.2, eq.(19) is accurately equivalent to the well-known condition

$$m_1^2 + m_2^2 \geq 2|m_3^2| \tag{20}$$

evaluated at any $Q > M_S$ and, in particular, at $Q = M_X$. If this is not satisfied the potential eq.(18) is always deeper than the realistic minimum.

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[2]The only possible exception are the stop soft mass terms $m_{Q_t}^2|Q_t|^2 + m_t^2|t|^2$ since the stop masses are given by $\sim (m_{Q_t,t}^2 + M_{top}^2 \pm \ \mathrm{mixing})$, but this possibility is barely consistent with the present bounds on squark masses.

UFB-2

If we include some additional field (besides H_2, H_1), this can only be justified in order to cancel (or keep under control) the D–terms in a more efficient way than just with H_1. It is easy to see by simple inspection that the best possible choice is a slepton L_i (along the ν_L direction), since it has the lightest mass without contributing to further quartic terms in V. Consequently, the relevant potential reads

$$V = m_1^2|H_1|^2 + m_2^2|H_2|^2 - 2|m_3^2||H_1||H_2| + m_{L_i}^2|L_i|^2 + \frac{1}{8}(g'^2 + g_2^2)(|H_2|^2 - |H_1|^2 - |L_i|^2)^2 .$$
(21)

It is now straightforward to write the deepest direction along the L_i, H_1 variables, namely[3]

$$|L_i|^2 = \frac{-4m_{L_i}^2}{g'^2 + g_2^2} + |H_2|^2 - |H_1|^2 \ ,$$
(22a)

$$|H_1| = |H_2|\frac{|m_3^2|}{m_1^2 - m_{L_i}^2} = |H_2|\frac{|m_3^2|}{\mu^2} \ ,$$
(22b)

provided that

$$|m_3^2| < \mu^2$$
(23a)

$$|H_2|^2 > \frac{4m_{L_i}^2}{(g'^2 + g_2^2)\left[1 - \frac{|m_3|^4}{\mu^4}\right]} \ ,$$
(23b)

otherwise the optimum value for L_i is $L_i = 0$, and we come back to the direction UFB-1. From (21), (22a), (22b) we can write the (tree-level) potential along the UFB-2 direction

$$V_{\text{UFB-2}} = \left[m_2^2 + m_{L_i}^2 - \frac{|m_3|^4}{\mu^2}\right]|H_2|^2 - \frac{2m_{L_i}^4}{g'^2 + g_2^2} \ .$$
(24)

From (24) it might seem that the potential is unbounded from below unless

$$m_2^2 + m_{L_i}^2 - \frac{|m_3|^4}{\mu^2} \geq 0 \ .$$
(25)

However, what should be really verified is that, for any value of $|H_2| < M_X$ satisfying (23b),

$$V_{\text{UFB-2}}(Q = \hat{Q}) > V_{\text{real min}}(Q = M_S) \ ,$$
(26)

where $V_{\text{real min}}$ is the value of the realistic minimum, given by eq.(10), and $V_{\text{UFB-2}}$ is evaluated at an appropriate scale $\hat{Q}$. More precisely $\hat{Q}$ must be of the same order as the most significant mass along this UFB-2 direction, which is $\hat{Q} \sim$ Max($g_2|H_2|$, $\lambda_{top}|H_2|$, M_S).

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[3]It is trivial to check that the remaining condition in order to get a true minimum in the tree-level potential of eq.(21), $\partial V/\partial H_2 = 0$, cannot be fulfilled. This result contradicts the usual statement that can be found in the literature, namely that (tree-level) spontaneous lepton number breaking, and therefore R–parity breaking, generating a majoron is possible in SUSY without introducing additional fields, since the scalar partner of the neutrino may acquire a non–vanishing VEV [10].

This direction is dangerous not only because in general the Higgses get too large VEVs but also because the breaking of lepton number through the VEV of the sneutrino leads to the existence of a majoron already excluded by experimental results [11].

Let us finally note that the last identity of eq.(22b) relies on the equality $m_1^2 - m_{L_i}^2 = \mu^2$, which only holds under the assumption of degenerate soft scalar masses for H_1 and L_i at M_X and in the approximation of neglecting the bottom and tau Yukawa couplings in the RGEs. Otherwise, one simply must replace μ^2 by $m_1^2 - m_{L_i}^2$ everywhere in eqs.(22–25).

UFB-3

The only remaining possibility is to take $H_1 = 0$. Then, the H_1 F–term can be cancelled with the help of the VEVs of d–type squarks of a particular generation, say d_{L_j}, d_{R_j}, without contributing to further quartic terms. More precisely

$$\left|\frac{\partial W}{\partial H_1}\right|^2 = \left|\mu H_2 + \lambda_{d_j} d_{L_j} d_{R_j}\right|^2 = 0 \ . \tag{27}$$

Taking the VEVs $d_{L_j} = d_{R_j} \equiv d$, the $SU(3)$ D–term remains vanishing. The main consequence of taking these VEVs as in eq.(27) is to modify the H_2 mass term from $m_2^2|H_2|^2$ to $(m_2^2 - \mu^2)|H_2|^2$. It is important to note that this trick cannot be used if $H_1 \neq 0$, as happens in the UFB–2 direction, since then the d_{L_j}, d_{R_j} F–terms would eventually dominate. Now, in order to cancel (or keep under control) the $SU(2)_L$ and $U(1)_Y$ D–terms we need the VEV of some additional field, which cannot be H_1 for the above mentioned reason. Once again the optimum choice is a slepton L_i along the ν_L direction, as in the UFB–2 case. Consequently, the relevant potential reads

$$V = (m_2^2 - \mu^2)|H_2|^2 + (m_{Q_j}^2 + m_{d_j}^2)|d|^2 + m_{L_i}^2|L_i|^2 + \frac{1}{8}(g'^2 + g_2^2)(|H_2|^2 + |d|^2 - |L_i|^2)^2. \tag{28}$$

This was the kind of possible UFB direction first noticed in the interesting work of ref.[5] taking a particular combination of the VEVs of $H_2, d_{L_j}, d_{R_j}, L_i$ (see eq.(6)), which is not the optimum one. It is straightforward to see that the deepest direction in the field–space is

$$|L_i|^2 = \frac{-4m_{L_i}^2}{g'^2 + g_2^2} + (|H_2|^2 + |d|^2) \ , \tag{29a}$$

$$d^2 = -\frac{\mu}{\lambda_{d_j}}H_2 \ , \tag{29b}$$

provided that

$$|H_2| > \sqrt{\frac{\mu^2}{4\lambda_{d_j}^2} + \frac{4m_{L_i}^2}{g'^2 + g_2^2}} - \frac{|\mu|}{2\lambda_{d_j}} \ , \tag{30}$$

otherwise the optimum value for L_i is $L_i = 0$. Now, from (28), (29a), (29b), we can write down the (tree-level) potential along the UFB-3 direction

$$V_{\text{UFB}-3} = \left[m_2^2 - \mu^2 + m_{L_i}^2\right]|H_2|^2 + \frac{|\mu|}{\lambda_{d_j}}\left[m_{Q_j}^2 + m_{d_j}^2 + m_{L_i}^2\right]|H_2| - \frac{2m_{L_i}^4}{g'^2 + g_2^2} \ . \tag{31}$$

If H_2 does not satisfy (30), then

$$V_{\text{UFB}-3} = \left[m_2^2 - \mu^2\right] |H_2|^2 + \frac{|\mu|}{\lambda_{d_j}} \left[m_{Q_j}^2 + m_{d_j}^2\right] |H_2| + \frac{1}{8}(g'^2 + g_2^2) \left[|H_2|^2 + \frac{|\mu|}{\lambda_{d_j}}|H_2|\right]^2 .$$
$$(32)$$

Analogously to the UFB–2 case, what should be demanded is that, for any value of $|H_2| < M_X$,

$$V_{\text{UFB}-3}(Q = \hat{Q}) > V_{\text{real min}}(Q = M_S) \ , \tag{33}$$

where $V_{\text{real min}}$ is the value of the realistic minimum, given by eq.(10), and $V_{\text{UFB}-3}$ is evaluated at an appropriate scale $\hat{Q}$. In this case $\hat{Q} \sim \text{Max}\left(g_3|d|,\ \lambda_{u_j}|d|,\ g_2|H_2|,\ \lambda_{top}|H_2|,\ g_2|L_i|,\ M_S\right)$. From eqs.(31–33), it is clear that the larger λ_{d_j} the more restrictive the constraint becomes. Consequently, the optimum choice of the $d-$type squark is the third generation one, i.e. $d_j = $ sbottom. We have considered anyway the three possibilities, confirming this expectative.

Finally, it is relevant to note that the job of the d_{L_j}, d_{R_j} squarks in eq.(27) can be done by e_{L_j}, e_{R_j} sleptons with $j \neq i$ (this was not noted in ref.[5]). Then everything between eq.(27) and eq.(33) remains identical with the substitutions

$$d \rightarrow e \ , \qquad\qquad \lambda_{d_j} \rightarrow \lambda_{e_j} \ , \qquad\qquad Q_j \rightarrow L_j \ . \tag{34}$$

This is true in particular for eq.(33) and eqs.(31,32), which represent the form of the UFB-3 bound. The appropriate scale, $\hat{Q}$, to evaluate $V_{\text{UFB}-3}$ is now given by $\hat{Q} \sim \text{Max}(g_2|e|, g_2|H_2|, \lambda_{top}|H_2|, g_2|L_i|, M_S)$. For the same reasons as before the optimum choice for the e_j slepton is the third generation one, i.e. $e_j = $ stau. In fact, this turns out to be the optimum choice for the UFB-3 direction (note e.g. that the second term in eq.(31) is now proportional to the slepton masses and thus smaller) and will represent, as we will see in sect.6, the *strongest* one of *all* the UFB and CCB constraints in the parameter space of the MSSM.

This completes the UFB directions and bounds to take into account in the MSSM.

4 Improved CCB constraints

These constraints arise from the existence of charge and color breaking (CCB) minima in the potential deeper than the realistic minimum. We have already mentioned the "traditional" CCB constraint [1] of eq.(5). Other particular CCB constraints have been explored in the literature [3, 4, 5, 12]. In this section we will perform a complete analysis of the CCB minima, obtaining a set of analytic constraints that represent the necessary and sufficient conditions to avoid the dangerous ones. As we will see, for certain values of the initial parameters, the CCB constraints "degenerate" into the previously found UFB constraints since the minima become unbounded from below directions. In this sense, the following CCB constraints comprise the UFB bounds of the previous section, which can be considered as special (but extremely important as we will see in sect.6) limits of the former.

On the other hand, we will introduce the one-loop radiative corrections in a consistent way, a fact that has not been properly considered up to now. Actually, as has

been explained in the Introduction, the radiative corrections to the potential can be reasonably approximated by zero *provided that* we are evaluating the tree-level potential at the appropriate scale. Therefore, it is still possible to perform the exploration of the CCB minima by using the tree-level potential. This simplifies enormously the analysis, which otherwise would be an impossible task. At the end of the day, however, it is crucial to substitute the correct scale (for more details see subsect.4.5). This procedure will allow us also to re-evaluate the restrictive power of the "traditional" CCB constraints [4], which will be shown in sect.6.

4.1   General properties

Let us enumerate a number of general facts which are relevant when one is looking for CCB constraints in the MSSM. The proof of the properties 1, 3, 5 below is left for the Appendix, giving here intuitive arguments of their validity.

1 The most dangerous, i.e. the deepest, CCB directions in the MSSM potential involve only one particular trilinear soft term of one generation (see eq.(2c)). This can be either of the leptonic type (i.e. $A_{e_i}\lambda_{e_i}L_iH_1e_i$) or the hadronic type (i.e. $A_{u_i}\lambda_{u_i}Q_iH_2u_i$ or $A_{d_i}\lambda_{d_i}Q_iH_1d_i$). Along one of these particular directions the remaining trilinear terms are vanishing or negligible. This is because the presence of a non-vanishing trilinear term in the potential gives a net negative contribution only in a region of the field space where the relevant fields are of order A/λ with λ and A the corresponding Yukawa coupling and soft trilinear coefficient; otherwise either the (positive) mass terms or the (positive) quartic F–terms associated with these fields dominate the potential. In consequence two trilinear couplings with different values of λ cannot efficiently "cooperate" in any region of the field space to deepen the potential. Accordingly, to any optimized CCB constraint there corresponds a unique relevant trilinear coupling.

2 One cannot say a priori which trilinear coupling gives the strongest constraints. In particular, contrary to what was claimed in [4] and used in [12], it is not true that the trilinear terms with bigger Yukawa couplings are the most important ones. This is easy to understand since, although the (negative[5]) trilinear terms, e.g. $A_{u_i}\lambda_{u_i}Q_iH_2u_i$, are in principle more important for larger λ_{u_i} couplings, the (positive) quartic terms, $\lambda_{u_i}^2\{|Q_iu_i|^2+|Q_iH_2|^2+|H_2u_i|^2\}$, are more important too. So there is a balance and one cannot predict which coupling size, large or small, will give the most restrictive constraint. We have examples in both senses.

3 If the trilinear term under consideration has a Yukawa coupling $\lambda^2 \ll 1$, which occurs in all the cases except for the top, then along the corresponding deepest CCB direction the D-term must be vanishing or negligible. Although this may seem quite intuitive, some authors, particularly in ref.[4], have argued that by taking VEVs of the u_L and u_R squarks much smaller than that of H_2, and other fields VEVs being zero (so that the $SU(2)_L \times U(1)_Y$ D-term is non-vanishing), a non-trivial CCB constraint appears. The trouble of their argument is that

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[4]For a recent partial analysis of this issue using the one-loop potential, see ref.[13].

[5]Recall that the phases of the fields can always be taken so that the trilinear scalar terms in (2c) are negative.

they fix $H_1 = 0$ by hand. However, this does not occur neither in the realistic minimum nor, necessarily, in any optimized CCB direction. We have redone their analysis in this point, allowing H_1 to participate in the game. Then, one obtains a modified constraint (that substitutes the one written in eq.(23) of ref.[4]), which turns out to be equivalent to require positive physical masses for the u_L and u_R squarks (for more details see the Appendix).

4 For a given trilinear coupling under consideration there are *two* different relevant directions to explore. Next, we illustrate them taking the trilinear coupling of the first generation, $A_u\lambda_u Q_u H_2 u_R$, as a guiding example, specifying how the directions are generalized to the other couplings.

Direction a)

$$H_2, Q_u, u_R \neq 0 \tag{35a}$$

$$|d_{L_j}|^2 = |d_{R_j}|^2 \tag{35b}$$

$$d_{L_j} d_{R_j} = -\frac{\mu}{\lambda_{d_j}} H_2 \tag{35c}$$

$$H_1 = 0 \quad \text{or} \quad \text{negligible} \tag{35d}$$

$$\text{Possibly} \quad L_i \neq 0 \tag{35e}$$

where Q_u takes the VEV along the u_L direction and d_{L_j}, d_{R_j} are d–type squarks such that

$$\lambda_{d_j} \gg \lambda_u \quad , \tag{36}$$

and whose VEVs are chosen to cancel the H_1 F–term

$$\left|\frac{\partial W}{\partial H_1}\right|^2 = \left|\mu H_2 + \lambda_{d_j} d_{L_j} d_{R_j}\right|^2 = 0 \quad . \tag{37}$$

From (36) and (35c) it follows[6] that $|d_{L_j}|^2 \ll |H_2|^2, |Q_u|^2, |u_R|^2$, thus the contribution of d_{L_j}, d_{R_j} to the D–terms and the mass soft–terms is negligible. The net effect of the d_{L_j}, d_{R_j} VEVs of eqs.(35b,35c) is therefore to decrease the H_2 squared mass from m_2^2 to[7] $m_2^2 - \mu^2$. This interesting fact was first observed in ref.[5]. The same job of the d_j squarks can be done by e_{L_j}, e_{R_j} sleptons provided that $\lambda_{e_j} \gg \lambda_u$. H_1 must be very small or vanishing, [eq.(35d)], otherwise the (positive) d_{L_j} and d_{R_j} F–terms, $\lambda_{d_j}^2 \left\{|H_1 d_{R_j}|^2 + |d_{L_j} H_1|^2\right\}$, would clearly dominate the potential. Note that this is also in agreement with the mentioned property 1, i.e. along a relevant CCB direction in the field-space only one trilinear scalar coupling can be non-negligible.

In addition to $H_2, Q_u, u_R, d_{L_j}, d_{R_j}$, other fields could take extra non-vanishing VEVs, but as in the above-explained UFB-2 direction (see sect.3) and for similar reasons, it turns out that the optimum choice is $L_i \neq 0$, eq.(35e), with the VEV

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[6]The VEVs of the H_2, Q_u, u_R fields are always of order A_u/λ_u, as we will see below.

[7]Note that $m_2^2 - \mu^2$ is simply the soft mass of H_2, since in the definition of m_2^2 is also absorved the H_1 F–term, $|\mu H_2|^2$ (see sect.2).

along the ν_L direction (this was not considered in ref.[5]). As we will see, in some special cases $\nu_L \neq 0$ can be advantageously replaced by[8] $e_L \neq 0$, $e_R \neq 0$. We will not consider this possibility for the moment.

Consequently, the tree-level scalar potential along this *(a)* direction takes the form

$$
\begin{aligned}
V \;=\;& \lambda_u^2 \left\{ |H_2 u_R|^2 + |Q_u H_2|^2| + |Q_u u_R|^2 \right\} \;+\; \mathrm{D-terms} \\
+\;& m_{Q_u}^2 |Q_u|^2 + m_u^2 |u_R|^2 + (m_2^2 - \mu^2)|H_2|^2 + m_{L_i}^2 |L_i|^2 \\
+\;& (A_u \lambda_u Q_u H_2 u_R + \mathrm{h.c.}) \;\;,
\end{aligned}
\tag{38}
$$

where we have neglected the contribution of d_{L_j}, d_{R_j} to the mass and D terms.

The generalization of this *(a)* direction to other couplings different from $A_u \lambda_u Q_u H_2 u_R$ is as follows. If the trilinear term under consideration is the charm one, i.e. $A_c \lambda_c Q_c H_2 c_R$, everything works as before with the obvious replacement $u \to c$ in eqs.(35–38). For the top trilinear term, however, this direction cannot be applied, since eq.(36) cannot be fulfilled. If the trilinear term is of the $A_{d_k} \lambda_{d_k} Q_k H_1 d_k$ type, everything is similar interchanging H_2 by H_1 and u by d_k. As we will see, for these couplings the presence of an extra VEV for a slepton occurs normally along the $e_L \neq 0$, $e_R \neq 0$ direction rather than $\nu_L \neq 0$. In any case, the sleptons must be chosen from generations satisfying $\lambda_{e_i} \ll \lambda_{d_k}$ in order to make the quartic F-terms associated with them negligible (this choice is always possible). Let us also note that the above consideration for the top trilinear coupling is analogously applicable for the bottom if $\tan\beta \gtrsim 4$. Finally, the direction *(a)* is generalized to the leptonic couplings, $A_{e_k} \lambda_{e_k} L_k H_1 e_k$, in a similar way to that of the $A_{d_k} \lambda_{d_k} Q_k H_1 d_k$ couplings. Now of course the role of the possible extra leptonic VEVs must be played by other sleptons, say L_i', e_{R_i}', from a lower generation than the leptonic coupling under consideration. This excludes the possibility of extra leptonic VEVs if the latter corresponds to the electron.

Direction b)

$$
H_2, Q_u, u_R, H_1 \neq 0 \;\;,
\tag{39a}
$$

$$
\mathrm{Possibly} \quad L_i \neq 0 \;\;,
\tag{39b}
$$

where Q_u takes the VEV along the u_L direction. Note that, according to the general property 1 (see also Appendix), once we allow H_1 to participate in the game, as reflected in eq.(39a), the remaining squark and slepton fields, apart from those involved in the trilinear coupling, must be vanishing. The only possible exception is again a slepton L_i VEV along the $\nu_L \neq 0$ direction. (Or, in some special cases, along the $e_L, e_R \neq 0$ direction. Then, since $H_1 \neq 0$, the associated Yukawa coupling must satisfy $\lambda_{e_i} \ll \lambda_u$ in order not to generate extra quartic F-terms; this requires $\tan\beta \lesssim 3$.)

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[8] e_L, e_R can be chosen from different generations in order to avoid the appearance of extra quartic F-terms. Alternatively, if $\lambda_u \gg \lambda_e$ (as happens if the lepton is of the first generation and $\tan\beta \lesssim 3$) these new F-terms are negligible. Working under the assumption of universality of the soft terms both choices are equivalent.

Therefore, along the *(b)* direction the tree-level scalar potential takes the form

$$
\begin{aligned}
V \;=\; & \lambda_u^2 \left\{ |H_2 u_R|^2 + |Q_u H_2|^2 + |Q_u u_R|^2 \right\} \\
& + \; (\mu \lambda_u Q_u u_R H_1^* + \text{h.c.}) \; + \; D - \text{terms} \\
& + \; m_{Q_u}^2 |Q_u|^2 + m_u^2 |u_R|^2 + m_2^2 |H_2|^2 + m_1^2 |H_1|^2 + m_{L_i}^2 |L_i|^2 \\
& + \; (A_u \lambda_u Q_u H_2 u_R + \text{h.c.}) \; + \; (\mu B H_1 H_2 + \text{h.c.}) \quad .
\end{aligned}
\tag{40}
$$

Notice that $(\mu \lambda_u Q_u u_R H_1^* + \text{h.c.})$ is a piece of the H_2 F-term, $|\partial W / \partial H_2|^2$. Recall that the $|\mu H_1|^2$ piece of this F-term has been absorbed in the definition of m_1^2 (see sect.2).

The direction *(b)* is generalized to the other trilinear couplings in a similar way as it was done for direction *(a)*. Let us mention that when dealing with these remaining couplings there are no restrictions at all on the value of $\tan \beta$. From previous arguments, for the top coupling the direction *(b)* is the only one to be taken into account.

5 Let us finally comment on the choice of the phases of the various fields involved in the previous *(a)* and *(b)* directions. Again, we continue using the trilinear coupling of the first generation $A_u \lambda_u Q_u H_2 u_R$ as a guiding example, but the following statements are trivially generalized to the other couplings.

If $H_1 = 0$, i.e. direction *(a)*, it is easy to see from (38) that the only term in the potential without a well-defined phase is the trilinear scalar term. Obviously, the fields involved in the coupling can take phases so that it becomes negative without altering other terms in (38). This clearly corresponds to the deepest direction in the field-space. Then, in eq.(38), we can write the trilinear term as

$$
-2 |A_u \lambda_u Q_u H_2 u_R| \quad .
\tag{41}
$$

If $H_1 \neq 0$ (direction *(b)*) there are clearly three terms in the potential of eq.(40) whose phases are in principle undetermined. These can be written as

$$
2 |A_u \lambda_u Q_u H_2 u_R| \cos \varphi_1 \; + \; 2 |\mu \lambda_u Q_u H_1 u_R| \cos \varphi_2 \; + \; 2 |\mu B H_1 H_2| \cos \varphi_3,
\tag{42}
$$

where φ_i are obvious combinations of the signs of A_u, B, μ, λ_u and the phases of the fields. Note that φ_1, φ_2, φ_3 are correlationated parameters. Now, it can be shown (see Appendix) that

- If $\text{sign}(A_u) = -\text{sign}(B)$, the three terms can be made negative simultaneously, so that after a convenient redefinition of the fields we can take $\varphi_1 = \varphi_2 = \varphi_3 = \pi$.

- If $\text{sign}(A_u) = \text{sign}(B)$ the previous choice is no longer possible. Then, for the vaste majority of the cases the deepest direction in the $(\varphi_1, \varphi_2, \varphi_3)$ space corresponds to take $\varphi_i = \varphi_j = \pi$, $\varphi_l = 0$, where φ_l corresponds to the smallest term (in absolute value) in eq.(42) and φ_i, φ_j are the other two angles. For the remaining cases this always corresponds to a direction very close to the deepest one.

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4.2 CCB constraints associated with the $Q_u H_2 u_R$ coupling

Using the previous general properties it is possible to completely classify the CCB constraints in the MSSM. According to property 1, there can only be one relevant trilinear coupling associated to an optimized CCB constraint. Now, as we did in the previous property 4, we will take the trilinear coupling of the first generation, $A_u \lambda_u Q_u H_2 u_R$, as a guiding example to explain the associated CCB bounds, specifying how they are generalized to the other couplings.

The bounds arise from the previously expounded *(a)* and *(b)*–directions, see eqs.(35) and (39) respectively. For a given choice of the initial parameters $m, M, A, B, \mu, \lambda_{top}$, compatible with electroweak breaking and $M_{\text{top}}^{\text{exp}}$, one can in principle write down the scalar potential (either eq.(38) or eq.(40)) at any scale and directly minimize it with respect to the scalar fields involved. Then, the possible CCB minima arising should be compared to the realistic minimum (10) in order to decide what is the deepest one. Of course, all this should be performed at the correct scale in order to incorporate the radiative corrections properly (recall that this scale depends itself on what are the relevant VEVs of the fields at the CCB minimum under consideration).

Unfortunately, despite the form of the potential in eqs.(38), (40) is much simpler than the general expression of eq.(1), it is still not possible to implement the previous program in a complete analytical way. The outcoming equations are in general so involved that they become useless for practical purposes. Alternatively, one could follow a numerical procedure, trying to find out (for each choice of the initial parameters) the corresponding CCB minima. This is, however, quite dangerous since there is still a considerable number of independent variables and the minima usually emerge from subtle cancellations between different terms, something that can easily escape a standard program of numerical minimization. In addition, with the numerical approach the final form for the CCB bounds is very uneasy to handle and we lose the track of the physical reasons behind it. Fortunately, it becomes now feasible to go quite far in the analytic examination of the general CCB minima, in some cases until the very end of the analysis, thus obtaining very useful constraints expressed in an analytical way. This is the kind of approach we have followed in the paper. As we will see, the final implementation of these constraints usually requires a complementary, but trivial, numerical task, namely the scanning of a certain variable in the range [0,1].

In order to write the CCB constraints it is helpful to express the various VEVs in terms of the H_2 one, using the following notation [4]

$$
\begin{aligned}
|Q_u| &= \alpha |H_2| \,, \quad |u_R| = \beta |H_2| \,, \\
|H_1| &= \gamma |H_2| \,, \quad |L_i| = \gamma_L |H_2| \,.
\end{aligned} \tag{43}
$$

E.g. the "traditional" direction, eq.(4), is recovered for the particular values $\alpha = \beta = 1$, $\gamma = \gamma_L = 0$.

We shall write now the form of the potential for the directions *(a)*, *(b)*, obtaining from its minimization the general form of the CCB bounds. It is convenient for this task to start with the *(b)* direction in the sign$(A_u) = -$sign(B) case, extending at the end the results to the sign$(A_u) = $sign$(B)$ case and to the *(a)* direction. The scalar potential along the direction *(b)*, see eq.(40), can be expressed as

$$
V = \lambda_u^2 F(\alpha, \beta, \gamma, \gamma_L) \alpha^2 \beta^2 |H_2|^4 - 2\lambda_u \hat{A}(\gamma) \alpha \beta |H_2|^3 + \hat{m}^2(\alpha, \beta, \gamma, \gamma_L) |H_2|^2 \,, \tag{44}
$$

where

$$F(\alpha,\beta,\gamma,\gamma_L) = 1 + \frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{f(\alpha,\beta,\gamma,\gamma_L)}{\alpha^2\beta^2} \ ,$$

$$f(\alpha,\beta,\gamma,\gamma_L) = \frac{1}{\lambda_u^2}\left\{\frac{1}{8}g_2^2\left(1 - \alpha^2 - \gamma^2 - \gamma_L^2\right)^2\right.$$

$$\left. + \frac{1}{8}g'^2\left(1 + \frac{1}{3}\alpha^2 - \frac{4}{3}\beta^2 - \gamma^2 - \gamma_L^2\right)^2 + \frac{1}{6}g_3^2\left(\alpha^2 - \beta^2\right)^2\right\} \ ,$$

$$\hat{A}(\gamma) = |A_u| + |\mu|\gamma \ ,$$

$$\hat{m}^2(\alpha,\beta,\gamma,\gamma_L) = m_2^2 + m_{Q_u}^2\alpha^2 + m_u^2\beta^2 + m_1^2\gamma^2 + m_{L_i}^2\gamma_L^2 - 2|m_3^2|\gamma \ . \tag{45}$$

(The L_i VEV has been taken along the direction ν_L since otherwise the D–terms cannot be eventually cancelled.) Then, minimizing V with respect to $|H_2|$ for fixed values of $\alpha,\beta,\gamma,\gamma_L$, we find, besides the $|H_2| = 0$ extremal (all VEVs vanishing), the following CCB solution

$$|H_2|_{ext} = |H_2(\alpha,\beta,\gamma,\gamma_L)|_{ext} = \frac{3\hat{A}}{4\lambda_u\alpha\beta F}\left\{1 + \sqrt{1 - \frac{8\hat{m}^2 F}{9\hat{A}^2}}\right\} \ . \tag{46}$$

It is easy to check that the solution with a minus sign in front of the square root in the previous equation corresponds to a maximum. Let us note that, as was stated above (see property 1 and footnote 6), the typical VEVs at a CCB minimum are indeed of order A/λ. The corresponding value of the potential is

$$V_{\text{CCB min}} = -\frac{1}{2}\alpha\beta|H_2|_{ext}^2\left(\hat{A}\lambda_u|H_2|_{ext} - \frac{\hat{m}^2}{\alpha\beta}\right) \ . \tag{47}$$

Eqs.(44–47) generalize those obtained in ref.[4].

Since the trilinear term of our guiding example has small coupling, $\lambda_u^2 \ll 1$, according to the above property 3 the D–terms should vanish. This implies

$$\alpha^2 - \beta^2 = 0 \ , \tag{48a}$$

$$1 - \alpha^2 - \gamma^2 - \gamma_L^2 = 0 \ . \tag{48b}$$

As a consequence $f(\alpha,\beta,\gamma,\gamma_L)$ becomes vanishing and $F = 1 + \frac{2}{\alpha^2}$. Let us note that eq.(48b) can only be fulfilled if $1 - \alpha^2 - \gamma^2 \geq 0$. In fact, playing only with the H_2, Q_u, u_R, H_1, L_i fields this is a necessary condition to cancel the D–terms. If $1 - \alpha^2 - \gamma^2 < 0$ the cancellation can only be achieved by including additional fields. By inspection, the best choice is to take the L_i VEV along the e_L direction plus an additional VEV $e_{R_j} = e_{L_i}$. Then the D–terms are cancelled and eq.(48b) becomes

$$1 - \alpha^2 - \gamma^2 + \gamma_L^2 = 0 \ . \tag{49}$$

In this case one has to replace $m_{L_i}^2$ by $m_{L_i}^2 + m_{e_j}^2$ in the definition of $\hat{m}^2$, eq.(45). We will not consider this possibility for the moment postponing for later the discussion of the only situation in which it could be relevant.

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The previous CCB minimum, eq.(47), will be negative[9] unless $\hat{A}^2 \leq F\hat{m}^2$, i.e.

$$(|A_u| + |\mu|\gamma)^2 \leq \left(1 + \frac{2}{\alpha^2}\right)\left[m_2^2 + (m_{Q_u}^2 + m_u^2)\alpha^2 + m_1^2\gamma^2 + m_{L_i}^2\gamma_L^2 - 2|m_3^2|\gamma\right] \quad (50)$$

where for convenience we have explicitly kept the dependence in the three variables α, γ, γ_L, which are subject to eq.(48b). Since $\lambda_u^2 \ll 1$, if (50) were not satisfied the corresponding CCB minimum of eq.(47) would be much deeper ($\propto -1/\lambda_u^2$) than the realistic one ($\propto -1/g_2^2$), eq.(10). Consequently, eq.(50) is the *general form* of the CCB bound for the *(b)*–direction when $\text{sign}(A_u) = -\text{sign}(B)$ and the Yukawa coupling is much smaller than one, as it is the case at hand. Let us remark that (50) should be satisfied for *any* choice of α, γ, γ_L obeying eq.(48b). E.g. the "traditional" bound, eq.(5), is recovered for the particular choice $\alpha = 1, \gamma = \gamma_L = 0$.

When $\text{sign}(A_u) = \text{sign}(B)$ one of the three terms $\{|A_u|, |\mu|\gamma, -2|m_3^2|\gamma\}$ in eqs.(45, 50) must flip the sign (see property 5 of the previous subsection).

For the *(a)*–direction all the equations (44-50) hold making $\gamma = 0$, $m_2^2 \to m_2^2 - \mu^2$. In particular eq.(50) with these replacements, i.e.

$$|A_u|^2 \leq \left(1 + \frac{2}{\alpha^2}\right)\left[m_2^2 - \mu^2 + (m_{Q_u}^2 + m_u^2)\alpha^2 + m_{L_i}^2\gamma_L^2\right] , \quad (51)$$

represents the general form of the CCB bounds for direction *(a)*.

Clearly, the strongest CCB constraints from (50) and (51) arise for particular values of α, γ, γ_L, which, in turn, depend on what are the values of various parameters involved in the expressions. This allows us to be more explicit about the final analytical form of the CCB constraints and to classify them below:

CCB-1

This bound arises by considering the direction *(a)* and thus the general condition (51). Then the strongest constraint is obtained by minimizing the right hand side of (51) with respect to α, keeping $\gamma_L^2 = 1 - \alpha^2$. This gives the following

1. If $m_2^2 - \mu^2 + m_{L_i}^2 > 0$ and $3m_{L_i}^2 - (m_{Q_u}^2 + m_u^2) + 2(m_2^2 - \mu^2) > 0$, then the optimized CCB-1 bound occurs for $\alpha = 1$, $\gamma_L = 0$, i.e.

$$|A_u|^2 \leq 3\left[m_2^2 - \mu^2 + m_{Q_u}^2 + m_u^2\right] \quad (52)$$

2. If $m_2^2 - \mu^2 + m_{L_i}^2 > 0$ and $3m_{L_i}^2 - (m_{Q_u}^2 + m_u^2) + 2(m_2^2 - \mu^2) < 0$, then the optimized bound occurs for $\alpha, \gamma_L \neq 0$, namely

$$|A_u|^2 \leq \left(1 + \frac{2}{\alpha^2}\right)\left[m_2^2 - \mu^2 + (m_{Q_u}^2 + m_u^2)\alpha^2 + m_{L_i}^2(1 - \alpha^2)\right] \quad (53)$$

with $\alpha^2 = \sqrt{\frac{2(m_{L_i}^2 + m_2^2 - \mu^2)}{m_{Q_u}^2 + m_u^2 - m_{L_i}^2}}$, $\gamma_L^2 = 1 - \alpha^2$.

3. If $m_2^2 - \mu^2 + m_{L_i}^2 < 0$, then the CCB-1 bound is automatically violated since there are many values of α that make the right hand side of (51) negative. In fact the minimization of the potential in this case gives $\alpha^2 \to 0$, and we are exactly led to the UFB-3 direction explained in sect.3, which represents the correct analysis in this instance.

---

[9] The mere existence of a CCB minimum is discarded by demanding $\hat{A}^2 < (8/9)F\hat{m}^2$, see eq.(46).

Let us mention that the bound (52) was first obtained in ref.[5]. However it seldom represents the optimized bound, as long as the condition for this (see above eq.(52)) will not normally be satisfied. Hence, eq.(53) will usually represent the (optimized) CCB-1 bound. Needless to say that the CCB-1 bound is always stronger than the "traditional" CCB bounds [1], see eq.(5).

Finally, in the very unlikely case that $3(m_{L_i}^2 + m_{e_j}^2) + (m_{Q_u}^2 + m_u^2) - 2(m_2^2 - \mu^2) < 0$, which only can take place in (very strange) non-universal cases, then the CCB-1 bound would be given by

$$|A_u|^2 \leq \left(1 + \frac{2}{\alpha^2}\right) \left[m_2^2 - \mu^2 + (m_{Q_u}^2 + m_u^2)\alpha^2 + (m_{L_i}^2 + m_{e_j}^2)(\alpha^2 - 1)\right] \qquad (54)$$

with $\alpha^2 = \sqrt{\frac{2(m_2^2 - \mu^2 - m_{L_i}^2 - m_{e_j}^2)}{m_{Q_u}^2 + m_u^2 + m_{L_i}^2 + m_{e_j}^2}}$, $\gamma_L^2 = \alpha^2 - 1$.

CCB-2

This bound arises from direction *(b)*, i.e. $\gamma \neq 0$, when $\text{sign}(A_u) = -\text{sign}(B)$. The corresponding CCB constraint is given by (50) with $\gamma_L^2 = 1 - \alpha^2 - \gamma^2$, that is

$$(|A_u| + |\mu|\gamma)^2 \leq \left(1 + \frac{2}{\alpha^2}\right) \left[m_2^2 + (m_{Q_u}^2 + m_u^2)\alpha^2 + m_1^2\gamma^2 \right.$$
$$\left. + m_{L_i}^2(1 - \alpha^2 - \gamma^2) - 2|m_3^2|\gamma\right] \qquad (55)$$

which should be handled in the following way:

1. Scan γ in the range $0 \leq \gamma \leq 1$

2. For each value of γ the optimum value of α^2, i.e. the one that minimizes the right hand side of (55), is in principle given by

$$\alpha_{ext}^4 = \frac{2\left[m_2^2 + m_1^2\gamma^2 + m_{L_i}^2(1 - \gamma^2) - 2|m_3^2|\gamma\right]}{m_{Q_u}^2 + m_u^2 - m_{L_i}^2} \qquad (56)$$

   Under the assumption of universality the denominator of (56) is always positive. On the other hand, the numerator should also be positive, otherwise the optimum value of α is $\alpha \to 0$ and we are exactly led to the UFB-2 direction explained in sect.3.

3. If $\alpha_{ext}^2 < 1 - \gamma^2$, then α_{ext}^2 is indeed the optimum value of α^2 to be substituted in (55).

4. If $\alpha_{ext}^2 > 1 - \gamma^2$, then the D–terms cannot be cancelled with $\alpha = \alpha_{ext}$ [see eq.(48b)]. This could be in principle circumvected by including a VEV for the e_{R_j} slepton, as explained around eq.(49). Then $\gamma_L^2 = \alpha^2 + \gamma^2 - 1$ and the $m_{L_i}^2(1 - \alpha^2 - \gamma^2)$ term in (55) must be replaced by $(m_{L_i}^2 + m_{e_j}^2)(\alpha^2 + \gamma^2 - 1)$. The new optimum value of α_{ext} would be in principle given by

$$\alpha_{ext}'^4 = \frac{2\left[m_2^2 + m_1^2\gamma^2 - (m_{L_i}^2 + m_{e_j}^2)(1 - \gamma^2) - 2|m_3|^2\gamma\right]}{m_{Q_u}^2 + m_u^2 + m_{L_i}^2 + m_{e_j}^2} \qquad (57)$$

   If $\alpha_{ext}'^2 > 1 - \gamma^2$, then $\alpha_{ext}'^2$ is indeed the optimum value of α^2 to be substituted in (55) together with the previous replacements. If $\alpha_{ext}'^2 < 1 - \gamma^2$, then the optimum value of α^2 is simply $\alpha^2 = 1 - \gamma^2$ (which is equivalent to $\gamma_L = 0$), which should be substituted in (55).

This bound, that also arises from direction *(b)*, is to be applied when $\text{sign}(A_u) = \text{sign}(B)$. It takes exactly the same form as the CCB-2 one (see above), but flipping the sign of one of the three terms $\{|A_u|, |\mu|\gamma, -2|m_3^2|\gamma\}$ in (55). Notice that, due to the form of (55) flipping the sign of $|A_u|$ or the sign of $|\mu|\gamma$ leads to the same result. Therefore, there are only two choices to examine: the first one writing $(|A_u| - |\mu|\gamma)^2$ in the left hand side of (55), the second one writing $+2|m_3^2|\gamma$ in the right hand side of (55) and hence in those of (56) and (57).

(Since one cannot know a priori what of the terms listed in eq.(42) is going to have the smallest absolute value at the CCB minimum, one cannot be sure from the beginning which one of the two choices will be the optimum one. Consequently, the fastest way to handle this is simply to perform the examination twice.)

Let us finish this subsection by noting that none of the previous CCB bounds depend on the size of the Yukawa coupling λ_u (except for the fact that $\lambda_u \ll 1$ has been assumed). However this fact will change as soon as we estimate the appropriate scale, Q, to evaluate them because the size of the tipical VEVs in the CCB minimum does depend on λ_u, see eq.(46). This issue will be examined in subsect.4.5.

4.3 Generalization to other couplings

The previous bounds CCB-1 – CCB-3 can be straightforwardly generalized to all the couplings with coupling constant $\lambda \ll 1$. This includes all the couplings apart from the top. There are however slight differences depending on the Higgs field (H_1 or H_2) involved in the coupling. Thus we expose the various generalizations in a separate way.

$\lambda_c Q_c H_2 c_R$

The CCB constraints associated with this coupling have exactly the same form as those for the $\lambda_u Q_u H_2 u_R$ coupling, i.e. the CCB-1 – CCB-3 bounds, with the obvious replacement $u \to c$.

$\lambda_d Q_u H_1 d_R$, $\lambda_s Q_c H_1 s_R$, $\lambda_b Q_t H_1 b_R$

When dealing with these couplings it is convenient to change the notation (43), expressing all the VEVs in terms of the H_1 one, i.e.

$$|Q_u|, |Q_c| \text{ or } |Q_t| = \alpha|H_1|, \quad |d_R|, |s_R| \text{ or } |b_R| = \beta|H_1|,$$
$$|H_2| = \gamma|H_1|, \quad |L_i| = \gamma_L|H_1|, \tag{58}$$

where Q_u, Q_c, Q_t take the VEVs along the d_L, s_L, b_L directions respectively. Then, all the results and equations of subsect.4.2, from eq.(44) until the end of the subsection, hold with the following replacements everywhere

$$H_1 \leftrightarrow H_2, \quad m_{L_i}^2 \leftrightarrow (m_{L_i}^2 + m_{e_j}^2),$$
$$m_1 \leftrightarrow m_2, \quad u \to d, s \text{ or } b. \tag{59}$$

Note in particular that if $1 - \alpha^2 - \gamma^2 > 0$, the cancellation of the D–terms requires equal VEVs for L_i (along the e_L direction) and e_{R_j}, while if $1 - \alpha^2 - \gamma^2 < 0$ the D–terms

can be cancelled just with $L_i \neq 0$ (along the ν_L direction). This works exactly in the opposite way to that of the $\lambda_u Q_u H_2 u_R$ case.

The modifications in the CCB-1 – CCB-3 bounds can be straightforwardly obtained. They remain the same with the previous eq.(59) substitutions.

$\lambda_e \mathbf{L_e H_1} e_\mathbf{R}$, $\lambda_\mu \mathbf{L_\mu H_1} \mu_\mathbf{R}$, $\lambda_\tau \mathbf{L_\tau H_1} \tau_\mathbf{R}$

The CCB bounds from these couplings have essentially the same form as the just mentioned d-type ones. All the results and equations of subsect.4.2, from eq.(44) until the end of the subsection, hold with the following replacements

$$
\begin{aligned}
H_1 &\leftrightarrow H_2 , & m_1 &\leftrightarrow m_2 \\
Q &\rightarrow L , & m_{L_i}^2 &\rightarrow (m_{L_i'}^2 + m_{e_j'}^2) , \\
u &\rightarrow e, \mu \text{ or } \tau , & (m_{L_i}^2 + m_{e_j}^2) &\rightarrow m_{L_i'}^2 .
\end{aligned}
\tag{60}
$$

Then L_e, L_μ, L_τ take the VEVs along the e_L, μ_L, τ_L directions respectively. The role of the sleptons L_i, e_{R_j} in the previous subsection is played now by two sleptons L_i', e_{R_j}' of a different generation than the trilinear coupling under consideration. In the bounds where both L_i' (along the direction e_L') and e_{R_j}' take non-vanishing VEVs, the associated Yukawa coupling, say λ_l', must be much smaller than the Yukawa coupling of the trilinear coupling under consideration, say λ_l, in order to avoid the appearance of large F–terms. Obviously this condition can always be satisfied except when the coupling under consideration is of the first generation (i.e. the electron one). Then this kind of extra VEVs cannot be used, so the optimum value for the "prime" sleptons is $e_L' = e_R' = 0$, i.e. $\gamma_L = 0$.

Under the assumption of universality it is easy to see that the CCB-1 bound will only take place in the possibility 1 [see condition above eq.(52)], while the CCB-2, CCB-3 bounds will always occur in the possibility 4 (note that the denominator of eq.(56) goes to zero).

4.4   The case of the top

Much of the expounded in subsect.4.2 about the $\lambda_u Q_u H_2 u_R$ coupling is still valid for the top one. More precisely, the eqs.(43–47) *hold* with the replacement $u \rightarrow t$. However, the top trilinear coupling represents a special case due to have the largest Yukawa coupling constant, λ_t. This is reflected in the three following differences:

- The D-terms along an optimized CCB direction are no longer vanishing or negligible, since $\lambda_t = O(1)$, which implies that the D–terms and the F–terms have orders of magnitude comparable [see property 3 in sect.4.1]. Consequently, eqs.(48) or (49) should not be imposed now.

- The direction *(a)* specified in eqs.(35) is no longer applicable due to the absence of d–type squarks such that $\lambda_{d_j} \gg \lambda_t$. Consequently, the only direction to take into account is the *(b)* one, eqs.(39), and the CCB-1 bound does not apply to the top case.

- Since $\lambda_t = O(1)$ it is no longer true that a negative minimum ($\propto -1/\lambda_t^2$) associated to the top trilinear coupling is necessarily much deeper than the realistic

minimum ($\propto -1/g_2^2$), thus destabilizing the standard vacuum, as can be easily seen by examining eqs.(46,47,10). Therefore, rather than the absence of a negative minimum, we must demand that the possible CCB minimum satisfies $V_{\text{CCB min}} > V_{\text{real min}}$, where $V_{\text{CCB min}}, V_{\text{real min}}$ are given by eqs.(47),(10).

In the following we will still use the $SU(3)$ D–term cancellation condition

$$|Q_t|^2 = |t_R|^2 \;\rightarrow\; \alpha^2 = \beta^2 \;, \tag{61}$$

taking the VEV of Q_t along t_L. This particular direction proves to be very close to the deepest one, simplifying substantially the subsequent analysis. The analogous approximation for the $SU(2) \times U(1)_Y$ D–terms is, however, not good (this comes from the smaller size of the associated gauge couplings), so we will allow them to be non-vanishing.

Since we have to analyze the potential along the direction (b), we must keep in mind that there are two different scenarios depending on the relative sign of A_t and B, see property 5 in subsect.4.1. In the following we will assume $\text{sign}(A_t) = -\text{sign}(B)$, which represents the simplest case. The extension of the results to the $\text{sign}(A_t) = \text{sign}(B)$ case is trivial and will be given at the end.

From (44, 45) we can optimize the value of $\gamma_L = |L_i|/|H_2|$. This is given by

$$(\gamma_L^2)_{ext} = 1 - \gamma^2 - \alpha^2 - \frac{4m_{L_i}^2}{(g'^2 + g_2^2)|H_2|^2} \;. \tag{62}$$

Notice that this value is only acceptable if $(\gamma_L^2)_{ext} > 0$, which, as we shall see, will have to be checked at the end of the examination. Assuming for the time being that indeed $(\gamma_L^2)_{ext} > 0$, the potential (with $\gamma_L = (\gamma_L)_{ext}$) is given from eq.(44) by

$$V = \lambda_t^2 F'(\alpha)\alpha^4|H_2|^4 - 2\lambda_t \hat{A}'(\gamma)\alpha^2|H_2|^3 + \hat{m}'^2(\alpha,\gamma)|H_2|^2 - \frac{2m_{L_i}^4}{g'^2 + g_2^2} \;, \tag{63}$$

with

$$
\begin{aligned}
F'(\alpha) &= 1 + \frac{2}{\alpha^2} + \frac{f'}{\alpha^4} \;;\; f' = 0 \;, \\
\hat{A}'(\gamma) &= |A_t| + |\mu|\gamma \;, \\
\hat{m}'^2(\alpha,\gamma) &= m_2^2 + (m_{Q_t}^2 + m_t^2)\alpha^2 + m_1^2\gamma^2 + m_{L_i}^2(1 - \alpha^2 - \gamma^2) - 2|m_3^2|\gamma. \tag{64}
\end{aligned}
$$

This can be handled in the following way:

1. Scan γ in the range $0 \le \gamma \le 1$

2. For each value of γ the optimum values of α^2, H_2 i.e. the ones that minimize the right hand side of (63), are given by

$$\alpha_{ext}^2 = \frac{\hat{A}'(\gamma)}{\lambda_t |H_2|_{ext}} - 1 - \frac{m_{Q_t}^2 + m_t^2 - m_{L_i}^2}{2\lambda_t^2 |H_2|_{ext}^2} \;, \tag{65}$$

$$|H_2|_{ext} = \frac{3\hat{A}'(\gamma)}{4\lambda_t \alpha_{ext}^2 F'(\alpha_{ext})} \left\{ 1 + \sqrt{1 - \frac{8\hat{m}'^2(\alpha_{ext},\gamma)F'(\alpha_{ext})}{9\hat{A}'^2(\gamma)}} \right\} \;. \tag{66}$$

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For each value of γ the coupled equations (65), (66) can be solved, e.g. by a numerical method. Then, the consistency of the procedure requires

$$\alpha_{ext}^2 > 0, \quad |H_2|_{ext} > 0, \quad (\gamma_L^2)_{ext} > 0 \ , \tag{67}$$

where $\alpha_{ext}, |H_2|_{ext}, (\gamma_L)_{ext}$ are given by eqs.(65), (66) and (62) respectively.

If (67) is fulfilled, then the corresponding value of the potential at the minimum is given by

$$V_{\text{CCB min}} = -\frac{1}{2}\alpha_{ext}^2 |H_2|_{ext}^2 \left(\lambda_t \hat{A}'(\gamma)|H_2|_{ext} - \frac{\hat{m}'^2(\alpha_{ext},\gamma)}{\alpha_{ext}^2} \right) - \frac{2m_{L_i}^4}{g'^2 + g_2^2} \ . \tag{68}$$

This value will be negative unless $\hat{A}'^2 \leq F'\hat{m}'^2$, i.e.

$$(|A_t| + |\mu|\gamma)^2 \ \leq \ \left(1 + \frac{2}{\alpha^2} \right) \left[m_2^2 + (m_{Q_t}^2 + m_t^2)\alpha^2 + m_1^2\gamma^2 \right.$$
$$\left. + m_{L_i}^2(1 - \alpha^2 - \gamma^2) - 2|m_3^2|\gamma \right] \ . \tag{69}$$

E.g. the "traditional" CCB bound of the type of eq.(5) is recovered for the particular choice $\alpha = 1, \gamma = 0$. However, as mentioned above, a negative minimum associated to the top trilinear coupling is not necessarily deeper than the realistic minimum. Consequently, the CCB bound to be imposed has the form

$$V_{\text{CCB min}} > V_{\text{real min}} \ , \tag{70}$$

where $V_{\text{CCB min}}$ and $V_{\text{real min}}$ are given by eqs.(68) and (10) respectively.

3. If (67) is not fulfilled, this means that there is no CCB minimum with $\gamma_L = (\gamma_L)_{ext}$. Then, necessarily, the optimum value of γ_L is

$$\gamma_L = 0 \ , \tag{71}$$

which implies

$$V = \lambda_t^2 F'(\alpha,\gamma)\alpha^4 |H_2|^4 - 2\lambda_t \hat{A}'(\gamma)\alpha^2 |H_2|^3 + \hat{m}'^2(\alpha,\gamma)|H_2|^2 \ , \tag{72}$$

The optimum values of α, H_2 are now given by

$$\alpha_{ext}^2 = \frac{8\lambda_t^2}{g'^2 + g_2^2 + 8\lambda_t^2} \left[\frac{\hat{A}'(\gamma)}{\lambda_t |H_2|_{ext}} - 1 - \frac{m_{Q_t}^2 + m_t^2}{2\lambda_t^2 |H_2|_{ext}^2} + \frac{g'^2 + g_2^2}{8\lambda_t^2}(1 - \gamma^2) \right] \tag{73}$$

$$|H_2|_{ext} = \frac{3\hat{A}'(\gamma)}{4\lambda_t \alpha_{ext}^2 F'(\alpha_{ext},\gamma)} \left\{ 1 + \sqrt{1 - \frac{8\hat{m}'^2(\alpha_{ext},\gamma)F'(\alpha_{ext},\gamma)}{9\hat{A}'^2(\gamma)}} \right\} \ . \tag{74}$$

with

$$F'(\alpha,\gamma) \ = \ 1 + \frac{2}{\alpha^2} + \frac{f'}{\alpha^4} \ ,$$
$$f' \ = \ \frac{g'^2 + g_2^2}{8\lambda_t^2}\left(1 - \alpha^2 - \gamma^2 \right)^2 \ ,$$
$$\hat{m}'^2(\alpha,\gamma) \ = \ m_2^2 + \left(m_{Q_t}^2 + m_t^2 \right)\alpha^2 + m_1^2\gamma^2 - 2|m_3^2|\gamma \ . \tag{75}$$

Consistency now requires

$$\alpha_{ext}^2 > 0, \quad |H_2|_{ext} > 0 \quad . \tag{76}$$

Otherwise there is *no* CCB minimum for the particular value of γ being scanned. If (76) is satisfied, then the value of the potential at the minimum is given by

$$V_{\text{CCB min}} = -\frac{1}{2}\alpha_{ext}^2 |H_2|_{ext}^2 \left(\hat{A}'(\gamma)\lambda_t |H_2|_{ext} - \frac{\hat{m}'^2(\alpha_{ext}, \gamma)}{\alpha_{ext}^2} \right) \quad . \tag{77}$$

and the CCB bound takes again the form

$$V_{\text{CCB min}} > V_{\text{real min}} \quad . \tag{78}$$

When $\text{sign}(A_t) = \text{sign}(B)$ the analysis is exactly the same but, as usual, one of the three terms proportional to $|A_t|$, $|\mu|\gamma$, $|m_3^2|\gamma$ in eqs.(64), (75) must flip its sign.

Let us finally note that if $\tan\beta$ is large ($\tan\beta \gtrsim 15$), then $\lambda_b = O(1)$ and the analysis of this subsection is also the correct one for the bottom, performing the substitutions

$$\begin{aligned}
H_1 &\leftrightarrow H_2 \,, & m_{L_i}^2 &\to (m_{L_i}^2 + m_{e_j}^2) \,, \\
m_1 &\leftrightarrow m_2 \,, & t &\to b \,.
\end{aligned} \tag{79}$$

4.5 The choice of the scale

As is well known (see e.g. ref. [14]) the *complete* effective potential, $V(Q, \lambda_\alpha(Q), m_\beta(Q), \phi(Q))$ (in short $V(Q,\phi)$), where Q is the renormalization scale, $\lambda_\alpha(Q), m_\beta(Q)$ are running parameters and masses, and $\phi(Q)$ are the generic classical fields, is scale-independent, i.e.

$$\frac{dV}{dQ} = 0 \quad . \tag{80}$$

This property allows in principle a different scale for each value of the classical fields, i.e. $Q = f(\phi)$. Denoting by $\langle\phi\rangle$ the VEVs of the ϕ–fields obtained from the minimization condition on V, it is clear that the two following minimization conditions

$$\frac{\partial V(Q = f(\phi), \phi)}{\partial\phi} = 0 \tag{81}$$

$$\left.\frac{\partial V(Q, \phi)}{\partial\phi}\right|_{Q=f(\phi)} = 0 \tag{82}$$

yield equivalent results for $\langle\phi\rangle$ (for a more detailed discussion see ref.[15]).

The previous results apply exactly *only* to the complete effective potential. In practice, however, we can only know V with a certain degree of accuracy in a perturbative expansion. In particular, at one-loop level

$$V_1 = V_o(Q, \phi) + \Delta V_1(Q, \phi) \tag{83}$$

where V_o is the (one-loop improved) tree-level potential and ΔV_1 is the one-loop radiative correction to the effective potential

$$\Delta V_1 = \sum_\alpha \frac{n_\alpha}{64\pi^2} M_\alpha^4 \left[\log\frac{M_\alpha^2}{Q^2} - \frac{3}{2}\right] \quad , \tag{84}$$

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with $M_\alpha^2(Q)$ being all the (in general field–dependent) tree-level squared mass eigenstates (see also eq.(8)). $V_1(Q, \phi)$ does *not* obey eq.(80) for any Q, but it is clear that in the region of Q of the order of the most significant masses appearing in (84), the logarithms involved in the radiative corrections, and hence the radiative corrections themselves, are minimized, thus improving the perturbative expansion. As a matter of fact, in that region of Q, V_1 is approximately scale-independent [6, 7], so eq.(80) is nearly satisfied. Consequently, by choosing an appropriate value of Q, eqs.(81) and eq.(82), plugging $V \to V_1$, produce essentially the same values of $\langle \phi \rangle$, although, of course, eq.(82) is much easier to handle. This statement can be numerically confirmed, see e.g. ref.[15].

Finally, choosing a Q scale, say $\hat{Q}$, such that $\partial \Delta V_1 / \partial \phi = 0$, we will get the same results from eq.(82) using V_1 or[10] V_o. On the other hand $\hat{Q}$ always belongs to the above-mentioned stability region since at $\hat{Q}$ the logarithms involved in ΔV_1, and ΔV_1 itself, are necessarily small, thus optimizing the perturbative expansion. For the CCB directions the equation $\partial \Delta V_1 / \partial \phi = 0$ amounts to a extremely involved condition but from the previous arguments it is sufficiently good for our calculation to take $\hat{Q}$ of the order of the most significant M_α mass appearing in (84) (the precise value is irrelevant), thus suppressing the relevant logarithms, and then use eq.(82) plugging $V \to V_o(\hat{Q})$. This was also the procedure proposed in ref.[6].

Turning back to our specific task, we have to choose the appropriate scale $\hat{Q}$ to evaluate the existence of CCB minima in the potential and the subsequent CCB bounds. Now in eq.(84), besides masses of order M_S, there appear other (field-dependent) masses. In general the latter will be much larger than M_S since the typical magnitude of the relevant fields in a CCB minimum is $O(M_S/\lambda)$. A more precise measure of the size of the most significant masses appearing in (84) comes from the explicit tree-level expresions for the VEVs of the relevant fields at the CCB minimum (see in particular eq.(46)) and from the inspection of what M_α masses they give rise to in the V_o potential. In this way we obtain the following estimations of the size of the appropriate scale, $\hat{Q}$, depending on the relevant trilinear coupling associated with the CCB bound under consideration

$$\lambda_u Q_u H_2 u_R, \lambda_c Q_c H_2 c_R, \lambda_t Q_t H_2 t_R: \quad \hat{Q}_{u,c,t} \sim \mathrm{Max}\left(M_S, g_3 \frac{A_{u,c,t}}{4\lambda_{u,c,t}}, \lambda_t \frac{A_{u,c,t}}{4\lambda_{u,c,t}} \right)$$

$$\lambda_d Q_u H_1 d_R, \lambda_s Q_c H_1 s_R, \lambda_b Q_t H_1 b_R: \quad \hat{Q}_{d,s} \sim \mathrm{Max}\left(M_S, g_3 \frac{A_{d,s}}{4\lambda_{d,s}} \right),$$

$$\hat{Q}_b \sim \mathrm{Max}\left(M_S, g_3 \frac{A_b}{4\lambda_b}, \lambda_t \frac{A_b}{4\lambda_b} \right)$$

$$\lambda_e L_e H_1 e_R, \lambda_\mu L_\mu H_1 \mu_R, \lambda_\tau L_\tau H_1 \tau_R: \quad \hat{Q}_{e,\mu,\tau} \sim \mathrm{Max}\left(M_S, g_2 \frac{A_{e,\mu,\tau}}{4\lambda_{e,\mu,\tau}} \right) \quad (85)$$

Moreover, for $\hat{Q}_{d,s}$, $\hat{Q}_{e,\mu,\tau}$, if we are considering the CCB-2,3 bounds, which involve $H_2 \neq 0$, we have to include $\lambda_t \frac{A_{d,s}}{4\lambda_{d,s}}$, $\lambda_t \frac{A_{e,\mu,\tau}}{4\lambda_{e,\mu,\tau}}$, respectively among the various quantities within the parenthesis above.

---

[10] Actually, this has been our procedure in sect.2 when analyzing the realistic minimum, $V_{\mathrm{real\ min}}$. We concluded there that a good choice of the scale in order to evaluate $V_{\mathrm{real\ min}}$ was $\hat{Q} = M_S$, where M_S (a certain average of the relevant M_α masses) was given by eq.(11).

Finally, let us note that a similar procedure for the choice of the $\hat{Q}$ scale was carried out in sect.3 for the UFB bounds.

Of course, the results for CCB and UFB bounds are quite stable against moderate variations of the $\hat{Q}$–scale.

5   Summary of UFB and CCB constraints

Here we summarize the two types of constraints, UFB and CCB, analyzed in sect.3 and sect.4 respectively, to which the reader is referred for further details.

5.1   UFB constraints

These constraints arise from directions in the field-space along which the (tree-level) potential becomes unbounded from below (UFB). It is interesting to note that usually this is only true at tree-level since radiative corrections eventually raise the potential for large enough values of the fields. This is the case of UFB-2,3 below.

UFB-1

The condition

$$m_1^2 + m_2^2 \geq 2|m_3^2| \tag{86}$$

must be verified at any scale $Q > M_S$ and, in particular, at the unification scale $Q = M_X$. M_S is the typical scale of SUSY masses (see e.g. eq.(11)).

UFB-2

For any value of $|H_2| < M_X$ satisfying

$$|H_2|^2 > \frac{4 m_{L_i}^2}{(g'^2 + g_2^2)\left[1 - \frac{|m_3|^4}{(m_1^2 - m_{L_i}^2)^2}\right]} \tag{87}$$

and provided that

$$|m_3^2| < m_1^2 - m_{L_i}^2 \tag{88}$$

the following condition must be verified:

$$V_{\text{UFB}-2}(Q = \hat{Q}) > V_{\text{real min}}(Q = M_S) \,, \tag{89}$$

where $V_{\text{real min}}$ is the value of the realistic minimum, given by eq.(10), $\hat{Q} \sim \text{Max}(g_2|H_2|, \lambda_{top}|H_2|, M_S)$, and

$$V_{\text{UFB}-2} = \left[m_2^2 + m_{L_i}^2 - \frac{|m_3|^4}{m_1^2 - m_{L_i}^2}\right]|H_2|^2 - \frac{2 m_{L_i}^4}{g'^2 + g_2^2} \,. \tag{90}$$

UFB-3

For any value of $|H_2| < M_X$ satisfying

$$|H_2| > \sqrt{\frac{\mu^2}{4 \lambda_{e_j}^2} + \frac{4 m_{L_i}^2}{g'^2 + g_2^2}} - \frac{|\mu|}{2 \lambda_{e_j}} \,, \tag{91}$$

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with $j \neq i$ the following condition must be verified:

$$V_{\text{UFB-3}}(Q = \hat{Q}) > V_{\text{real min}}(Q = M_S) , \tag{92}$$

where $V_{\text{real min}}$ is given by eq.(10), $\hat{Q} \sim \text{Max}(g_2|e|, \lambda_{top}|H_2|, g_2|H_2|, g_2|L_i|, M_S)$ with $|e| = \sqrt{\frac{|\mu|}{\lambda_{e_j}}|H_2|}$ and $|L_i|^2 = -\frac{4m_{L_i}^2}{g'^2 + g_2^2} + (|H_2|^2 + |e|^2)$, λ_{e_j} is an e-type Yukawa coupling and

$$V_{\text{UFB-3}} = (m_2^2 - \mu^2 + m_{L_i}^2)|H_2|^2 + \frac{|\mu|}{\lambda_{e_j}}(m_{L_j}^2 + m_{e_j}^2 + m_{L_i}^2)|H_2| - \frac{2m_{L_i}^4}{g'^2 + g_2^2} . \tag{93}$$

If $|H_2|$ does not satisfy eq.(91), the constraint is still given in the form (92), but with

$$V_{\text{UFB-3}} = (m_2^2 - \mu^2)|H_2|^2 + \frac{|\mu|}{\lambda_{e_j}}(m_{L_j}^2 + m_{e_j}^2)|H_2| + \frac{1}{8}(g'^2 + g_2^2)\left[|H_2|^2 + \frac{|\mu|}{\lambda_{e_j}}|H_2|\right]^2 . \tag{94}$$

From (92), (93), (94), it is clear that the larger λ_{e_j} the more restrictive the constraint becomes. Consequently, the optimum choice of the e–type slepton should be the third generation one, i.e. $e_j = \text{stau}$.

It is interesting to mention that the previous constraint (92) with the following replacements

$$e \to d , \qquad \lambda_{e_j} \to \lambda_{d_j} , \qquad L_j \to Q_j , \tag{95}$$

must also be imposed. Now i may be equal to j (the optimum choice is $d_j = \text{sbottom}$) and $\hat{Q} \sim \text{Max}\ (g_2|H_2|, \lambda_{top}|H_2|, g_3|d|, \lambda_{u_j}|d|, g_2|L_i|, M_S)$. However, the optimum condition is the first one with the sleptons (note e.g. that the second term in eq.(93) is proportional to the slepton masses and thus smaller) and will represent, as we will see in sect.6, the *strongest* one of *all* the UFB and CCB constraints in the parameter space of the MSSM.

5.2 CCB constraints

These constraints arise from the existence of charge and color breaking (CCB) minima in the potential deeper than the realistic minimum. As was explained in subsect.4.1 and Appendix, the most dangerous, i.e. the deepest, CCB directions in the MSSM potential involve only one particular trilinear soft term of one generation. Then, for each trilinear soft term we will write below the three possible (optimized) types of constraints that emerge. Following the notation of the previous section, they are named CCB-1,2,3.

$\lambda_u Q_u H_2 u_R$

The following constraints must be evaluated at the scale $\hat{Q} \sim \text{Max}\left(M_S, g_3 \frac{A_u}{4\lambda_u}, \lambda_t \frac{A_u}{4\lambda_u}\right)$.

CCB-1

1. If $m_2^2 - \mu^2 + m_{L_i}^2 > 0$ and $3m_{L_i}^2 - (m_{Q_u}^2 + m_u^2) + 2(m_2^2 - \mu^2) > 0$, then the optimized CCB-1 bound is

$$|A_u|^2 \leq 3\left[m_2^2 - \mu^2 + m_{Q_u}^2 + m_u^2\right] \tag{96}$$

2. If $m_2^2 - \mu^2 + m_{L_i}^2 > 0$ and $3m_{L_i}^2 - (m_{Q_u}^2 + m_u^2) + 2(m_2^2 - \mu^2) < 0$, then the optimized CCB-1 bound is

$$|A_u|^2 \leq \left(1 + \frac{2}{\alpha^2}\right) \left[m_2^2 - \mu^2 + (m_{Q_u}^2 + m_u^2)\alpha^2 + m_{L_i}^2(1 - \alpha^2)\right] \qquad (97)$$

with $\alpha^2 = \sqrt{\frac{2(m_{L_i}^2 + m_2^2 - \mu^2)}{m_{Q_u}^2 + m_u^2 - m_{L_i}^2}}$.

3. If $m_2^2 - \mu^2 + m_{L_i}^2 < 0$, then the CCB-1 bound is automatically violated. In fact the minimization of the potential in this case gives $\alpha^2 \to 0$, and we are exactly led to the UFB-3 direction shown above, which represents the correct analysis in this instance.

Let us mention that the bound (96) seldom represents the optimized bound, as long as the condition for this (see above eq.(96)) will not normally be satisfied. Hence, eq.(97) will usually represent the (optimized) CCB-1 bound.

Finally, in the very unlikely case that $3(m_{L_i}^2 + m_{e_j}^2) + (m_{Q_u}^2 + m_u^2) - 2(m_2^2 - \mu^2) < 0$, which only can take place in (very strange) non-universal cases, then the CCB-1 bound would be given by

$$|A_u|^2 \leq \left(1 + \frac{2}{\alpha^2}\right) \left[m_2^2 - \mu^2 + (m_{Q_u}^2 + m_u^2)\alpha^2 + (m_{L_i}^2 + m_{e_j}^2)(\alpha^2 - 1)\right] \qquad (98)$$

with $\alpha^2 = \sqrt{\frac{2(m_2^2 - \mu^2 - m_{L_i}^2 - m_{e_j}^2)}{m_{Q_u}^2 + m_u^2 + m_{L_i}^2 + m_{e_j}^2}}$.

CCB-2

This second constraint applies whenever $\text{sign}(A_u) = -\text{sign}(B)$. The general form of the CCB-2 constraint is

$$\begin{aligned}
(|A_u| + |\mu|\gamma)^2 \leq \left(1 + \frac{2}{\alpha^2}\right) [& m_2^2 + (m_{Q_u}^2 + m_u^2)\alpha^2 + m_1^2\gamma^2 \\
& + m_{L_i}^2(1 - \alpha^2 - \gamma^2) - 2|m_3^2|\gamma]
\end{aligned} \qquad (99)$$

which should be handled in the following way:

1. Scan γ in the range $0 \leq \gamma \leq 1$

2. For each value of γ the optimum value of α^2, i.e. the one that minimizes the right hand side of (99), is in principle given by

$$\alpha_{ext}^4 = \frac{2\left[m_2^2 + m_1^2\gamma^2 + m_{L_i}^2(1 - \gamma^2) - 2|m_3^2|\gamma\right]}{m_{Q_u}^2 + m_u^2 - m_{L_i}^2} \qquad (100)$$

Under the assumption of universality the denominator of (100) is always positive. On the other hand, the numerator should also be positive, otherwise the optimum value of α is $\alpha \to 0$ and we are exactly led to the UFB-2 direction explained above.

3. If $\alpha_{ext}^2 < 1 - \gamma^2$, then α_{ext}^2 is the optimum value of α^2 to be substituted in (99).

4. If $\alpha_{ext}^2 > 1 - \gamma^2$ and $\tan\beta \lesssim 3$, then the $m_{L_i}^2(1 - \alpha^2 - \gamma^2)$ term in (99) must be replaced by $(m_{L_i}^2 + m_{e_j}^2)(\alpha^2 + \gamma^2 - 1)$. The new optimum value of α_{ext} would be in principle given by

$$\alpha_{ext}'^4 = \frac{2\left[m_2^2 + m_1^2\gamma^2 - (m_{L_i}^2 + m_{e_j}^2)(1 - \gamma^2) - 2|m_3|^2\gamma\right]}{m_{Q_u}^2 + m_u^2 + m_{L_i}^2 + m_{e_j}^2} \tag{101}$$

If $\alpha_{ext}'^2 > 1 - \gamma^2$, then $\alpha_{ext}'^2$ is the optimum value of α^2 to be substituted in (99) together with the previous replacement. If $\alpha_{ext}'^2 < 1 - \gamma^2$, then the optimum value of α^2 is simply $\alpha^2 = 1 - \gamma^2$, which should be substituted in (99).

5. If $\alpha_{ext}^2 > 1 - \gamma^2$ and $\tan\beta > 3$, then the optimum value of α^2 is simply $\alpha^2 = 1 - \gamma^2$, which should be substituted in (99).

CCB-3

This bound is the equivalent to the CCB-2 one, but when $\mathrm{sign}(A_u) = \mathrm{sign}(B)$. It has exactly the same form as CCB-2 but flipping the sign of one of the three terms $\{|A_u|, |\mu|\gamma, -2|m_3^2|\gamma\}$ in (99). Notice that, due to the form of (99) flipping the sign of $|A_u|$ or the sign of $|\mu|\gamma$ leads to the same result. Therefore, there are only two choices to examine: the first one writing $(|A_u| - |\mu|\gamma)^2$ in the left hand side of (99), the second one writing $+2|m_3^2|\gamma$ in the right hand side of (99) and hence in those of (100) and (101).

$\lambda_c Q_c H_2 c_R$

The CCB constraints associated with this coupling have exactly the same form as those for the $\lambda_u Q_u H_2 u_R$ coupling, i.e. the CCB-1 – CCB-3 bounds, with the obvious replacement $u \to c$ (this is also valid for the scale $\hat{Q}$). Now, there is no constraint on $\tan\beta$ and, therefore, possibility 4 in CCB-2,3 can be applied for any value of $\tan\beta$ and possibility 5 should not be taken into account.

$\lambda_d Q_u H_1 d_R$, $\lambda_s Q_c H_1 s_R$, $\lambda_b Q_t H_1 b_R$

Now the scale is given by: $\hat{Q}_{d,s} \sim \mathrm{Max}\left(M_S, g_3 \frac{A_{d,s}}{4\lambda_{d,s}}\right), \hat{Q}_b \sim \mathrm{Max}\left(M_S, g_3 \frac{A_b}{4\lambda_b}, \lambda_t \frac{A_b}{4\lambda_b}\right)$.

The CCB-1 bounds, eqs.(96,97,98), remain the same with the following replacements

$$\begin{aligned}
m_1 &\leftrightarrow m_2 , \\
m_{L_i}^2 &\leftrightarrow (m_{L_i}^2 + m_{e_j}^2) , \\
u &\to d, s \text{ or } b .
\end{aligned} \tag{102}$$

For the bottom coupling the CCB-1 bound is only valid if $\tan\beta \lesssim 4$.

Concerning the CCB-2,3 bounds, they remain the same with the previous (102) substitutions. Moreover, for the estimation of $\hat{Q}_{d,s}$ we have to include $\lambda_t \frac{A_{d,s}}{4\lambda_{d,s}}$ among the various quantities within the parenthesis above. Now, there is no constraint on $\tan\beta$ and therefore possibility 4 in CCB-2,3 can be applied for any value of $\tan\beta$, disregarding possibility 5.

$$\lambda_e \mathbf{L_e H_1 e_R} , \ \lambda_\mu \mathbf{L_\mu H_1 \mu_R} , \ \lambda_\tau \mathbf{L_\tau H_1 \tau_R}$$

The scale is given by: $\hat{Q}_{e,\mu,\tau} \sim \text{Max}\left(M_S, g_2 \frac{A_{e,\mu,\tau}}{4\lambda_{e,\mu,\tau}}\right)$.

The CCB bounds remain the same with the following replacements

$$
\begin{aligned}
m_1 &\leftrightarrow m_2 , & m_{L_i}^2 &\to \left(m_{L_i'}^2 + m_{e_j'}^2\right) , \\
u &\to e, \mu \text{ or } \tau , & \left(m_{L_i}^2 + m_{e_j}^2\right) &\to m_{L_i'}^2 , \\
Q &\to L .
\end{aligned}
\tag{103}
$$

where L_i', e_{R_j}' are two sleptons of a different generation than the trilinear coupling under consideration. When both extra sleptons appear in the bounds, the associated Yukawa coupling, say λ_l', must be much smaller than the Yukawa coupling of the trilinear coupling under consideration, say λ_l. Obviously this condition can always be satisfied except when the coupling under consideration is of the first generation (i.e. the electron one). In that case $\alpha^2 = 1 - \gamma^2$.

Here there is no constraint on $\tan \beta$ and therefore possibility 4 in CCB-2,3 can be applied for any value of $\tan \beta$ and possibility 5 should not be taken into account. Moreover, for the estimation of $\hat{Q}_{e,\mu,\tau}$ if we are considering the CCB-2,3 bounds we have to include $\lambda_t \frac{A_{e,\mu,\tau}}{4\lambda_{e,\mu,\tau}}$ among the various quantities within the parenthesis above.

Under the assumption of universality it is easy to see that the CCB-1 bound will only take place in the possibility 1 [see condition above eq.(96)], while the CCB-2, CCB-3 bounds will occur in the possibility 4 [note that the denominator of eq.(100) goes to zero].

$\lambda_t \mathbf{Q_t H_2 t_R}$

The CCB-1 bound does not apply to the top case. Moreover, since $\lambda_t = O(1)$ it is not true that a negative minimum associated to the top trilinear coupling is necessarily much deeper than the realistic minimum, thus destabilizing the standard vacuum, as was the case of the previous couplings. Therefore, rather than the absence of a negative minimum, we must demand that the possible CCB minimum satisfies $V_{\text{CCB min}} > V_{\text{real min}}$.

When $\text{sign}(A_t) = -\text{sign}(B)$ (i.e. CCB-2), the potential is given by

$$
V = \lambda_t^2 F'(\alpha)\alpha^4 |H_2|^4 - 2\lambda_t \hat{A}'(\gamma)\alpha^2 |H_2|^3 + \hat{m}'^2(\alpha,\gamma)|H_2|^2 - \frac{2m_{L_i}^4}{g'^2 + g_2^2} , \tag{104}
$$

with

$$
\begin{aligned}
F'(\alpha) &= 1 + \frac{2}{\alpha^2} + \frac{f'}{\alpha^4} ; \ f' = 0 , \\
\hat{A}'(\gamma) &= |A_t| + |\mu|\gamma , \\
\hat{m}'^2(\alpha,\gamma) &= m_2^2 + (m_{Q_t}^2 + m_t^2)\alpha^2 + m_1^2\gamma^2 + m_{L_i}^2(1 - \alpha^2 - \gamma^2) - 2|m_3^2|\gamma. \tag{105}
\end{aligned}
$$

This should be handled in the following way:

1. Scan γ in the range $0 \leq \gamma \leq 1$

2. For each value of γ the optimum values of α^2, H_2 i.e. the ones that minimize the right hand side of (104), are given by

$$\alpha_{ext}^2 = \frac{\hat{A}'(\gamma)}{\lambda_t |H_2|_{ext}} - 1 - \frac{m_{Q_t}^2 + m_t^2 - m_{L_i}^2}{2\lambda_t^2 |H_2|_{ext}^2} \ , \tag{106}$$

$$|H_2|_{ext} = \frac{3\hat{A}'(\gamma)}{4\lambda_t \alpha_{ext}^2 F'(\alpha_{ext})} \left\{ 1 + \sqrt{1 - \frac{8\hat{m}'^2(\alpha_{ext}, \gamma) F'(\alpha_{ext})}{9\hat{A}'^2(\gamma)}} \right\} \ . \tag{107}$$

For each value of γ the coupled equations (106), (107) can be solved, e.g. by a numerical method. Then, the consistency of the procedure requires

$$\alpha_{ext}^2 > 0, \quad |H_2|_{ext} > 0, \quad 1 - \gamma^2 - \alpha_{ext}^2 - \frac{4m_{L_i}^2}{(g'^2 + g_2^2)|H_2|_{ext}^2} > 0 \ . \tag{108}$$

If (108) is fulfilled, then the corresponding value of the potential at the minimum is given by

$$V_{\text{CCB min}} = -\frac{1}{2} \alpha_{ext}^2 |H_2|_{ext}^2 \left(\lambda_t \hat{A}'(\gamma)|H_2|_{ext} - \frac{\hat{m}'^2(\alpha_{ext}, \gamma)}{\alpha_{ext}^2} \right) - \frac{2m_{L_i}^4}{g'^2 + g_2^2} \ . \tag{109}$$

Consequently, the CCB bound has the form

$$V_{\text{CCB min}}(Q = \hat{Q}) > V_{\text{real min}}(Q = M_S) \ , \tag{110}$$

where $V_{\text{CCB min}}$ and $V_{\text{real min}}$ are given by eqs.(109) and (10) respectively; the value of the scale M_S was explained in UFB-1 above and $\hat{Q} \sim \text{Max}\left(M_S, g_3 \frac{A_t}{4\lambda_t}, \lambda_t \frac{A_t}{4\lambda_t}\right)$.

3. If (108) is not fulfilled, then the potential is given by

$$V = \lambda_t^2 F'(\alpha, \gamma)\alpha^4 |H_2|^4 - 2\lambda_t \hat{A}'(\gamma)\alpha^2 |H_2|^3 + \hat{m}'^2(\alpha, \gamma)|H_2|^2 \ , \tag{111}$$

The optimum values of α, H_2 are now given by

$$\alpha_{ext}^2 = \frac{8\lambda_t^2}{g'^2 + g_2^2 + 8\lambda_t^2} \left[\frac{\hat{A}'(\gamma)}{\lambda_t |H_2|_{ext}} - 1 - \frac{m_{Q_t}^2 + m_t^2}{2\lambda_t^2 |H_2|_{ext}^2} + \frac{g'^2 + g_2^2}{8\lambda_t^2}(1 - \gamma^2) \right] \tag{112}$$

$$|H_2|_{ext} = \frac{3\hat{A}'(\gamma)}{4\lambda_t \alpha_{ext}^2 F'(\alpha_{ext}, \gamma)} \left\{ 1 + \sqrt{1 - \frac{8\hat{m}'^2(\alpha_{ext}, \gamma) F'(\alpha_{ext}, \gamma)}{9\hat{A}'^2(\gamma)}} \right\} \ , \tag{113}$$

with

$$F'(\alpha, \gamma) = 1 + \frac{2}{\alpha^2} + \frac{f'}{\alpha^4} \ ,$$

$$f' = \frac{g'^2 + g_2^2}{8\lambda_t^2} \left(1 - \alpha^2 - \gamma^2\right)^2 \ ,$$

$$\hat{m}'^2(\alpha, \gamma) = m_2^2 + \left(m_{Q_t}^2 + m_t^2\right)\alpha^2 + m_1^2\gamma^2 - 2|m_3^2|\gamma \ . \tag{114}$$

29

Consistency now requires

$$\alpha_{ext}^2 > 0, \quad |H_2|_{ext} > 0 \quad . \tag{115}$$

Otherwise there is *no* CCB minimum for the particular value of γ being scanned. If (115) is satisfied, then the value of the potential at the minimum is given by

$$V_{\text{CCB min}} = -\frac{1}{2}\alpha_{ext}^2 |H_2|_{ext}^2 \left(\hat{A}'(\gamma)\lambda_t |H_2|_{ext} - \frac{\hat{m}'^2(\alpha_{ext}, \gamma)}{\alpha_{ext}^2} \right) \quad . \tag{116}$$

and the CCB bound takes again the form

$$V_{\text{CCB min}}(Q = \hat{Q}) > V_{\text{real min}}(Q = M_S) \quad . \tag{117}$$

When $\text{sign}(A_t) = \text{sign}(B)$ (i.e. CCB-3) the analysis is exactly the same but, as usual, one of the three terms proportional to $|A_t|$, $|\mu|\gamma$, $|m_3^2|\gamma$ in eqs.(105), (114) must flip its sign.

Let us finally note that if $\tan\beta$ is large ($\tan\beta \gtrsim 15$), then $\lambda_b = O(1)$ and the analysis of this subsection is also the correct one for the bottom, performing the substitutions

$$\begin{aligned}
H_1 &\leftrightarrow H_2 \, , & m_{L_i}^2 &\to (m_{L_i}^2 + m_{e_j}^2) \, , \\
m_1 &\leftrightarrow m_2 \, , & t &\to b \, .
\end{aligned} \tag{118}$$

6 Constraints on the parameter space

In the previous sections, a complete analysis of all the potentially dangerous unbounded from below (UFB) and charge and color breaking (CCB) directions has been carried out. In particular, the analytical form of the constraints obtained on the parameter space of the MSSM has been summarized in sect.5. Now, we will analyze numerically those constraints. We will see that they are very important and, in fact, there are *extensive regions* in the parameter space which are forbidden.

Our analysis will be quite general in the sense that we will consider the whole parameter space of the MSSM, m, M, A, B, μ, with the only assumption of universality[11]. Actually, universality of the soft SUSY-breaking terms at M_X is a desirable property not only to reduce the number of independent parameters, but also for phenomenological reasons, particularly to avoid flavour-changing neutral currents (see, e.g. ref.[16]). As discussed in sect.2, the requirement of correct electroweak breaking fixes one of the five independent parameters of the MSSM, say μ, so we are left with only four parameters (m, M, A, B). Although we will perform the numerical analysis on this space, it is worth noticing that particularly interesting values of B can be obtained from Supergravity (SUGRA). In this sense we will first consider two values of B as guiding examples to get an idea of how strong the different constraints are and then we will vary B in order to obtain the most general results. Hence, let us first justify, theoretically and phenomenologically, the two specific values of B.

The particular values of the soft terms depend on the type of Supergravity theory from which the MSSM derives and, in general, on the mechanism of SUSY-breaking.

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[11]Let us remark, however, that the constraints found in previous sections are general and they could also be applied for the non-universal case.

But, in fact, is still possible to learn things about soft terms without knowing the details of SUSY-breaking [17]. Let us consider the simple case[12] of canonical kinetic terms for hidden and observable matter fields (i.e. a Kähler potential $K = \sum_\alpha |\phi_\alpha|^2$). Then, irrespective of the SUSY-breaking mechanism, the scalar masses are automatically universal. Furthermore, if the observable part of the superpotential W is assumed to be as in eq.(3), μ being an initial parameter, then the B term and the universal A terms are automatically generated and they are related to each other (assuming that Yukawa couplings and μ are hidden field independent [17]) by the well known relation [18]

$$B = A - m \ . \tag{119}$$

Finally, if the gauge kinetic function is the same for the different gauge groups of the theory $f_a = f$ (where a is associated with $SU(3)$, $SU(2)_L$ and $U(1)_Y$), the gaugino masses are also universal. This SUGRA theory is attractive for its simplicity and for the natural explanation that it offers to the universality of the soft terms. However, this scenario has a serious drawback. It is well known that, in order to get appropriate $SU(2)_L \times U(1)_Y$ breaking, the μ parameter has to be of the same order of magnitude (M_W) as the soft SUSY-breaking terms discussed above. This is in general unexpected since the μ term is a SUSY term whereas the soft terms are originated after SUSY-breaking. In principle, the natural scale of μ would be the Planck mass. The unnatural smallness of the μ parameter is the so-called μ problem. We will briefly explain here three interesting scenarios considered in SUGRA in order to solve the problem, illustrating them in the case of canonical kinetic terms:

(a) In ref.[19] was pointed out that the presence of a non-renormalizable term in the superpotential, $\lambda W H_1 H_2$, characterized by the coupling λ, yields dynamically a μ parameter when the hidden sector part of W acquires a VEV, namely $\mu = m_{3/2}\lambda$, where $m_{3/2}$ is the gravitino mass. The fact that μ is of the electroweak scale order is a consequence of our assumption of a correct SUSY-breaking scale $m_{3/2} = O(M_W)$. Now, with this solution to the μ problem, the B parameter can be straightforwardly evaluated. The simple result (in the case of λ independent of the hidden fields [17]) is

$$B = 2m \ . \tag{120}$$

For this mechanism to work, the $\mu H_1 H_2$ term in eq.(3) must be initially absent (otherwise the natural scale for μ would be the Planck mass), a fact that remarkably enough, is automatically guaranteed in the framework of Superstring theory as we will see below.

(b) In refs.[20, 19] it was shown that if a term, $Z H_1 H_2 + h.c.$, characterized by the coupling Z is present in the Kähler potential, an effective low-energy B term is naturally generated. In the case of Z independent of the hidden fields, this mechanism for solving the μ problem is equivalent [19] to the previous one *(a)* and therefore the value of B is again given by eq.(120). Now, the size of μ is $\mu = m_{3/2}Z$.

(c) In ref.[21] the observation was made that in the framework of any SUSY-GUT, starting again with $\mu = 0$, an effective μ term is generated by the integration of the heavy degrees of freedom. The prediction for B is once more given by eq.(120).

The solutions discussed here in order to solve the μ problem are *naturally present* in Superstring theory. In ref.[19] was first remarked that the $\mu H_1 H_2$ term is naturally

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[12]We will assume from now on a vanishing cosmological constant.

absent as already mentioned above. The reason is that in SUGRA theories coming from Superstring theory mass terms for light fields are forbidden in the superpotential. Then a realistic example where non-perturbative SUSY-breaking mechanisms like gaugino-squark condensation induce superpotentials of the type *(a)* was given. In ref.[22] the same kind of superpotential was obtained using pure gaugino condensation in the context of orbifold models. The alternative mechanism *(b)* in which there is an extra term in the Kähler potential originating a μ-term is also naturally present in some large classes of four-dimensional Superstrings [23, 24, 22]. In Superstring theory, neither the kinetic terms are in general canonical nor the couplings (Yukawas, λ, Z) and the mass term (μ) are independent of hidden fields. However, it is still possible to obtain (the phenomenologically desirable) universal soft terms in the so-called dilaton-dominated limit [23, 25]. This limit is not only interesting because of that, but also because it is quite *model independent* (i.e. for any compactification scheme the results for the soft terms are the same). It is also remarkable, that in this limit once again the value of B for the two mechanisms *(a)*, *(b)* coincides [17] with that of eq.(120). If, alternatively, we just assume that a small ($\sim M_W$) dilaton-independent mass μ is present in the superpotential, then the result for B is now given [25] by eq.(119) as in the case of canonical kinetic terms.

From the above analysis, it is clear that eqs.(119,120) give us two values of B very interesting from the theoretical and phenomenological point of view. Thus, we will consider, for the moment, in our numerical study of the UFB and CCB constraints both possibilities. In fact, the value of μ is also fixed once we choose a particular mechanism for solving the μ problem, e.g. mechanisms *(a)*, *(b)* (see above). However, this value still depends on the couplings λ and Z which are in general model dependent[13], so we prefer to eliminate μ in terms of the other parameters by imposing appropriate symmetry-breaking at the weak scale as mentioned above. Let us now turn to the numerical results.

In Fig.1 we have presented in detail the case $B = A - m$ with $m = 100$ GeV, to get an idea of how strong the different constraints are, plotting the excluded regions in the remaining parameter space (A/m, M/m). It is worth noticing here that even before imposing CCB and UFB constraints, the parameter space is strongly restricted by the experiment. As already mentioned in sect.2, not for all the parameter space it is possible to choose the boundary condition of λ_{top} so that the experimental mass of the top is reproduced, since the RG infrared fixed point of λ_{top} puts an upper bound on M_{top}, namely $M_{\text{top}} \lesssim 197 \sin \beta$ GeV [9], where $\tan \beta = v_2/v_1$. In this way, the upper and lower darked regions are forbidden because $M_{\text{top}}^{\text{phys}} = 174$ GeV cannot be reached. Furthermore, the small central darked region is also forbidden because there is no value of μ capable of producing the correct electroweak breaking.

Fig.1a shows the region excluded by the "traditional" CCB bounds of the type of eq.(5), evaluated at an *appropriate* scale (see subsect.4.5). For a point in the parameter space to be excluded we have also demanded that the corresponding CCB minimum is deeper than the realistic one (this is especially relevant for the bounds coming from the top trilinear term). Clearly, the "traditional" bounds, when correctly evaluated, turn out to be very weak. In fact, only the leptonic (circles) and the d–type (dia-

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[13]For an analysis of the MSSM from Superstring theory taking into account a particular value of Z coming from orbifold compactifications, and therefore a fixed value of μ, see ref.[26].

monds) terms do restrict, very modestly, the parameter space. Let us recall here that it has been a common (incorrect) practice in the literature to evaluate these traditional bounds at all the scales between M_X and M_W, thus obtaining very important (and of course overestimated) restrictions in the parameter space. Fig.1b shows the region excluded by our "improved" CCB constraints obtained in sect.4 and summarized in sect.5. Comparing Figs.1a and 1b it is clear that the excluded region becomes dramatically increased. Notice also that all the trilinear couplings (except the top one in this case) give restrictions, producing areas constrained by different types of bounds simultaneously. The restrictions coming from the UFB constraints, obtained in sect.3 and summarized in sect.5, are shown in Fig.1c. By far, the most restrictive bound is the UFB–3 one (small filled squares). Indeed, the UFB–3 constraint is the *strongest* one of *all* the UFB and CCB constraints, excluding extensive areas of the parameter space, as is illustrated in the figure. In our opinion, this is a most remarkable result. Finally, in Fig.1d we summarize all the constraints plotting also the excluded region due to the (conservative) experimental bounds on SUSY particle masses (filled diamonds) of eq.(16). More precisely, this forbidden area comes from too small masses for the gluino, lightest chargino, lightest neutralino, left sbottom, and left and right u, c squarks. The allowed region left at the end of the day (white) is quite small.

Figs.2a, 2b, 2c give, in a summarized way, the same analysis as that of Fig.1, but for three different values of m ($m = 100$ GeV, $m = 300$ GeV, $m = 500$ GeV). For the plots with m bigger than 100 GeV the gluino, lightest stop, lightest chargino and lightest neutralino are responsible for the excluded region due to experimental bounds on masses. The ants indicate regions which are excluded by negative squared mass eigenvalues, in this case the lightest stop. The figures show a clear trend in the sense that the larger the value of m, the less restrictive the constraints become. This is mainly due to the effect of the UFB-3 constraint (note the almost exact m–invariance of the CCB bounds). Anyway, extensive areas in the parameter space are forbidden in all cases.

The same conclusions are obtained for the other (theoretically and phenomenologically well-motivated) value of B, $B = 2m$. The results in this scenario are shown in Fig.3, where the whole darked region is forbidden because $M_{\text{top}}^{\text{phys}} = 174$ GeV cannot be reached. Unlike the Fig.2, now in some cases the left sbottom may also get a negative squared mass eigenvalue.

Finally, in Figs.4a, 4b we generalize the previous analyses by varying the value of B for different values of m, namely $m = 100$ GeV, $m = 300$ GeV. The final allowed regions from all types of bounds in the parameter space of the MSSM are shown. Both figures exhibit a similar trend. For a particular value of m, the larger the value of B the smaller the allowed region becomes. More precisely, the maximum allowed value of B is $B = 2.5m$ for $m = 100$ GeV and $B = 3.5m$ for $m = 300$ GeV. This fact comes mainly from the enhancement of the forbidden areas by the UFB-3 constraint and the requirement of $M_{\text{top}}^{\text{phys}} = 174$ GeV. Both facts are due to the decreasing of $\tan \beta$ as B grows. Then higher top Yukawa couplings are needed in order to reproduce the experimental top mass. On the one hand, this cannot be always accomplished due to the infrared fixed point limit on the top mass. On the other hand, the larger the top Yukawa coupling, the stronger the UFB-3 bound becomes. For negative values of B the corresponding figures can easily be deduced from the previous ones, taking into that they are invariant under the transformation $B, A, M \rightarrow -B, -A, -M$.

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From the various figures it is clear that the CCB and UFB constraints *put important bounds* not only on the value of A, but also on the values of B and M, which is an interesting novel fact.

7   Conclusions

Although the possible existence of dangerous charge and color breaking minima in the supersymmetric standard model has been known since the early 80's, a complete study of this crucial issue was still lacking. This was due to two reasons: First, the complexity of the SUSY scalar potential, V, caused that only particular directions in the field-space were considered, thus obtaining necessary but not sufficient conditions to avoid dangerous charge and color breaking minima. Second, the radiative corrections to V were not normally included in a proper way.

In the present paper we have carried out a complete analysis of all the potentially dangerous directions in the field-space of the MSSM, obtaining the corresponding constraints on the parameter space. These are completely general and can be applied to the non-universal case. The constraints turn out to be very important and, in fact, there are *extensive regions* in the parameter space which are forbidden, increasing the predictive power of the theory.

The constraints can be clasified in two types. First, the ones associated with the existence of charge and color breaking (CCB) minima in the potential deeper than the realistic minimum. Second, the constraints associated with directions in the field-space along which the potential becomes unbounded from below (UFB). It is worth mentioning here that the unboundedness is only true at tree-level since radiative corrections eventually raise the potential for large enough values of the fields, but still these minima can be deeper than the realistic one and thus dangerous.

We have performed a complete analysis of both types of directions obtaining *new and very restrictive bounds*, expressed in an analytic way, that represent a set of necessary and sufficient constraints. They are summarized in sect.5. For certain values of the initial parameters the CCB constraints "degenerate" into the UFB constraints since the minima become unbounded from below directions. In this sense, the CCB constraints comprise the UFB bounds, which can be considered as special (but extremely important) limits of the former.

We have also taken into account the radiative corrections to V in a proper way. To this respect, let us remember that, usually, the scalar potential is considered at tree-level, improved by one-loop RGEs, so that all the parameters appearing in it are running with the renormalization scale, Q. Then it is often demanded that the CCB and UFB constraints are satisfied at any scale between M_X and M_Z. However, this is not correct since the tree-level scalar potential is strongly Q-dependent and the one-loop radiative corrections to it are crucial to make the potential stable against variations of the scale. Using the scale independence of V, instead of minimizing the complete one-loop potential, which would be an impossible task, we have demanded that the previous (tree-level-like) bounds are satisfied at the renormalization scale, Q, at which the one-loop correction to the potential is essentially negligible. This simplifies enormously the analysis, producing equivalent results. We have also given explicit expressions of the appropriate scale to evaluate the different types of bounds.

The usual lack in the literature of an optimum scale to evaluate the constraints implies that their restrictive power has normally been overestimated. E.g., the "traditional" CCB bounds (see eq.(5)) when (incorrectly) analyzed at M_X are very strong. However, we have seen that when correctly evaluated, they turn out to be very weak (see Fig.1a). The new CCB constraints obtained here are much more restrictive and, in fact, the excluded region becomes dramatically increased (see Fig.1b). On the other hand, the restrictions coming from the new UFB constraints are by far the most important ones, excluding extensive areas of the parameter space (see e.g. Fig.1c).

We have performed a numerical analysis of how our UFB and CCB constraints put restrictions on the whole parameter space of the MSSM. As already mentioned they are very strong producing important bounds not only on the value of A (soft trilinear parameter), but also on the values of B (soft bilinear parameter) and M (gaugino masses). This is a new and interesting feature. This analysis is summarized in Figs.2–4.

Finally, let us mention that all the constraints that has been obtained here come from the requirement that the standard vacuum is the global minimum of the theory. Although the possibility of living in a metastable vacuum with a lifetime larger than the present age of the Universe [2] does not seem specially attractive, it cannot be excluded. Since the constraints on the parameter space found in this paper are very strong, this dynamical question deserves further analysis [27].

Appendix

In subsect.4.1 we have enumerated five general properties concerning charge and color breaking (CCB) minima in the MSSM. Properties 1, 3, 5 remained to be proved, which is the aim of this appendix.

Let us however first notice that some of the properties (in particular the 1 and 3 ones) can be intuitively understood by a simple consideration. Suppose we consider a region in the field-space where only one trilinear scalar term is non-negligible. Denoting by ϕ the typical size of the relevant VEVs at a CCB minimum, we can schematically write the relevant terms in the potential as

$$V \sim Nm^2\phi^2 - 2A\lambda\phi^3 + N'\lambda^2\phi^4 \; + \; \mathrm{D-terms} \; , \tag{121}$$

where $N, N' = O(1)$ (typically $N, N' \sim 3$), $m, A \sim M_S$ (i.e. the scale of SUSY breaking) and λ is the Yukawa coupling (note that with a convenient choice of the field phases, the trilinear scalar term can always be made negative as in (121)). Ignoring for the moment the D−terms, it is clear that V will only be negative in the range

$$\frac{Nm^2}{2A\lambda} < \phi < \frac{2A}{N'\lambda} \; , \tag{122}$$

which implies

$$\phi \sim \frac{M_S}{\lambda} \; . \tag{123}$$

Now, if $\lambda \ll 1$, then $\phi \gg M_S$. In that case it is clear that the D−terms must be essentially cancelled (i.e. property 3), otherwise they would contribute a positive amount of order $g^2|\phi|^4$ that would dominate the potential (121). Furthermore, from (123), it follows that two trilinear scalar terms with different Yukawa couplings cannot efficiently "cooperate" to improve the CCB bounds (i.e. property 1): the potential can only be negative in two separate regions in the field-space given by eq.(123) applied to each coupling. In any of these regions, the presence of the extra trilinear term plus the associated mass and F terms can only yield a positive contribution to the potential. An explicite example of this argument can be found below eq.(37)

Property 1

As we have already mentioned, according to this property the most dangerous CCB directions in the MSSM potential involve only one particular trilinear soft term.

Since it is not possible to get an analytical formulation of the general CCB minima with all the fields and couplings in the game, the proof of the previous statement can only come from an exhaustive analysis of all the ways in which two or more different trilinear scalar terms could cooperate to improve the CCB bounds. Next we consider all the cases in a separate way[14].

$\lambda \mathbf{QH_2u} + \lambda' \mathbf{Q'H_2u'} \; ; \; \lambda \ll \lambda'$

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[14]We simplify somewhat the notation (in an obvious way) to go more straightforwardly through the arguments. Likewise, in some specific points we will use the assumption of universality to simplify the arguments, but these can easily be extended with slight modifications to more general cases.

Here we consider the simultaneous presence in the Lagrangian of two different couplings of the u type and the corresponding terms in the scalar potential from the associated D–terms, F–terms and soft terms. According to the notation of the heading, the pair of quarks $\{u, u'\}$ may represent $\{u, c\}$, $\{u, t\}$ or $\{c, t\}$. It is convenient for our analysis to roughly divide the field-space in the three following regions

a) $Qu \ll Q'u'$

b) $Qu \sim Q'u'$

c) $Qu \gg Q'u'$

where $Q, u, Q', u' > 0$ without loss of generality. Let us examine the CCB issue in each zone separately, taking for simplicity $Q = u$, $Q' = u'$.

a) All terms in the potential involving Q and/or u are negligible, so the only significant term is the λ' one. Therefore the (a) area is irrelevant for property 1.

b) In this region $A\lambda Q H_2 u \ll A'\lambda' Q' H_2 u'$, so, again, property 1 cannot be disproved here. We can check however that the region (b) is anyway irrelevant for CCB bounds. The only terms in V where the presence of Q, u is relevant are

$$(m_Q^2 + m_u^2)Q^2 \ + \ D - \text{terms} \ , \tag{124}$$

where we have used $Q = u$.

In the case where $\lambda = \lambda_u$, $\lambda' = \lambda_c$, it happens that, very accurately, $m_Q^2 = m_{Q'}^2$, $m_u^2 = m_{u'}^2$. Therefore Q^2 occurs in V only through the combination $\hat{Q}^2 \equiv Q^2 + Q'^2$. Along any direction with $Q^2/Q'^2 =$ const. the relevant terms in the potential can be writen as

$$-2A'\hat{\lambda} H_2 \hat{Q}^2 + (m_Q^2 + m_u^2)\hat{Q}^2 + \hat{\lambda}^2 \hat{Q}^4 + 2\hat{\lambda}^2 H_2^2 \hat{Q}^2 (1 + \frac{Q^2}{Q'^2}) + D - \text{terms} + \cdots \ , \tag{125}$$

where $\hat{\lambda} = \lambda' Q'^2/\hat{Q}^2$ and the D–terms are a function of $\hat{Q}^2$. Therefore everything occurs as if there were a single coupling $\hat{\lambda} \hat{Q} H_2 \hat{u}$, except for the additional (positive) term proportional to $\frac{Q^2}{Q'^2}$. Recalling now that in the case of a coupling $\ll 1$, the general CCB bound does not depend on the value of the coupling itself[15], it is clear that the optimum direction arises for $\frac{Q^2}{Q'^2} = 0$. Thus the (b) region is irrelevant.

When $\lambda' = \lambda_t$, the previous argument is not valid, but it is still true from (124) that the same role of Q can be played by a slepton L with exactly the same VEV along the ν_L direction. Then the D–terms are exactly the same but, since $m_L^2 < m_Q^2 + m_u^2$, it is clear that the potential becomes deeper. Consequently, the (b) region does never correspond to an improved CCB bound.

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[15] For more details, see sect.4, e.g. eqs.(50), (51).

c) In this region the effect of Q', u' in the mass terms is negligible and it is convenient to look at the potential "from the point of view" of $\lambda Q H_2 u$ as the relevant coupling. The relevant terms of the potential are

$$
\begin{aligned}
V &= \left(m_Q^2 + m_u^2\right) Q^2 - 2AH_2\left(\lambda Qu + \lambda' Q'u'\right) + m_2^2 H_2^2 + m_1^2 H_1^2 - 2m_3^2 H_1 H_2 \\
&+ \left|\mu H_1 + \lambda Qu + \lambda' Q'u'\right|^2 + 2\left|\lambda H_2 Q\right|^2 + 2\left|\lambda' H_2 Q'\right|^2 \\
&+ \frac{1}{8}(g^2 + g'^2)\left|H_2^2 - H_1^2 - Q^2\right|^2 \;,
\end{aligned}
\tag{126}
$$

where for simplicity we have taken $A = A'$.

For $\lambda' Q' \sim \lambda Q$, or smaller, it is clear that the only non-negligible term involving Q' is $2\left|\lambda' H_2 Q'\right|^2$, which is positive. Thus, a value of Q' of this order can never be useful to make the potential deeper.

For greater values of Q', in particular $\lambda' Q'u' \sim \lambda Qu$, there appear new relevant terms in the potential involving Q', u', as can be seen from (126). In this case the potential (126) can be reformulated as if it was derived from a single coupling $\hat{\lambda} Q H_2 u$ with $\hat{\lambda} \equiv \lambda(1 + \lambda' Q'^2 / \lambda Q^2)$, except for the terms

$$
2\left|\lambda H_2 Q\right|^2 + 2\left|\lambda' H_2 Q'\right|^2 = 2\left|\hat{\lambda} H_2 Q\right|^2 \frac{1 + \frac{\lambda' Q'^2}{\lambda Q^2}\frac{\lambda'}{\lambda}}{\left(1 + \frac{\lambda' Q'^2}{\lambda Q^2}\right)^2} \;,
\tag{127}
$$

which appear instead of the $2\left|\hat{\lambda} H_2 Q\right|^2$ term. Since $\lambda' \gg \lambda$, it is clear that as long as $\frac{\lambda' Q'^2}{\lambda Q^2} \ll \frac{\lambda'}{\lambda}$ (which, by definition, always occurs in the (c) region), (127) is bigger than $2\left|\hat{\lambda} H_2 Q\right|^2$, so the CCB bounds obtained in this region are less stringent than those obtained by consideration of a unique coupling $\lambda Q H_2 u$ (recall that for small couplings the form of the CCB bound does not depend on the value of the coupling itself).

$\lambda \mathbf{Q H_1 d} + \lambda' \mathbf{Q' H_1 d'}$; $\lambda \ll \lambda'$

This case can be analyzed along similar lines than the previous heading, with analogous results.

$\lambda \mathbf{Q H_1 d} + \lambda' \mathbf{Q' H_2 u'}$; $\lambda \ll \lambda'$

Again, we divide the field-space in the three regions

a) $Qd \ll Q'u'$

b) $Qd \sim Q'u'$

c) $Qd \gg Q'u'$

with $Q, d, Q', u' > 0$.

a) Similarly to the previous heading, this case is irrelevant.

b) In this region the trilinear scalar term $\lambda A Q H_1 d$ is negligible, so property 1 cannot be disproved. Let us note anyway that the only relevant terms involving the Q, d fields are the mass terms and the D–terms. Hence their role can be more profitably played by sleptons, which have lower masses. More precisely, a single slepton L (taken along the ν_L direction) will be needed if $H_2^2 - Q'^2 - H_1^2 > 0$, while two sleptons, L (along l_L), l_R, will be needed if $H_2^2 - Q'^2 - H_1^2 < 0$. (Furthermore the leptonic coupling λ_l must be $\lambda_l \ll \lambda'$, so a choice that always works is to take the slepton from the first generation.)

c) Analogously to the previous heading, this region is more conveniently seen "from the point of view" of $\lambda Q H_1 d$ as the relevant coupling. The only relevant terms in the potential involving Q', u' are

$$-2A'\lambda' Q'^2 H_2 + \left| \mu H_1 + \lambda' Q'^2 \right|^2 + 2 \left| \lambda' H_2 Q' \right|^2 \tag{128}$$

(recall we are taking $Q' = u'$). Clearly, if $Q' \neq 0$, only the first two terms can be useful to make the potential deeper. The first term will only be significant if $H_2 \sim H_1$, but then the (positive) third term dominates the potential. Therefore we conclude that $Q' \neq 0$ can only be relevant for the CCB bounds if $H_2 = 0$ or negligible. Then, the Q' value can be optimally adjusted so that

$$\left| \mu H_1 + \lambda' Q'^2 \right|^2 = 0 \ . \tag{129}$$

Of course, this possibility has been considered in the analysis of the CCB bounds (see CCB–1 bound in the main text). In any case, note that, since $H_2 = 0$, the *only* relevant trilinear scalar term is $\lambda Q H_2 d$, in agreement with property 1.

$\boldsymbol{\lambda Q H_1 d + \lambda' Q' H_2 u'} \ ; \ \boldsymbol{\lambda \gg \lambda'}$

This case is completely analogous to the previous one interchanging $Q \leftrightarrow Q'$, $d \leftrightarrow u'$, $H_1 \leftrightarrow H_2$, $\lambda \leftrightarrow \lambda'$,

$\boldsymbol{\lambda L H_1 l + \textbf{other couplings}}$

The analysis is completely similar to that of $\lambda Q H_1 d$ + other couplings in the three previous headings. The only exception is that when $\lambda L H_1 l$ corresponds to the electron coupling there is no slepton, say L', with smaller Yukawa coupling, to play with (the existence of such an slepton is used when analyzing the (b) region above). However, this is irrelevant in practice since

- The leptonic couplings of μ, τ turn out to give more stringent CCB restrictions than the electron one, as can be seen in the text (see sect.6).

- For all the leptonic couplings the direction with $L' \neq 0$ is never the most dangerous one.

Two couplings with $\lambda \sim \lambda'$

This case represents the only possible exception to property 1.

A paradigmatic example would be to consider the bottom and tau couplings

$$\lambda_b Q H_1 b + \lambda_\tau L H_1 \tau \ . \tag{130}$$

In the extreme (and non–realistic) case that $\lambda_b = \lambda_\tau$, $m_Q^2 = m_L^2$, $m_b^2 = m_\tau^2$, $A_b = A_\tau$ (at the correct scale), it is easy to see that for a given value of $|\phi|^2 \equiv |Q|^2 + |L|^2$ the potential is independent of the particular values of $|Q|^2, |L|^2$. In practice, however, the previous equalities do not hold, in particular, typically $m_Q^2 > m_L^2$, $m_b^2 > m_\tau^2$. Hence, it becomes profitable to use just one of the two VEVs, typically $|L|^2$, as it is confirmed by the numerical results (see sect.6). Consequently, in this case property 1 holds.

Other examples can arise for particular values of $\tan\beta$. For example

$$\lambda_u Q H_2 u + \lambda_l L H_1 l \ . \tag{131}$$

can have $\lambda_u \sim \lambda_l$ for particular choices of (u,l) and particular values of $\tan\beta$ (e.g. for $(u,l) = (c,\tau)$ and $\tan\beta \sim 2$).

Playing just with the fields appearing in (131), it is possible to arrive to an optimized CCB condition

$$\left(|A_u| + |A_l| \gamma \frac{\lambda_l}{\lambda_u} \frac{\gamma_l^2}{\alpha^2} + |\mu|\gamma + |\mu| \frac{\lambda_l}{\lambda_u} \frac{\gamma_l^2}{\alpha^2} \right)^2$$
$$< \left[1 + \frac{2}{\alpha^2} + \left(\frac{\lambda_l}{\lambda_u} \right)^2 \left(\left(\frac{\gamma_l}{\alpha} \right)^4 + 2 \frac{\gamma^2 \gamma_l^2}{\alpha^4} \right) \right]$$
$$\times \left[m_2^2 + (m_Q^2 + m_u^2)\alpha^2 + m_1^2 \gamma^2 + (m_L^2 + m_l^2)\gamma_l^2 - 2|m_3^2|\gamma \right] \tag{132}$$

where $\gamma = H_2/H_1$, $\alpha^2 = Q^2/H_2^2$, $\gamma_l^2 = L^2/H_2^2$ and $1 - \gamma^2 - \gamma_l^2 - \alpha^2 = 0$. Actually, eq.(132) holds if sign(A)=–sign(B). In the opposite case we have to change the sign either of the $\propto |m_3^2|$ or of the $\propto |\mu|$ terms in the previous equation. We have not used this type of condition in the examination of the CCB bounds of the MSSM (see sections 4–6).

Property 3

In the general property 3 of subsect.4.1 it was stated that if the trilinear term under consideration has a Yukawa coupling $\lambda^2 \ll 1$, which occurs in all the cases except for the top, then the corresponding deepest CCB direction occurs for vanishing (or negligible) D–terms. Next, we prove this property taking for definiteness the trilinear coupling

$$\lambda Q H_2 u \tag{133}$$

as the relevant coupling and considering (a priori) non-vanishing VEVs for the fields H_2, Q (taken along the u_L direction), u, H_1, parameterized as

$$|Q| = \alpha|H_2|, \ \ |u| = \beta|H_2|, \ \ |H_1| = \gamma|H_2| \tag{134}$$

(a non-vanishing VEV for a slepton could be also included in the analysis). For simplicity we will focuss on the $SU(2) \times U(1)$ D–terms, so we will assume for the moment

$$\alpha = \beta \tag{135}$$

and thus $Q = u$. Then the corresponding scalar potential has the form

$$
\begin{aligned}
V \;=\;& \left(m_Q^2 + m_u^2\right)|Q|^2 + m_2^2|H_2|^2 + m_1^2|H_1|^2 - \left(m_3^2 H_1 H_2 \;+\; \text{h.c.}\right) \\
& +\; 2\left|\lambda H_2 Q\right|^2 + \left|\lambda Q^2\right|^2 \\
& +\; \left(\lambda A Q^2 H_2 + \lambda \mu Q^2 H_1^* \;+\; \text{h.c.}\right) \\
& +\; \frac{1}{8}(g^2 + g'^2)\left[|H_2|^2 - |H_1|^2 - |Q|^2\right]^2 \;.
\end{aligned}
\tag{136}
$$

The strategy of our proof is to suppose that the values of the fields are in such a way that D–terms$\neq 0$, and then show that no CCB minimum can arise in this situation.

The first consideration is that if D–terms$\neq 0$, then, necessarily, all the terms involving λ are irrelevant. For the quartic F–terms (second line of (136)), this is obvious since $\lambda^2 \ll (g^2 + g'^2)$. The trilinear terms (third line of (136)) can only be competitive with the D–terms if the generic value of the involved fields (say ϕ) is $|\phi| \lesssim \frac{\lambda}{g^2} M_S$ (recall that $A, \mu = O(M_S)$). In that case, both the trilinear and the D–terms are negligible compared to the mass terms. Let us also note that if the values of the fields are tuned in such a way that the D–terms are non-vanishing but small enough to be comparable with the rest of the terms, then it is always favoured to slightly modify those values so that D–terms$\rightarrow 0$ (or negligible), since this is accomplished with almost no cost in the rest of the terms. Consequently, in any case we can write the scalar potential as

$$
V = |H_2|^2 \hat{m}^2(\alpha, \gamma) + |H_2|^4 \frac{1}{8}(g^2 + g'^2)\left[1 - \alpha^2 - \gamma^2\right]^2
\tag{137}
$$

with

$$
\hat{m}^2(\alpha, \gamma) = m_2^2 + \left(m_Q^2 + m_u^2\right)\alpha^2 + m_1^2\gamma^2 - 2m_3^2\gamma \;.
\tag{138}
$$

If α, γ are such that $\hat{m}^2(\alpha, \gamma) < 0$, then V has a minimum in the H_2 direction at

$$
|H_2|^2_{\min} = \frac{-4\hat{m}^2(\alpha, \gamma)}{(g^2 + g'^2)\left[1 - \alpha^2 - \gamma^2\right]^2} \;,
\tag{139}
$$

$$
V_{\min}(\alpha, \gamma) = \frac{-2|\hat{m}(\alpha, \gamma)|^4}{(g^2 + g'^2)\left[1 - \alpha^2 - \gamma^2\right]^2} \;.
\tag{140}
$$

It is important to stress that this is *not* necessarily a CCB minimum (in fact it will never be) since we have still to minimize with respect to α, γ and we could well find $\alpha = 0$ in that process. Actually, the realistic minimum, $V_{\text{real min}}$ (see eq.(10)), is a particular case of (140), more precisely

$$
V_{\text{real min}} = V_{\min}(\alpha = 0, \gamma = \gamma_{\text{real}}) = -\frac{\left\{\left[\,(m_1^2 + m_2^2)^2 - 4|m_3|^4\,\right]^{1/2} - m_1^2 + m_2^2\right\}^2}{2\left(g^2 + g'^2\right)}
\tag{141}
$$

with

$$
(\gamma_{\text{real}})^{-1} = \tan\beta = \frac{m_1^2 + m_2^2}{2m_3^2} + \sqrt{\left(\frac{m_1^2 + m_2^2}{2m_3^2}\right)^2 - 1} \;.
\tag{142}
$$

Of course, one has to demand

$$
V_{\min}(\alpha, \gamma) > V_{\text{real min}} \;,
\tag{143}
$$

well understood that (143) does *not* necessarily mean that we are comparing the relative depth of two minima of V, since $V_{\min}(\alpha, \gamma)$ may not correspond to an actual minimum.

A necessary condition for (143) to be satisfied is that

$$\left.\frac{\partial V_{\min}(\alpha, \gamma)}{\partial \alpha^2}\right|_{\alpha=0, \gamma=\gamma_{\text{real}}} > 0 \ . \tag{144}$$

This condition was worked out in ref.[4], but without including γ in the game (the authors took $\gamma = 0$). Now, it is clear that (144) corresponds to the requirement that $V_{\text{real min}}$ is an actual minimum in the whole field-space (not just in H_1, H_2). As it was mentioned in sect.2, this is simply equivalent to demand all the scalar mass eigenvalues to be positive. If this is demanded from the beginning (as it should be), eq.(144) is a redundant condition. In fact (144) has the explicite form

$$m_Q^2 + m_u^2 > \frac{1}{2}\left|\left[(m_1^2 + m_2^2)^2 - 4|m_3|^4\right]^{1/2} - m_1^2 + m_2^2\right| \ , \tag{145}$$

which is equivalent to require that the sum of the two mass eigenvales of the u–mass matrix is positive. For our later convenience, let us note that (145) implies

$$m_Q^2 + m_u^2 + m_2^2 > 0 \ . \tag{146}$$

In order to study the relevance of (143) we must consider the *minimum* of $V_{\min}(\alpha, \gamma)$ in the α, γ variables. It is interesting to check that $\alpha = 0, \gamma = \gamma_{\text{real}}$ does correspond to a minimum (the realistic one). However, there might be other minima. A necessary condition to have a minimum is $\hat{m}^2(\alpha, \gamma) < 0$, which implies

$$m_2^2 + m_1^2\gamma^2 - 2m_3^2\gamma < 0 \ . \tag{147}$$

Using $m_1^2 + m_2^2 > 2m_3^2$ (eq.(13)) and $m_1^2 > m_2^2$, it is clear that (147) can only be satisfied in a certain range of values of γ:

$$0 \leq \gamma_{inf} \leq \gamma \leq \gamma_{sup} < 1 \ . \tag{148}$$

Now we can write the minimization condition for α

$$\begin{aligned}
\frac{\partial V_{\min}(\alpha, \gamma)}{\partial \alpha^2} &= \frac{-4\hat{m}^2(\alpha, \gamma)}{[1 - \alpha^2 - \gamma^2]^3}\left[m_2^2 + m_1^2\gamma^2 - 2m_3^2\gamma + (m_Q^2 + m_u^2)(1 - \gamma^2)\right] \\
&= \frac{-4\hat{m}^2(\alpha, \gamma)}{[1 - \alpha^2 - \gamma^2]^3}\hat{m}^2(\alpha^2 = 1 - \gamma^2, \gamma) \ , \tag{149}
\end{aligned}$$

where the quantity $\hat{m}^2(\alpha^2 = 1 - \gamma^2, \gamma)$ satisfies

$$\begin{aligned}
\hat{m}^2(\alpha^2 = 1 - \gamma^2, \gamma = 0) &> 0 \\
\hat{m}^2(\alpha^2 = 1 - \gamma^2, \gamma = 1) &> 0 \ . \tag{150}
\end{aligned}$$

To analyze (149) we can distinguish two cases

a) $m_1^2 < m_Q^2 + m_u^2$

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In this case (which is the usual one) $\hat{m}^2(\alpha^2 = 1 - \gamma^2, \gamma)$ is a monotonically decreasing function in the range $0 \le \gamma \le 1$, so from (150) it follows that

$$\hat{m}^2(\alpha^2 = 1 - \gamma^2, \gamma) > 0 \tag{151}$$

in all this range. Then, since $\hat{m}^2(\alpha^2 > 1 - \gamma^2, \gamma) > \hat{m}^2(\alpha^2 = 1 - \gamma^2, \gamma)$, it is clear from the condition $\hat{m}^2(\alpha, \gamma) < 0$ that (149) is only meaningful in a certain range

$$0 \le \alpha^2 \le \alpha_{sup}^2 < 1 - \gamma^2 \ . \tag{152}$$

Hence, it follows from (151), (149) that $\frac{\partial V_{\min}(\alpha, \gamma)}{\partial \alpha^2} > 0$ in all the range (152), and therefore the optimum value of α is always $\alpha = 0$. Consequently, there are *no* CCB minima.

b) $m_1^2 > m_Q^2 + m_u^2$

This is a rather unusual, but still possible case. Now $\hat{m}^2(\alpha^2 = 1 - \gamma^2, \gamma)$ is not monotonically decreasing in the range $0 \le \gamma \le 1$, but it has a minimum. However, if

$$m_2^2 + m_Q^2 + m_u^2 - \frac{m_3^4}{m_1^2 - m_Q^2 - m_u^2} > 0 \ , \tag{153}$$

it is still true that $\hat{m}^2(\alpha^2 = 1 - \gamma^2, \gamma) > 0$ in all the $0 \le \gamma \le 1$ range. Then, the argument follows exactly as in the previous case *(a)*. If (153) is not satisfied, then there is a segment of γ values where $\hat{m}^2(\alpha^2 = 1 - \gamma^2, \gamma) < 0$ (only the part of the segment overlaping (148) is relevant). For these values of γ it is clear from (149) that $\frac{\partial V_{\min}(\alpha, \gamma)}{\partial \alpha^2} < 0$ for $\alpha^2 < 1 - \gamma^2$ and $\frac{\partial V_{\min}(\alpha, \gamma)}{\partial \alpha^2} > 0$ for $\alpha^2 > 1 - \gamma^2$. Therefore *there is* a CCB minimum at $\alpha^2 = 1 - \gamma^2$, *but* this is precisely the point where D–terms$= 0$. Note also from (140) that at this point $V \to -\infty$, but this is not right since if D–terms$= 0$, we cannot neglect the terms involving λ any more.

Finally, had we included the $SU(3)$ D–term in the game (relaxing eq.(135)), it is easy to convince yourself that the whole argument would have followed analogously.

Property 5

The last property concerns the optimum choice of the phases of the fields involved in the scalar potential when analyzing CCB minima. Taking again $\lambda Q H_2 u$ as the relevant coupling, the relevant terms in the superpotential are

$$W = \epsilon_{ij} \lambda H_{2i} Q_j u + \mu \epsilon_{ij} H_{1i} H_{2j} \ , \tag{154}$$

The corresponding terms in the scalar potential without a definite phase, say $V_{\rm ph}$, are

$$\begin{aligned} V_{\rm ph} &= (A\lambda \epsilon_{ij} H_{2i} Q_j u + {\rm h.c.}) \\ &+ (B\mu \epsilon_{ij} H_{1i} H_{2j} + {\rm h.c.}) - (\mu^* \lambda H_{1i}^* Q_i u + {\rm h.c.}) \end{aligned} \tag{155}$$

We will take λ, μ, A, B as real numbers for simplicity and also because their phases are quite constrained by limits on the electric dipole moment of the neutron since they give large one-loop contributions to this CP-violating quantity. The following

results are independent of the signs of μ, λ, as well as on the form in which the two $SU(2)$ contractions in (154) are defined. This comes from the fact that all these signs can be re-absorbed in phase redefinitions of the fields involved. Along the direction $H_1^o, H_2^o, u_L, u_R \neq 0$ at which the CCB minima appear (see text), V_{ph} can be re-writen as

$$
\begin{aligned}
V_{\text{ph}} = \quad & -2\,|A\lambda H_2 Q u|\ \text{sign}(A)\,\text{sign}(\lambda)\ \cos(\alpha + \beta) \\
& + 2\,|B\mu H_1 H_2|\ \text{sign}(B)\,\text{sign}(\mu)\ \cos(\alpha + \gamma) \\
& - 2\,|\mu\lambda H_1^* Q u|\ \text{sign}(\mu)\,\text{sign}(\lambda)\ \cos(\beta - \gamma)\ ,
\end{aligned}
\tag{156}
$$

where $\alpha = \text{phase}(H_2^o)$, $\beta = \text{phase}(u_L u_R)$, $\gamma = \text{phase}(H_1^o)$.

Of course, if $H_1 = 0$, the only non-vanishing term in (156) is the one proportional to A, which, for exploring minima of the potential, can always be writen as

$$
V_{\text{ph}} = -2\,|A\lambda H_2 Q u|\ .
\tag{157}
$$

If $H_1 \neq 0$ and $\text{sign}(A) = -\,\text{sign}(B)$, it is straightforward to check from (156) that α, β, γ can be taken so that the three terms become negative, which of course corresponds to the deepest direction in V_{ph}, i.e.

$$
V_{\text{ph}} = \quad -2\,|A\lambda H_2 Q u| - 2\,|B\mu H_1 H_2| - 2\,|\mu\lambda H_1^* Q u|\ .
\tag{158}
$$

If $H_1 \neq 0$ and $\text{sign}(A) = \text{sign}(B)$, the previous direction (158) is no longer available. Then V_{ph} can be expressed as

$$
V_{\text{ph}} = C_1 \cos(\varphi_1) + C_2 \cos(\varphi_2) + C_3 \cos(\varphi_1 - \varphi_2)\ ,
\tag{159}
$$

where $C_i > 0$ are the three absolute values of eq.(156), ordered for convenience so that

$$
C_1 \geq C_2 \geq C_3\ ,
\tag{160}
$$

and the φ_i phases are certain independent combinations of α, β, γ and the signs of A, B, λ, μ. For fixed values of C_i, the minimization in the φ_1, φ_2 variables gives the following result:

- If

$$
\frac{C_2}{C_3} \geq 1 + \frac{C_2}{C_1}
\tag{161}
$$

  (this is by far the most usual case), then the minimum in the φ_i space lies on

$$
\varphi_1 = \pi,\ \varphi_2 = \pi\ ,
\tag{162}
$$

  i.e. in this case V_{ph} can simply be expressed as

$$
V_{\text{ph}} = -C_1 - C_2 + C_3\ .
\tag{163}
$$

- If

$$\frac{C_2}{C_3} \leq 1 + \frac{C_2}{C_1} \quad , \tag{164}$$

then the optimum choice of phases is given by

$$\left| \frac{\sin \varphi_2}{\sin \varphi_1} \right| = \frac{C_1}{C_2}$$

$$\left| \frac{\sin \varphi_1}{\sin(\varphi_1 - \varphi_2)} \right| = \frac{C_3}{C_1} \quad , \tag{165}$$

which substituted in (159) gives

$$V_{\text{ph}} = -\frac{1}{2} C_1 C_2 C_3 \left(\frac{1}{C_1^2} + \frac{1}{C_2^2} + \frac{1}{C_3^2} \right) \quad . \tag{166}$$

Clearly, (164) is much more unlikely than (161) and harder to handle (compare eqs.(162,163) with eqs.(165, 166). Furthermore, in the rare cases corresponding to (164), eqs.(162,163) still provide a very good approximation[16] to the actual minimum of V_{ph}. In conse-quence, we have always used eq.(163) as the optimum direction of V_{ph} when sign$(A) =$ sign(B).

Finally, let us point out that all the previous results about the choice of phases translate unchanged to the cases in which the relevant coupling is of the $\lambda Q H_1 d$ or $\lambda L H_1 e$ types.

---

[16]The worst situation occurs for $C_1 = C_2 = C_3$, where the actual minimum of V_{ph} is $-3C_1/2$, while eq.(163) gives $-C_1$.

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Figure Captions

Fig. 1 Excluded regions in the parameter space of the Minimal Supersymmetric Standard Model, with $B = A - m$, $m = 100$ GeV and $M_{\text{top}}^{\text{phys}} = 174$ GeV. The central darked region is excluded because there is no solution for μ capable of producing the correct electroweak breaking. The upper and lower darked regions are excluded because it is not possible to reproduce the experimental mass of the top. a) The circles and diamonds indicate regions excluded by the "traditional" Charge and Color Breaking constraints associated with the e and d-type trilinear terms respectively. b) The same as (a) but using our "improved" Charge and Color Breaking constraints. The triangles correspond to the u-type trilinear terms. c) The crosses, squares and small filled squares indicate regions excluded by the Unbounded From Below-1,2,3 constraints respectively. d) The previous excluded regions together with the one arising from the experimental lower bounds on supersymmetric particle masses (filled diamonds).

Fig. 2 Excluded regions in the parameter space of the Minimal Supersymmetric Standard Model, with $B = A - m$ and $M_{\text{top}}^{\text{phys}} = 174$ GeV, for different values of m. The central darked region is excluded because there is no solution for μ capable of producing the correct electroweak breaking. The upper and lower darked regions are excluded because it is not possible to reproduce the experimental mass of the top. The small filled squares indicate regions excluded by our Unbounded From Below constraints. The circles indicate regions excluded by our "improved" Charge and Color Breaking constraints. The filled diamonds indicate regions excluded by the experimental lower bounds on supersymmetric particle masses. The ants indicate regions excluded by negative scalar squared mass eigenvalues.

Fig. 3 The same as Fig. 2 but with $B = 2m$. Now, the whole darked region is excluded because it is not possible to reproduce the experimental mass of the top.

Fig. 4 Contours of allowed regions in the parameter space of the Minimal Supersymmetric Standard Model, with $M_{\text{top}}^{\text{phys}} = 174$ GeV and different values of B and m, by the whole set of constraints.