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$O(N)$ and RP^{N-1} Models in Two Dimensions

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Abstract

I provide evidence that the 2D RP^{N-1} model for $N \geq 3$ is equivalent to the $O(N)$-invariant non-linear σ-model in the continuum limit. To this end, I mainly study particular versions of the models, to be called constraint models. I prove that the constraint RP^{N-1} and $O(N)$ models are equivalent for sufficiently weak coupling. Numerical results for the step-scaling function of the running coupling $\bar{g}^2 = m(L)L$ are presented. The data confirm that the constraint $O(N)$ model is in the same universality class as the $O(N)$ model with standard action. I show that the differences in the finite size scaling curves of RP^{N-1} and $O(N)$ models observed by Caracciolo et al. can be explained as a boundary effect. It is concluded, in contrast to Caracciolo et al., that RP^{N-1} and $O(N)$ models share a unique universality class.

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1 Introduction

Motivated by the close analogy with non-Abelian lattice gauge theories in four dimensions non-linear σ-models in two dimensions have been studied intensively during the last 20 years. Most important both types of models were found to be asymptotically free [1].

Starting from the early eighties so called RP^{N-1} models were discussed. The spins of these models are elements of the real projective space in N dimensions. This space can be thought of as a sphere S^{N-1} where opposite points are identified. Hence in perturbation theory the RP^{N-1} model is equivalent with the $O(N)$-invariant σ-model. The fact however that the real projective space is not simply connected gives rise to topological defect structures similar to vortices in the 2D XY model. The questions discussed in the literature are whether these defects induce a phase transition at a finite coupling or whether these non-perturbative effects survive in the weak coupling limit.

The lattice action of the RP^{N-1} model mostly discussed is

$$S = -\beta \sum_{<xy>} (\vec{s}_x \vec{s}_y)^2 \ , \tag{1}$$

where $< xy >$ is a pair of nearest neighbour points on the lattice and $\vec{s}$ is a unit vector in R^N. An alternative way to identify $-\vec{s}_x$ and $\vec{s}_x$ is to introduce an Z_2 gauge field

$$S = -\beta \sum_{<xy>} z_{<xy>} \vec{s}_x \vec{s}_y \ , \tag{2}$$

where z takes the values 1 or -1.

Similar models have been introduced to describe orientational phase transitions in nematic liquid crystals [2]. These models have mainly been studied in three dimensions, where a weak first order phase transition is found (see ref. [3] and references given in [4]).

The numerical study of RP^{N-1} models in 2D gave rise to much controversy. Some authors [5, 6, 7] find that their results are consistent with a phase transition at a finite coupling, while others doubt the existence of a phase transition but still see strong crossover effects between the strong and the weak coupling regime [3, 8, 4].

Recently Caracciolo et al. [9, 10, 11] argued that there is no phase transition in the RP^{N-1} models. They claimed, based on a finite size scaling

analysis, that the RP^{N-1} models however have a weak coupling limit distinct from that of the $O(N)$-invariant σ-models. They claim even further, that a whole sequence of universality classes can be obtained from mixed models.

In the following I will give evidence that rules out the scenario presented in [9, 10, 11]. For a particular type of the action of the $O(N)$ and the RP^{N-1} model I will show that the models are exactly equivalent for sufficiently small coupling. I discuss the scaling properties of vortices of the RP^{N-1} model with standard action in the weak coupling regime. I give numerical results for the step scaling function introduced in ref. [12] that supports that the constraint model gives the same universal results as the standard action. I show that the differences in the finite size scaling curves for the RP^2 and the $O(3)$ model found in refs. [10, 11] can be explained as a boundary effect.

2   The Constraint $O(N)$ and RP^{N-1} Models

Let me first define the models. The field variable $\vec{s}_x$ is in both cases a unit vector in R^N. In the case of the $O(N)$-invariant model the Boltzmann weight of a configuration is equal to 1 if

$$\vec{s}_x \vec{s}_y > C \tag{3}$$

for all nearest neighbour pairs of sites $< xy >$. Else the Boltzmann weight is equal to 0.

In the case of the RP^{N-1} model $-\vec{s}_x$ and $\vec{s}_x$ are identified and the constraint on the field configuration is given by

$$|\vec{s}_x \vec{s}_y| > C \tag{4}$$

for all nearest neighbours $< xy >$. Equivalently one might introduce a gauge field $z_{<xy>}$ taking the values -1 or 1.

$$z_{<xy>} \vec{s}_x \vec{s}_y > C \ . \tag{5}$$

In the following I shall show that the constraint $O(N)$-invariant model and the constraint RP^{N-1} model are equivalent for $C > \cos(\pi/4)$. Let us consider a lattice where all closed paths are contractible, i.e. all closed paths can be shrunk to an elementary plaquette by removing single plaquettes sequentially. A hyper-cubical square lattice with open boundary conditions is an example for such a lattice.

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Consider the class of 2^V, where V is the number of lattice points, configurations that arise from a given configuration $\vec{s}_x$ by taking either $+\vec{s}_x$ or $-\vec{s}_x$ at each lattice point. Since

$$|\vec{s}_x\vec{s}_y| = |(-\vec{s}_x)\vec{s}_y| = |\vec{s}_x(-\vec{s}_y)| = |(-\vec{s}_x)(-\vec{s}_y)| \tag{6}$$

all configurations in such a class are either allowed or forbidden RP^{N-1} configurations. Obviously a class of configurations that is forbidden under the RP^{N-1} constraint contains no configuration that is allowed under the $O(N)$ constraint (with the same C). In the following I will demonstrate that for $C > \cos(\pi/4)$ a class of configurations that is allowed under the RP^{N-1} constraint contains exactly 2 configurations allowed under the $O(N)$ constraint, and therefore the partition functions are equal up to a trivial factor 2^{V-1}.

Take one configuration out of an allowed class of RP^{N-1} configurations. Pick one site x. Replace the spins on the other sites by

$$\vec{s}_y{}' = \vec{s}_y \prod_{<uv> \in \text{ path}(x,y)} \text{sign}(\vec{s}_u\vec{s}_v) \ . \tag{7}$$

The result of this construction is independent of the paths chosen if

$$\prod_{<uv> \in \text{closed path}} \text{sign}(\vec{s}_u\vec{s}_v) = 1 \tag{8}$$

for all closed paths on the lattice. For elementary loops consisting of four lattice-points this is the case for $C > \cos(\pi/4)$. All other paths can be successively built up out of elementary loops, since we have chosen a simply connected lattice topology. When adding an elementary loop the sign of a loop is conserved since the sign of the product of the new links in the path is the same as for the old links. Hence the sign of any closed path is 1 for $C > \cos(\pi/4)$.

The idea behind this proof has been discussed for an action similar to that in eq. (2) by Caselle and Gliozzi [6]. However for that action the rigorous proof for a pure gauge in the weak coupling limit is missing.

In Monte Carlo simulations one typically uses periodic boundary conditions, which leads to the lattice topology of a torus. Here loops exist that wind around the torus and hence cannot be contracted to an elementary loop.

In order to avoid configurations that are allowed under the RP^{N-1} constraint but not for $O(N)$ one has to require $C > \cos(\pi/L)$ where L is the extension of the lattice in units of the lattice spacing. It is important to note that such boundary effects might well survive the continuum limit in a finite size scaling analysis. However this boundary effect can be reproduced by proper boundary conditions imposed upon the $O(N)$ model. For $C > \cos(\pi/4)$ a constraint RP^{N-1} model on a periodic lattice is equivalent to a constraint $O(N)$ model with fluctuating boundary conditions. Fluctuating boundary conditions mean that in the partition function one sums over periodic as well as anti-periodic boundary conditions. In the case of anti-periodic boundary conditions one identifies $\vec{s}(0, y) = -\vec{s}(L, y)$, $\vec{s}(L + 1, y) = -\vec{s}(1, y)$ $\vec{s}(x, 0) = -\vec{s}(x, L)$ and $\vec{s}(x, L + 1) = -\vec{s}(x, 1)$.

3 Scaling of the Vortex Density for the Standard Actions

For the standard actions of the RP^{N-1} model similar arguments apply. In the limit $\beta \to \infty$ the energy of a vortex should win against the entropy and vortices should play no role in the continuum limit of the theory.

Let us identify a frustrated plaquette in eq. (2) with the center of a vortex. The classical solution of the $\vec{s}$-field for a fixed gauge field with two frustrated plaquettes has an energy proportional to $\ln r$ where r is the distance in between these two frustrated plaquettes. Hence one can find a finite r_0 such that the energy is larger than $2b_0 + \epsilon$, where b_0 is the leading coefficient in the perturbative β-function. Therefore the density of vortex pairs with a distance larger than r_0 dies out faster than the square of the inverse correlation length. Hence they can not play a role in the continuum limit of the theory.

4 Numerical Results for the Constraint Models

In this section I show that the constraint $O(N)$ model reproduces universal results of the $O(N)$-invariant σ-model. Therefore I compute the step scaling function of ref. [12] for three different values of the running coupling and compare the result with that of ref. [12] obtained with the standard action. Furthermore I estimate the correlation length at $C = \cos(\pi/4)$ using the

running coupling and also measure the correlation length for both the $O(N)$ and the RP^{N-1} model for $C < \cos(\pi/4)$ to check the importance of defects in the generation of the mass in the RP^{N-1} case.

The running coupling of ref. [12] is defined by

$$\bar{g}^2 = \frac{2}{N-1} m(L)L \ , \tag{9}$$

where $m(L)$ is the mass gap on a lattice with extension L in spatial direction. The β-function for the running coupling $\bar{g}^2$ is given by [12]

$$\beta(g^2) = -\frac{N-2}{2\pi}\bar{g}^4 - \frac{N-2}{(2\pi)^2}\bar{g}^6 - \frac{(N-1)(N-2)}{(2\pi)^3}\bar{g}^8 ... \ . \tag{10}$$

The step scaling function $\sigma(s, u)$ is the discrete version of the β-function. It gives the value of the coupling after change of L by a factor of s starting from a coupling u.

The simulation was done using the evident modification of the single cluster algorithm [13]. A bond $< xy >$ is called deleted if after the reflection of one of the spins $\vec{s}_x$ or $\vec{s}_y$ the constraint $\vec{s}_x\vec{s}_y < C$ is still satisfied.

A proof of ergodicity is given in the appendix. The simulation results listed in table 1 are based on about 10^7 single cluster updates. The correlation function was measured using the cluster-improved estimator [15] . The mass was extracted from the correlation function at distance L and $2L$.

Fitting the data of table 1 to an Ansatz

$$\Sigma(2, u, a/L) = \sigma(2, u) + c\ (a/L)^2 \ . \tag{11}$$

I obtain $\sigma(2, 1.0595) = 1.2589(10)$ from $L/a \geq 16$, $\sigma(2, 0.8166) = 0.9150(8)$ from $L/a \geq 8$ and $\sigma(2, 0.7383) = 0.8159(8)$ from $L/a \geq 8$. These results can be compared with the step scaling function obtained in ref. [12] $\sigma(2, 1.0595) = 1.2641(20)$, $\sigma(2, 0.8166) = 0.9176(8)$ and $\sigma(2, 0.7383) = 0.8166(9)$. The slight disagreement (about 2 standard deviations) might well be explained by deviations of the corrections to finite size scaling from the fit-Ansatz chosen.

The exact prediction for the mass gap given by [14]

$$\frac{m}{\Lambda_{\overline{MS}}} = \frac{8}{e} \tag{12}$$

for $N = 3$ and the conversion factor for the Λ parameters

$$\Lambda = \frac{e^{-\Gamma'(1)}}{4\pi}\Lambda_{\overline{MS}} \tag{13}$$

given in ref. [12] allows us to give an estimate for the infinite volume correlation length based on the measurement of the correlation length on a finite lattice. Taking the Monte Carlo result for the running coupling given in table 1 I obtain $\xi = 0.7 \times 10^5$ as estimate for the correlation length at $C = 0.55$ and $\xi = 0.6 \times 10^9$ as estimate at $C = \cos(\pi/4)$, where the $O(3)$ and RP^2 constraint models become identical.

In addition I performed some simulations for both the constraint $O(N)$ model and the constraint RP^{N-1} model at smaller C values such that the correlation length ξ is much smaller than the lattice size L. Here I adopted the definitions used in refs. [9, 10, 11].

The correlation function in the vector-channel is defined by

$$G_v(x, y) = \langle \vec{s}_x \vec{s}_y \rangle \ . \tag{14}$$

Since naively this quantity vanishes identically under the symmetries of the RP^{N-1} model one considers the tensor channel with the correlation function

$$G_t(x, y) = \langle (\vec{s}_x \vec{s}_y)^2 \rangle - \frac{1}{N} \ . \tag{15}$$

Starting from these definitions of the correlation function one obtains the susceptibility

$$\chi = \frac{1}{V} \sum_{x,y} G(x, y) \tag{16}$$

and

$$F = \frac{1}{V} \sum_{x,y} \cos(\frac{2\pi}{L}k(x - y))) \ G(x, y) \tag{17}$$

with $k = (1, 0)$ or $k = (0, 1)$.

The second moment correlation length is now defined as

$$\xi = \frac{\sqrt{\chi/F - 1}}{2\sin(\pi/L)} \ . \tag{18}$$

In table 2 some results for the constraint $O(3)$ model are given. It is remarkable that already for $C = 0$ the correlation length is larger than ten.

The ratio of the correlation length in the vector and the tensor channel is about $\xi_v/\xi_t = 3.3(1)$.

In table 3 my results for the constraint RP^2 model are summarized. At $C = 0.55$ there is a factor of about 10^3 in between the correlation lengths of the $O(3)$ and the RP^2 model. This means that it is practically impossible to see the true asymptotic behaviour of the constraint RP^2 model in a computer simulation.

5 Finite Size Scaling and Universality

In this section I shall demonstrate that the difference in the finite size scaling curves observed in [9] and [10] can be explained in part by the boundary effect discussed above. I simulated the $O(3)$-invariant model with the standard action on a square lattice using fluctuating boundary conditions in both lattice-directions. For the updates of the boundary conditions I used the boundary flip algorithm proposed in ref. [16] for the Ising model and generalized to $O(N)$ vector models in ref. [17].

I performed runs at $\beta = 1.4$, $\beta = 1.5$ and $\beta = 1.6$. The true correlation lengths for these β values are $\xi = 6.90(1)$, $\xi = 11.09(2)$ and $\xi = 19.07(6)$, respectively [15]. I used lattice sizes ranging from $L = 6$ to $L = 128$. Throughout I performed 100000 measurements. I performed a measurement after one boundary-flip update for each direction and roughly *cluster-size/lattice-size* standard single cluster updates. In figure 1 the dimensionless quantity $\chi_t(2L)/\chi_t(L)$ is plotted as a function of ξ/L. I give the results for fluctuating boundary conditions (circles) and for comparison the results with periodic boundary conditions (diamonds). In order to compare with fig. 2 of ref. [11] one has to take the factor $\xi_v/\xi_t = 3.3(1)$ into account. My result for periodic boundary conditions is consistent with that given in fig. 2b of ref. [11]. The fluctuating boundary conditions remove the characteristic dip visible in the finite size scaling curve for periodic boundary conditions. The finite size scaling curve for periodic boundary conditions looks qualitatively much like that of fig. 2a of ref. [11] (RP^2 like models). However the slope of the curve in fig. 2a of ref. [11] is much steeper for large ξ/L than that of fig. 1. This shows that the results given in fig. 2a of ref. [11] are effected by strong corrections to scaling due to vortices.

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6 Is There a Phase Transition?

In order to understand the phase-structure of the RP^{N-1} models one might, in analogy with the KT scenario of the XY model [18], discuss the RG-flow of the models in a 2 dimensional parameter space. In addition to the coupling g^2 one might introduce a coupling parameter μ for the plaquette-term, controlling the density of vortices.

$$S = -\frac{1}{g^2} \sum_{<xy>} z_{xy} \vec{s}_x \vec{s}_y + \ln \mu \sum_p z_p \ , \tag{19}$$

where $z_p = \prod_{<xy> \in p} z_{xy}$.

I will make no attempt here to derive the RG flow-equations. However certain qualitative features and their consequences seem to be evident:

a) For $(g^2, 0)$ the standard β-function of the $O(N)$ model is recovered.

b) Vortices cause disorder. Therefore a non-vanishing μ should amount to a positive contribution in the derivative of g^2 with respect to the logarithm of the cutoff scale and hence accelerate the flow towards strong coupling.

Statement a) rules out that a possible phase transition in RP^{N-1} is KT like, since the fixed-point of the KT-transition is purely Gaussian. Furthermore statement b) rules out any fixed-point that might occur at a finite μ.

Still we have to explain why Monte Carlo simulations and strong coupling expansions seem to be in favour of a phase transition. It seems plausible that in analogy with the KT-flow equations μ is irrelevant for small coupling g^2 but becomes relevant above some threshold value g_t^2. That means above g_t^2 the RG-trajectories are driven off from the renormalized trajectory of the $O(N)$-invariant model.

7 Conclusions

I have proven that the constraint $O(N)$ and constraint RP^{N-1} model become equivalent for $C > \cos(\pi/4)$. Using the renormalized coupling $\bar{g}^2 = m(L)L$ I estimated the correlation length at $C = \cos(\pi/4)$ to be about $\xi = 0.6 \cdot 10^9$ for both models with $N = 3$. For C-values being smaller, such that $\xi << 1000$, the models display huge differences. This means that the asymptotic behaviour of the constraint RP^2 practically can not be observed in a computer simulation. I argue that a similar scenario holds for models

with a standard action. As $\beta \to \infty$ vortices in the RP^{N-1} model vanish and the RP^{N-1} becomes equivalent to an $O(N)$ model by the virtue of a gauge fixing.

On lattices with periodic boundary conditions one has to notice that paths winding around the lattice are not contractible. The effect of such loops in the RP^{N-1} model amount to fluctuating boundary conditions in the equivalent $O(N)$ model. I demonstrated numerically that this fact partially explains the differences found in the finite size scaling curves for the $O(3)$ and RP^2 models observed in refs. [10, 11].

8 Note added

My conclusions are confirmed by the work of F. Niedermayer, P. Weisz and D.-S. Shin [19], which I found today on the hep-lat bulletin-board.

9 Appendix

In the following I prove that the single cluster algorithm applied to the constraint $O(N)$-invariant model is ergodic.

It is sufficient to show that any allowed configuration can be transformed in a finite number of cluster-update steps to the configuration $\vec{s} = (1, 0, ..., 0)$ for all sites.

Let us consider a $N = 2$ (XY) model with a bond dependent constraint $C_{<xy>}$. Assume that the spins are distributed in an angle range $[0, \alpha_k]$ with respect to the 1-axis. (The largest range to start with is $[0, 2\pi]$.)

Take a reflection axis which has an angle $\alpha_k/2$ with the 1-axis. Per construction none of the sites x with $\phi_x > \alpha_k/2$ is connected via a frozen bond with a site y with $\phi_y < \alpha_k/2$. Hence all spins can be moved into the range $[0, \alpha_k + 1]$ with $\alpha_{k+1} = \alpha_k/2$ using a finite number (smaller or equal the number of sites) of cluster updates. Iterating this process, in a finite number of steps all spins can be put into the range $[0, \min \arccos(C_{<xy>})]$. Now take for each site a reflection axis with $\alpha_x = \phi_x/2$. Per construction all these clusters are single site clusters.

We hence constructed a sequence of a finite number of cluster updates that transforms an arbitrary configuration to the $s = (1, 0)$ for all sites configuration.

For general N this procedure can be iterated. Consider the Nth and $(N-1)^{th}$ component as an embedded XY model. Remove the Nth component. Go ahead untill $s = (1, 0, ..., 0)$ for all sites is reached.

Acknowledgements

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Figure captions

Figure 1: The dimensionless quantity $\chi_t(2L)/\chi_t(L)$ is given as a function of ξ_v/L for the $O(3)$ model. The data points with the circles are obtained with fluctuation boundary conditions while those with diamonds are obtained with periodic boundary conditions.

Table 1: The renormalized coupling $\bar{g}^2$ from the constraint O(3) model. C gives the value of the constraint. L/a and L'/a are the lattice extensions in spacial direction.

C	L/a	L'/a	$\bar{g}^2(L)$	$\bar{g}^2(L')$
0.0515	4	8	1.0595(2)	1.2623(3)
0.0820	5	10	1.0595(2)	1.2564(3)
0.1047	6	12	1.0595(2)	1.2542(3)
0.1225	7	14	1.0595(2)	1.2540(2)
0.1371	8	16	1.0595(2)	1.2542(4)
0.1607	10	20	1.0595(2)	1.2545(3)
0.1786	12	24	1.0595(2)	1.2542(4)
0.2058	16	32	1.0595(2)	1.2559(3)
0.2255	20	40	1.0595(2)	1.2569(3)
0.2637	32	64	1.0595(2)	1.2582(6)
0.1992	4	8	0.8166(2)	0.9358(3)
0.2413	6	12	0.8166(2)	0.9234(2)
0.2663	8	16	0.8166(2)	0.9186(3)
0.2835	10	20	0.8166(2)	0.9171(2)
0.2967	12	24	0.8166(2)	0.9167(3)
0.2538	4	8	0.7383(2)	0.8373(2)
0.2924	6	12	0.7383(2)	0.8246(2)
0.3147	8	16	0.7383(2)	0.8197(3)
0.3299	10	20	0.7383(2)	0.8191(4)
0.3416	12	24	0.7383(2)	0.8174(3)
0.55	16	32	0.4491(2)	0.4759(4)
$1/\sqrt{2}$	16	32	0.2654(1)	0.2746(1)

Table 2: The second moment correlation length in the vector (ξ_v) and tensor (ξ_t) channel for the constraint $O(3)$-invariant vector model for various values of the constraint C.

C	L	ξ_v	ξ_t
0.00	64	11.20(5)	3.29(6)
0.10	128	23.3(2)	6.9(2)
0.2255	400	76.7(4)	24.0(6)

Table 3: The second moment correlation length in the tensor channel ξ_t for the constraint RP^2 model for various values of the constraint C.

C	L	ξ_t
0.50	64	4.72(2)
0.51	64	5.66(2)
0.52	128	7.10(6)
0.53	128	9.06(5)
0.55	128	16.52(7)