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Doppler Peaks: A Fingerprint of Topological Defects.

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Abstract

The fluctuations in the cosmic microwave background (CMB) on large angular scales (> few degrees) are caused by perturbations in the gravitational field via the Sachs–Wolfe effect. On intermediate scales, $0.1^\circ \lesssim \theta \lesssim 2^\circ$, the dominant contribution is due to coherent oscillations in the baryon radiation plasma before recombination. Unless the universe is reionized at some redshift $z > 50$, these oscillations lead to the 'Doppler peaks' in the angular power spectrum. In structure formation scenarios based on inflation the position of the first peak is typically at $\ell \sim 200$, with a height which is $4-6$ times that of the Sachs–Wolfe 'plateau'. Here we present a corresponding study for perturbations induced by global textures. We find that the first Doppler peak is reduced to an amplitude comparable to that of the Sachs–Wolfe contribution, and that it is shifted to $\ell \sim 350$. We believe that our analysis can be easily extended to other types of global topological defects and general global scalar fields.

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Presently there are two main classes of models to explain the origin of large scale structure formation. Initial perturbations can either be due to quantum fluctuations of a scalar field during an inflationary era[1], or they may be seeded by topological defects formed during a symmetry breaking phase transition in the early universe[2]. The CMB anisotropies are a powerful tool to discriminate among these models by purely linear analysis. Usually CMB anisotropies are parameterized in terms of C_ℓ's, defined as the coefficients in the expansion of the angular correlation function

$$\langle \frac{\delta T}{T}(\mathbf{n})\frac{\delta T}{T}(\mathbf{n}')\rangle\Big|_{(\mathbf{n}\cdot\mathbf{n}'=\cos\vartheta)} = \frac{1}{4\pi}\sum_\ell (2\ell+1)C_\ell P_\ell(\cos\vartheta).$$

For scale invariant spectra of perturbations $\ell(\ell+1)C_\ell$ is constant on large angular scales, say $\ell \lesssim 50$. Both inflation and topological defect models lead to approximately scale invariant spectra on large scales.

Large scale CMB anisotropies are mainly caused by inhomogeneities in the spacetime geometry via the Sachs–Wolfe effect[3]. On smaller angular scales ($0.1^\circ \lesssim \theta \lesssim 2^\circ$) the dominant contribution comes from coherent oscillations in the baryon–radiation plasma prior to recombination. On even smaller scales the anisotropies are damped due to the finite thickness of the recombination shell, as well as by photon diffusion during recombination (Silk damping).

Disregarding Silk damping, gauge invariant linear perturbation analysis leads to[4]

$$\frac{\delta T}{T} = \left[-\frac{1}{4}D_g^{(r)} - V_j n^j - \Psi + \Phi\right]_i^f + \int_i^f (\Psi' - \Phi')d\tau\ , \tag{1}$$

where Φ and Ψ are quantities describing the perturbations in the geometry and $\mathbf{V}$ denotes the peculiar velocity of the radiation fluid with respect to the overall Friedman expansion. $D_g^{(r)}$ specifies the intrinsic density fluctuation in the radiation fluid. There are several gauge invariant variables which describe density fluctuations; they all differ on super–horizon scales but coincide inside the horizon. Below we use another variable, D_r, for the radiation density fluctuation[5]. Since the coherent oscillations giving rise to the Doppler peaks act only on sub–horizon scales, the choice of this variable is irrelevant for our calculation.

Φ , Ψ and $D_g^{(r)}$ in Eq. (1) determine the anisotropies on large angular scales[1], and have been calculated for both inflation and defect models [6, 7, 8, 9]. Generically,

[1]One might think that $D_g^{(r)}$ leads *just* to coherent oscillations of the baryon radiation fluid, but this is not the case. Note that, e.g., for adiabatic CDM models without source term one can derive $(1/4)D_g^{(r)} = -(5/3)\Psi$ on super–horizon scales. Since for CDM perturbations $\Phi = -\Psi$ and $\Psi' \simeq 0$,

a scale invariant spectrum is predicted and thus the Sachs–Wolfe calculations yield mainly a normalization for the different models. On the other hand the amplitude of the Doppler peak, which most probably will be measured in the near future, might be an important discriminating tool between them. In this *Letter* we present a computation for the Doppler contribution from global topological defects; in particular we perform our analysis for π_3–defects, textures [10], in a universe dominated by cold dark matter (CDM). We believe that our main conclusion remains valid for all global defects.

The Doppler contribution to the CMB anisotropies is given by

$$\left[\frac{\delta T}{T}(\mathbf{x},\mathbf{n})\right]^{Doppler} = \frac{1}{4}D_r(\mathbf{x}_{rec},t_{rec}) + \mathbf{V}(\mathbf{x}_{rec},t_{rec})\cdot\mathbf{n}, \qquad (2)$$

where $\mathbf{x}_{rec} = \mathbf{x} - \mathbf{n}t_0$. In the previous formula $\mathbf{n}$ denotes a direction in the sky and t is the conformal time, with t_0 and t_{rec} the present and recombination times, respectively. To evaluate Eq. (2) we calculate D_r and $\mathbf{V}$ at t_{rec}. We consider a two–fluid system: baryons plus radiation, which prior to recombination are tightly coupled, and CDM. The evolution of the perturbation variables in a flat background, $\Omega = 1$, is described by[5]

$$\begin{array}{lcl} V_r' + \frac{a'}{a}V_r &=& k\Psi + k\frac{c_s^2}{1+w}D_r \\ V_c' + \frac{a'}{a}V_c &=& k\Psi \\ D_r' - 3w\frac{a'}{a}D_r &=& (1+w)[3\frac{a'}{a}\Psi - 3\Phi' - kV_r - \frac{9}{2}\left(\frac{a'}{a}\right)^2 k^{-1}(1+\frac{w\rho_r}{\rho})V_r] \\ D_c' &=& 3\frac{a'}{a}\Psi - 3\Phi' - kV_c - \frac{9}{2}\left(\frac{a'}{a}\right)^2 k^{-1}(1+\frac{w\rho_r}{\rho})V_c \ , \end{array} \qquad (3)$$

where subscripts $_r$ and $_c$ denote the baryon–radiation plasma and CDM, respectively; D, V are density and velocity perturbations; $w = p_r/\rho_r$, $c_s^2 = p_r'/\rho_r'$ and $\rho = \rho_r + \rho_c$. The only place where the seeds enter this system is through the potentials Ψ and Φ. These potentials can be split into a part coming from standard matter and radiation, and a part due to the seeds, $\Psi = \Psi_{(c,r)} + \Psi_s$ and $\Phi = \Phi_{(c,r)} + \Phi_s$, where Ψ_s and Φ_s

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the usual Sachs–Wolfe result $\delta T/T = (1/3)\Psi(\mathbf{x}_{rec},t_{rec})$ is recovered. Neglecting $D_g^{(r)}$, the result would be 2Ψ and therefore wrong by a factor of 6!

are determined by the energy momentum tensor of the seeds. In this way, finally the seed source terms below arise[4].

From Eqs. (3) we derive two second order equations for D_r and D_c, namely

$$\begin{aligned} D_r'' + \frac{a'}{a}[1+3c_s^2-6w+F^{-1}\rho_c]D_r' - \frac{a'}{a}\rho_c F^{-1}(1+w)D_c' & \\ +4\pi Ga^2[\rho_r(3w^2-8w+6c_s^2-1) - 2F^{-1}w\rho_c(\rho_r+\rho_c) & \\ +\rho_c(9c_s^2-7w) + \frac{k^2}{4\pi Ga^2}c_s^2]D_r - 4\pi Ga^2\rho_c(1+w)D_c &= (1+w)S\ ; \end{aligned} \quad (4)$$

$$\begin{aligned} D_c'' + \frac{a'}{a}[1+(1+w)F^{-1}\rho_r(1+3c_s^2)]D_c' - \frac{a'}{a}(1+3c_s^2)F^{-1}\rho_r D_r' & \\ -4\pi Ga^2\rho_c D_c - 4\pi Ga^2\rho_r(1+3c_s^2)[1-2(\rho_r+\rho_c)F^{-1}w]D_r &= S\ , \end{aligned} \quad (5)$$

where $F \equiv k^2(12\pi Ga^2)^{-1} + \rho_r(1+w) + \rho_c$ and S denotes a source term, which in general is given by $S = 4\pi Ga^2(\rho+3p)^{seed}$. In our case, where the seed is described by a global scalar field ϕ, we have $S = 8\pi Ga^2(\phi')^2$. From numerical simulations one finds that the integral of $a^2|\phi'|^2$ over a shell of radius k, can be modeled by[9]

$$a^2\langle|\phi'|^2\rangle(k,t) = \frac{\frac{1}{2}A\eta^2}{\sqrt{t}[1+\alpha(kt)+\beta(kt)^2]}, \quad (6)$$

with η denoting the symmetry breaking scale of the phase transition leading to texture formation. The parameters in (6) are $A \sim 3.3$, $\alpha \sim -0.7/(2\pi)$ and $\beta \sim 0.7/(2\pi)^2$. On super–horizon scales, where the source term is important, this fit is accurate to about 10%. As we argue later, analytical estimates support this finding. On small scales the accuracy reduces to a factor of 2. By using this fit in the calculation of D_r and D_c from Eqs. (4), (5) we effectively neglect the time evolution of phases of $(\phi')^2$; the incoherent evolution of these phases may smear out subsequent Doppler peaks[11], but will not affect substantially the height of the first peak.

From D_r and D_r' we calculate the Doppler contribution to the C_ℓ's according to

$$C_\ell = \frac{2}{\pi}\int dk\left[\frac{k^2}{16}|D_r(k,t_{rec})|^2 j_\ell^2(kt_0) + (1+w)^{-2}|D_r'(k,t_{rec})|^2(j_\ell'(kt_0))^2\right], \quad (7)$$

where j_ℓ denotes the spherical Bessel function of order ℓ and j_ℓ' stands for its derivative with respect to the argument. As we will see below, the angular power spectrum

$\ell(\ell+1)C_\ell$ yields the Doppler peaks.

In order to solve Eqs. (4), (5) we need to specify initial conditions. For a given scale k we choose the initial time t_{in} such that the perturbation is super–horizon and the universe is radiation dominated. In this limit the evolution equations reduce to

$$D_r'' - \frac{2}{t^2}D_r = \frac{4}{3}\frac{A\epsilon}{\sqrt{t}} \; ; \tag{8}$$

$$D_c'' + \frac{3}{t}D_c' - \frac{3}{2t}D_r' - \frac{3}{2t^2}D_r = \frac{A\epsilon}{\sqrt{t}} \; , \tag{9}$$

with particular solutions

$$D_r = -\frac{16}{15}\epsilon A t^{3/2} \; ; \; D_c = -\frac{4}{7}\epsilon A t^{3/2} \; . \tag{10}$$

In the above equations we have introduced $\epsilon \equiv 4\pi G\eta^2$, the only free parameter in the model. We consider perturbations seeded by the texture field, and therefore it is incorrect to add a homogeneous growing mode to the above solutions. With these initial conditions, Eqs. (4), (5) are easily integrated numerically, leading to the spectra for $D_r(k, t_{rec})$ and $D_r'(k, t_{rec})$ [*see*, Fig. 1].

Integrating Eq. (7), we obtain the Doppler contribution to the CMB anisotropies [*see*, Fig. 2]. For $\ell < 1000$, we find three peaks located at $\ell = 365$, $\ell = 720$ and $\ell = 950$. Silk damping, which we have not taken into account here, will decrease the relative amplitude of the third peak with respect to the second one; however it will not affect substantially the height of the first peak.

Our most important results regard the amplitude and position of the first Doppler peak, for which we find

$$\ell(\ell+1)C_\ell \Big|_{\ell\sim 360} = 5\epsilon^2 \; . \tag{11}$$

It is interesting to notice that the position of the first peak is displaced by $\Delta\ell \sim 150$ towards smaller angular scales than in inflationary models [6]. This might be due to the difference in the growth of super–horizon perturbations, which is $D_r \propto t^{3/2}$ in our case, and $D_r \propto t^2$ for inflationary models.

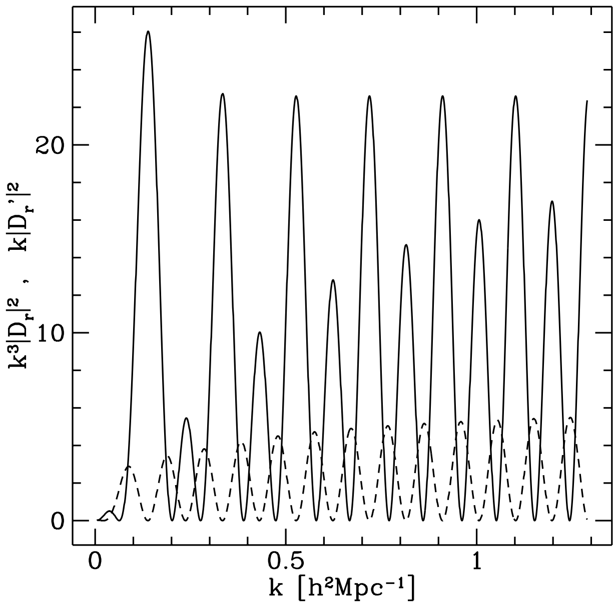

Figure 1: The dimensionless power spectra, $k^3|D_r|^2$ (solid line) and $k|D_r'|^2$ (dashed line) in units of $(A\epsilon)^2$, are shown as functions of k. These are exactly the quantities which enter in the expression for the C_ℓ's. We set $h = 0.5$, $\Omega_B = 0.05$ and $z_{rec} = 1100$.

One may understand the height of the first peak from the following analytic estimate: matching the sub–horizon with the super–horizon solutions of Eq. (5), in the matter dominated era, one finds

$$D_c \sim -\frac{2}{5}A\epsilon(k/2\pi)^{1/2}t^2 \ .$$

From Eq. (4) we then obtain in this limit $D_r \approx A\epsilon k^{-3/2}$. Plugging this latter value into Eq. (7) we get roughly

$$\ell(\ell+1)C_\ell \sim (A\epsilon)^2 \ ,$$

for the height of the first peak.

Let us now compare our value for the Doppler peak with the level of the Sachs–Wolfe plateau [7, 8, 9],

$$\ell(\ell+1)C_\ell\Big|_{SW} \sim (6-14)\epsilon^2. \qquad (12)$$

It is apparent from Eqs. (11) and (12) that the Doppler contribution from textures is substantially smaller than for inflationary models.

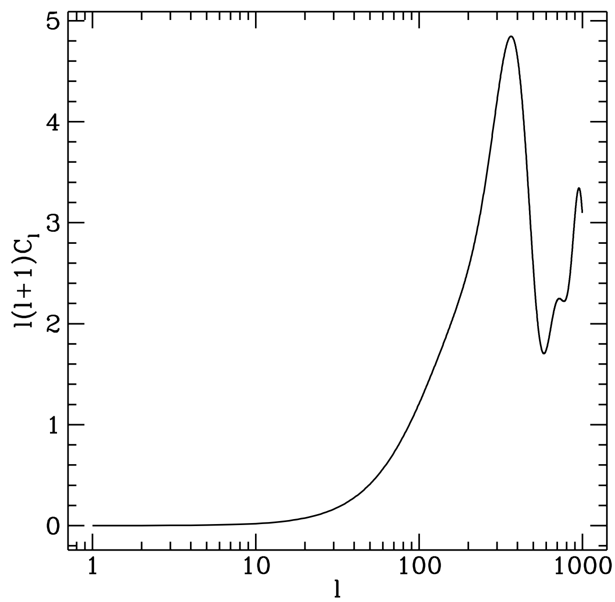

Figure 2: The angular power spectrum for the Doppler contribution to the CMB anisotropies is shown in units of ϵ^2. We set cosmological parameters $h = 0.5$, $\Omega_B = 0.05$ and $z_{rec} = 1100$.

Normalizing with *COBE*–DMR experiment[12]

$$\ell(\ell+1)C_\ell\Big|_{COBE} \sim 8 \times 10^{-10} \quad ,$$

one finds the symmetry breaking energy scale, and from this $\epsilon \sim 10^{-5}$. This value for ϵ depends of course on the numerical simulations[7, 8, 9].

We believe that our result, stating that the first Doppler peak has a height comparable to the Sachs–Wolfe plateau, is valid for all global defects. This depends crucially on the $1/\sqrt{t}$ behavior of $(a\phi')^2 = \dot\phi^2$ on large scales (cf. Eq. (6)), which is a generic feature of global defects: on super–horizon scales, $\dot\phi^2(k)$ represents white noise superimposed on the average given by $\dot\phi^2(k=0) \propto \sqrt{V}/t^2$. Since there are $N = (L/t)^3$ independent patches in a simulation of linear size L, the amplitude of $\dot\phi^2(k)$ is proportional to $L^{3/2}/(t^2\sqrt{N}) \propto 1/\sqrt{t}$. (Notice that this argument does not apply for local cosmic strings.)

Based on our analysis we conclude that if the existence of Doppler peaks is indeed confirmed and the height of the first peak is larger than about twice the level of the

Sachs–Wolfe plateau, namely

$$\ell(\ell+1)C_\ell\Big|_{peak} > 2\times 10^{-9} \quad ,$$

and if the first peak is positioned at $\ell < 300$, *then* global topological defects are ruled out. On the other hand, if the first Doppler peak is positioned at $\ell \sim 350$ and its height is below the above value, global defects are strongly favored if compared to inflationary models. To our knowledge this is the first clear fingerprint within present observational capabilities, to distinguish among these two competing models of structure formation.

As we were completing our work, a preprint[15] on the same issue, but following a different approach, came to our attention. The authors calculate the Doppler peaks from cosmic textures in the synchronous gauge. A main assumption of that analysis, which is not shared by us, is that spatial gradients in the scalar field are frozen outside the horizon, and therefore time derivatives are negligible. Even though we basically agree with the shape and position of their Doppler peaks, we draw a stronger conclusion regarding the difference between texture and inflationary power spectra, since we also calculate the height of the Doppler peaks, for which these authors do not present any estimates.

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