

Neutrino pair synchrotron radiation from relativistic electrons in strong magnetic fields

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ABSTRACT

The emissivity for the neutrino pair synchrotron radiation in strong magnetic fields has been calculated both analytically and numerically for high densities and moderate temperatures, as can be found in neutron stars. Under these conditions, the electrons are relativistic and degenerate. We give here our results in terms of an universal function of a single variable. For two different regimes of the electron gas we present a simplified calculation and compare our results to those of Kaminker et al. Agreement is found for the classical region, where many Landau levels contribute to the emissivity, but some differences arise in the quantum regime. One finds that the emissivity for neutrino pair synchrotron radiation is competitive, and can be dominant, with other neutrino processes for magnetic fields of the order $B \sim 10^{14} - 10^{15}G$. This indicates the relevance of this process for some astrophysical scenarios, such as neutron stars and supernovae.

Subject headings : Stars : Magnetic Fields—Stars: Neutron.

1. INTRODUCTION

Estimates of the magnetic field strength at the surface of neutron stars are obtained from several different scenarios: theoretical models of pulsar emission (Ruderman 1972), the accretion flow in binary X-ray sources (Ghosh and Lamb 1978) and observation of features in the spectra of pulsating X-ray sources which have been interpreted as cyclotron lines (Trümper et al. 1978, Wheaton et al. 1979, Gruber et al. 1980, Mihara et al. 1990). For a recent review of all these topics the interested reader is addressed to the book by Michel (Michel 1991). In a sample of more than 300 pulsars the range of values of the surface magnetic field strength runs into the interval: $10.36 \leq \log B \text{ (Gauss)} \leq 13.33$ (Manchester and Taylor 1981).

Very recently, several authors (Duncan and Thomson 1992, Thomson and Duncan 1993, Bisnovatyi-Kogan and Mosheenko 1992, Bisnovatyi-Kogan 1993) have proposed two different physical mechanisms leading to an amplification of some initial magnetic field in a collapsing star. Fields as strong as $B \sim 10^{14} - 10^{16} G$, or even more, might be generated in new-born neutron stars.

According to (Bisnovatyi-Kogan and Mosheenko 1992, Bisnovatyi-Kogan 1993), a mirror-asymmetric magnetic field distribution might arise in a rapidly and differentially rotating proto-neutron star having, originally, both a toroidal and a poloidal component. The field amplification due to differential rotation leads to the formation of an additional toroidal field from the poloidal one by twisting of the field lines. After the first 20 seconds of the life of a new-born neutron star (basically, its Kelvin-Helmholtz epoch) the induced toroidal magnetic field could be as huge as $B \sim 10^{15} - 10^{17} G$.

In a second scenario (Duncan and Thomson 1992, Thomson and Duncan 1993), a dynamo action in a differentially rotating and convective young neutron star is responsible

for the strengthening of some initial dipole field up to values of $B \sim 10^{12} - 3 \times 10^{13}G$ if the convective episodes arose during the main-sequence stage or to $B \sim 10^{14} - 10^{15}G$ if the dipole field is generated after collapse.

In presence of a strong magnetic field the so-called neutrino-pair synchrotron radiation process becomes allowed :

$$[e^-] \xrightarrow{\vec{B}} [e^-] + \nu + \bar{\nu} \quad (1)$$

This reaction has been studied by a few groups (Landstreet 1967, Canuto et al. 1970, Yakovlev and Tschaepé 1981, Vidaurre 1990, Kaminker et al. 1991, Kaminker et al. 1992, Kaminker and Yakovlev 1993) for different regimes of the electron plasma. The calculation of the corresponding emissivity, both analytically or numerically, is far from obvious. In fact, this calculation appears as a multiple integral and summation over the variables and quantum numbers involving the wave functions of the initial and final electron and the corresponding statistical weights. The wave function integrals lead to Laguerre or Bessel functions with a complicated behaviour, so that approximations in order to simplify the expressions are sometimes delicate. This in fact has led to errors in the past literature.

In a recent paper, Kaminker et al. (Kaminker et al. 1991) have studied the neutrino emissivity of the above process (1) for moderately high magnetic fields $B \sim 10^{12} - 10^{14}G$ and high densities, in the degenerate-relativistic regime for the electrons. They claim that, within these conditions, the emissivity is independent of the electron density. So far, to our knowledge, there are no numerical tests which confirm these results.

Given all these circumstances, together with the interest of the problem, both from the theoretical and astrophysical point of view, we have considered useful to reexamine the existing results for the above regime. Therefore, we have performed a numerical study of this process at high densities $\rho Y_e \geq 10^7 g/cm^3$, where Y_e is the electron fraction per baryon and ρ the matter density in c.g.s. units, and moderate temperatures $T < 10^9 K$, for

values of the magnetic field strength $B \leq 10^{16}G$. Under these conditions, the electrons are relativistic and degenerate. We have found a result which is in agreement with the one of Kaminker et al. when the electron gas is in the classical regime. We also obtain agreement for the corresponding analytical expressions, which we present in a simpler way than these authors. For the quantum regime, however, we give analytical formulae which show the correct dependence on B for large magnetic fields, in contrast to those given by Kaminker et al. This fact would be particularly important if some of the magnetic field amplification mechanisms described above were physically realizable in nature.

The influence of the neutrino-pair synchrotron radiation in presence of strong magnetic fields merits to be examined in different astrophysical scenarios, such as the delayed mechanism of type II Supernovae, neutrino cooling of proto-neutron stars during the Kelvin epoch or the secular cooling of the neutron star. Furthermore, the neutrinos originated from the combined effect of one of the proposed mechanisms to enhance the magnetic field and the neutrino-pair synchrotron radiation process could be envisaged as a signature of the mechanism itself.

This paper is organized as follows. In section 2 we discuss the calculation of the emissivity for the synchrotron process and present our main results. These results are illustrated with more detail in section 3 and 4 for two different regimes of the electron gas. We end in section 5 with some conclusions and remarks.

2. CALCULATION OF THE EMISSIVITY

The emissivity for the process (1) can be written as

$$\varepsilon_\nu = \frac{G^2 e B}{3(2\pi)^6} \sum_{n=1}^{\infty} \sum_{n'=0}^{n-1} \int_{-\infty}^{+\infty} dp_z \int_{q^2 \leq \omega^2} d^3q \omega$$

$$\times A f(E) [1 - f(E')] \quad (2)$$

where G is the Fermi coupling constant. The initial (final) electron, of mass m , has an energy $E = \sqrt{m^2 + p_\perp^2 + p_z^2}$ ($E' = \sqrt{m^2 + p'_\perp^2 + p_z'^2}$) characterized by the Landau quantum number n (n') and momentum p_z (p'_z) along the B-direction. We have introduced $p_\perp^2 = 2eBn$ and $p'_\perp^2 = 2eBn'$, which correspond to the classical transverse momenta of the electron. In the above equation, q is the four-momentum transfer and $\omega = E - E'$ the energy which is carried away by the neutrino pair. $f(E)$ is the Fermi-Dirac distribution function : $f(E) = [\exp(\frac{E-\mu}{T}) + 1]^{-1}$, where μ is the electron chemical potential. The integration region over \vec{q} is restricted by :

$$q^2 = \omega^2 - \vec{q}^2 \geq 0. \quad (3)$$

The expression for A in the equation of the emissivity can be found in (Vidaurre 1990, Kaminker et al. 1992). In the relativistic limit, the relevant expression for A is :

$$\begin{aligned} A = & \frac{(C_V^2 + C_A^2)}{EE'} \left\{ [-2(EE' - p_z p'_z)]^2 \right. \\ & + (EE' - p_z p'_z)(p_t^2 - 2q_\perp^2) - \frac{q_\perp^2}{2}(q_\perp^2 - p_t^2) \left. \right] \Psi(u) \\ & + p_t^2 [(EE' - p_z p'_z) - \frac{1}{2}(p_t^2 - q_\perp^2)] \Phi(u) \left. \right\} \end{aligned} \quad (4)$$

where

$$\begin{aligned} \Psi &= \frac{n'!}{n!} u^s e^{-u} \left[\frac{u}{n'} (L_{n'-1}^{s+1})^2 + \frac{n}{u} (L_{n'}^{s-1})^2 \right] \\ \Phi &= \frac{n'!}{n!} u^s e^{-u} \left[\frac{n}{n'} (L_{n'-1}^s)^2 + (L_{n'}^s)^2 \right] \end{aligned} \quad (5)$$

$L_{n_1}^{n_2}$ are Laguerre functions with argument $u = \frac{q_\perp^2}{2eB}$ (q_\perp is the component of \vec{q} orthogonal to the magnetic field) and $s = n - n'$. C_V (C_A) is the effective vector (axial) coupling of the neutrino pair to the electron current, coming from both the Fierz reordered charged current (for electron neutrinos) and neutral current (for all neutrino species) weak interactions. In Eq. (4) we have dropped a term proportional to $(C_V^2 - C_A^2)$, which disappears in the extreme relativistic limit, and the interference term proportional to $C_V C_A$, which does not contribute to the integrated emissivity in Eq. (2). The argument goes as follows. The corresponding integrand, for $C_V C_A$, is odd under the simultaneous change of sign of the longitudinal momenta of the initial and final electrons, so that a symmetric integration of both p_z and q_z in Eq.(2) cancels this asymmetric term. We have defined $p_t^2 = p_\perp^2 + p'_\perp^2$. If one takes for the electroweak mixing angle $\sin^2\theta_W = 0.23$, then $C_V^2 + C_A^2 = 1.6748$.

One has the relationship

$$\omega^2 - q_z^2 = 2m^2 + p_t^2 - 2(EE' - p_z p'_z) \approx p_t^2 - 2(EE' - p_z p'_z) \quad (6)$$

where the latter approximation corresponds to considering relativistic electrons.

By substituting in Eq. (4) we obtain

$$A = \frac{(C_V^2 + C_A^2)}{2EE'} (\omega^2 - q_z^2 - q_\perp^2) [p_t^2 (\Psi - \Phi) - (\omega^2 - q_z^2 - q_\perp^2) \Psi] \quad (7)$$

As mentioned above, for the range of temperatures and densities we are interested in, the electrons are degenerate. The product of distribution functions appearing in (2) will then restrict the energies to $E, E' \sim \mu$. Moreover, one can write the following identity :

$$f(E) [1 - f(E')] = \mathcal{B}(\omega) [f(E') - f(E)] \quad (8)$$

where $\mathcal{B}(\omega) = [\exp(\omega/T) - 1]^{-1}$ is a Bose-Einstein distribution function with zero chemical potential. From the last equation, it is apparent that the energy difference ω will be restricted to a few times T and will be much lower than the relevant values of E and E' , if the electrons are degenerate. Similarly, one can easily estimate that q_z and q_\perp will contribute as $\sim T$, whereas p_t , p_z and p'_z contribute as $\sim \mu$. This allows us to neglect the last term in Eq. (7). In this case one gets

$$A = \frac{(C_V^2 + C_A^2)}{2EE'} p_t^2 (\omega^2 - q_z^2 - q_\perp^2) \Theta \quad (9)$$

with $\Theta = \Psi - \Phi$. We have tested numerically that the complete expression Eq.(4) or Eq.(7) gives approximately the same result as Eq.(9) for the physical conditions we are considering here.

We can also perform the following approximations, in agreement with the above discussion :

$$\begin{aligned} n + n' &= \frac{p_t^2}{2eB} = \frac{(E + E')^2 + \omega^2 - 2(p_z^2 + p_z'^2)}{4eB} \approx \frac{(E + E')^2 - 4p_z^2}{4eB} \\ \omega &= \frac{2seB + p_z^2 - p_z'^2}{E + E'} \approx \frac{2seB + 2p_z q_z}{E + E'} \end{aligned} \quad (10)$$

Due to the Pauli principle, the quantity $f(E)[1 - f(E')]$ will be nonzero only when E and E' are within a narrow interval around μ of width $\omega \sim T$. With this in mind, we have replaced all the slowly varying functions in (2) by their value around $E, E' \sim \mu$.

The latter equation, together with the phase space restriction (3), requires that q_z and q_\perp must lie inside the elliptical domain (Kaminker et al. 1991)

$$q_\perp^2 + \frac{p_\perp^2}{\mu^2} (q_z - \chi_z)^2 \leq \chi_\perp^2 \quad (11)$$

where $\chi_z = \frac{seBp_z}{p_\perp^2}$ and $\chi_\perp = \frac{seB}{p_\perp}$.

We have numerically calculated the emissivity of the synchrotron process (1) for values of the electron density $\rho Y_e \geq 10^7 g/cm^3$, moderate temperatures $T < 10^9 K$, and magnetic fields $B \leq 10^{16} G$. The results, for this range of values, can be expressed in a compact way as :

$$\varepsilon_\nu = 1.47 \cdot 10^{14} B_{13}^2 T_9^5 f(x) \text{ erg/cm}^3/\text{s} \quad (12)$$

where B_{13} is the magnetic field in units of $10^{13} G$, T_9 is the temperature in units of $10^9 K$, and $x = \frac{\mu T}{eB}$ is a dimensionless variable. This result is useful in order to perform analytical calculations. We have plotted in Fig. 1 the function $f(x)$. As can be seen from this figure, $f(x)$ first increases as x grows, and is almost constant for large values of x . This behaviour corresponds to different physical regimes of the electron plasma, and will be explained in the next sections.

3. Large x Regime

Let us discuss the behaviour of the emissivity corresponding to large values of the parameter x defined above. This situation corresponds, for example, to sufficiently high electron densities for a fixed magnetic field and temperature. The number of Landau levels which are involved in Eq. (2) is $n_{max} = \frac{\mu^2}{2eB}$. On the other hand, the maximum of $s = n - n'$ can be estimated from Eq. (10) as $s_{max} \sim \frac{\mu T}{eB} = x$. We then have $n_{max} \gg s_{max} \gg 1$ for $\frac{\mu}{T} \gg 1$. According to this idea, we have used the following approximations for the Laguerre polynomials in Eq. (5) :

$$\Psi \rightarrow J_{s-1}^2 + J_{s+1}^2$$

$$\Phi \rightarrow 2J_s^2 \quad (13)$$

with $J_s(a)$ a Bessel function and $a = \sqrt{2(n + n')u}$. One can prove that $a \leq s$ always. The approximation shown by Eq. (13) can be used if $n \gg 1$ and $s \ll n$. Because it is time saving and more suitable numerically than Eq. (5), we made use of it in our numerical computation of the emissivity, whenever large values of n (and $s \ll n$) were encountered. By changing the sum over n and n' to a sum over n' and s , and integrating the angle of \vec{q} around the \vec{B} direction, one arrives to the following expression :

$$\begin{aligned} \varepsilon_\nu = & \frac{G^2 eB}{3(2\pi)^5} (C_V^2 + C_A^2) \sum_{n'=0}^{\infty} \sum_{s=1}^{\infty} \int_{-\infty}^{+\infty} dp_z \int_{q^2 \leq \omega^2} dq_z dq_\perp q_\perp \omega \\ & \times \left(1 - \frac{p_z^2}{\mu^2}\right) (\omega^2 - q_z^2 - q_\perp^2) \mathcal{B}(\omega) [f(E') - f(E)] \Theta(a) \end{aligned} \quad (14)$$

The argument of Θ can be written as $a = p_\perp q_\perp / (eB)$, with $p_\perp = \sqrt{\mu^2 - p_z^2}$, and $\omega = \frac{seB + p_z q_z}{\mu}$. Further approximations can be made in the above equation for degenerate electrons, if one considers the distribution functions $f(E)$ and $f(E')$ as step functions in the energy. Within this assumption, the sum over n' can be done explicitly. One gets

$$\sum_{n'=0}^{\infty} [f(E') - f(E)] = \frac{\omega}{2eB} (E + E') \simeq \frac{\mu\omega}{eB} \quad (15)$$

In deriving Eq.(15), we have used the fact that s is lower than n_{max} . By inserting the latter expression into Eq. (14) we obtain

$$\begin{aligned} \varepsilon_\nu = & \frac{G^2 \mu}{3(2\pi)^5} (C_V^2 + C_A^2) \sum_{s=1}^{\infty} \int_{-\infty}^{+\infty} dp_z \int_{q^2 \leq \omega^2} dq_z dq_\perp q_\perp \omega^2 \\ & \times \left(1 - \frac{p_z^2}{\mu^2}\right) (\omega^2 - q_z^2 - q_\perp^2) \mathcal{B}(\omega) \Theta(a) \end{aligned} \quad (16)$$

Next we make use of recurrence relationships for the Bessel functions and obtain the formula :

$$\Theta(a) = 2(J'_s)^2 + 2(s^2/a^2 - 1)J_s^2 \quad (17)$$

For the large values of s involved here, one can use the following approximation to the Bessel functions (Gradshteyn and Ryzhik 1980):

$$J_s(a) \approx \frac{1}{\pi} \sqrt{\frac{2(s-a)}{3a}} K_{1/3}(z) \quad (18)$$

where $z = \frac{[2(s-a)]^{3/2}}{3\sqrt{x}}$ and $K_{1/3}$ is the modified Bessel function, which can be further approximated as :

$$K_{1/3}(z) \approx \sqrt{\frac{\pi}{2z}} \exp(-z) \quad (19)$$

In this way one obtains, after some algebra,

$$\Theta(a) \approx \frac{2}{\pi} s^{-4/3} (2z)^{1/3} \exp(-2z) \quad (20)$$

As can be seen from Fig. 2, the latter equation provides a reasonable approximation to Eq.(17). In this figure, we have plotted $\Theta(a)$ as obtained from Eq.(20) (dotted line) and from Eq.(17) (solid line) for $s = 200$. Another important feature is that only values of the argument a close to s will contribute. This can be understood from Eq.(20), due to the exponential behaviour, which effectively limits z . In fact, if we define σ as the 'width' of the exponential, one has $(1 - a/s) < (\frac{\sigma}{s})^{2/3}$. By substituting into Eq.(11) one obtains that important values of q_z and q_\perp are restricted to

$$\begin{aligned} \left(1 - \frac{q_{\perp}}{\chi_{\perp}}\right) &\leq \left(\frac{\sigma}{s}\right)^{2/3} \\ |q_z - \chi_z| &\leq \Delta(q_{\perp}) \leq \sqrt{2} \frac{\mu\chi_z}{p_z} \left(\frac{\sigma}{s}\right)^{1/3} \end{aligned} \quad (21)$$

We have defined $\Delta(q_{\perp}) = \frac{\mu\chi_{\perp}}{p_{\perp}} \sqrt{1 - \frac{q_{\perp}^2}{\chi_{\perp}^2}}$. Thus q_z is restricted to a narrow interval of width $\Delta(q_{\perp})$ around χ_z . This allows us to perform the integral over q_z approximately. To the first order in $\Delta(q_{\perp})$ one can write

$$\int_{\chi_z - \Delta(q_{\perp})}^{\chi_z + \Delta(q_{\perp})} dq_z \longrightarrow 2\Delta(q_{\perp}) \quad (22)$$

with the replacement $q_z \rightarrow \chi_z$ in the integrand of Eq. (16). This means that ω^2 will be replaced by $\chi_z^2 + \chi_{\perp}^2$. The integral over q_{\perp} is then immediate. Since the total number of level differences s is large, one can substitute the sum over s by an integral over the continuous variable $t = s/x$. Thus by changing $\sum_s \rightarrow x \int_0^{\infty} dt$ and performing the remaining integrals one finally obtains

$$\varepsilon_{\nu} = \frac{G^2}{\pi^6} (C_V^2 + C_A^2) \zeta(5) (eB)^2 T^5 = 1.16 \cdot 10^{15} B_{13} T_9^5 \text{ erg/cm}^3/\text{s} \quad (23)$$

This equation implies that the emissivity does not depend on the electron density in this regime, in agreement with the result previously found in Ref. (Kaminker et al. 1991). In fact, our Eq. (23) is close to the one obtained in this reference. It is also in good agreement with the values of $f(x)$ obtained numerically (and plotted in Fig. 1), as can be seen by comparing Eq.(23) with Eq.(12).

A numerical fit which reproduces the behaviour of $f(x)$ for $x > 2$ to better than 4% is given by

$$f(x) = \frac{-5.0224 - 8.1289x + 9.2892x^2}{1.0293 + 2.0605x + 1.0727x^2} \quad (24)$$

4. Low x Regime

We now address the question of whether an increase of the magnetic field will always give a larger neutrino emission. In Fig. 3 we present the corresponding emissivity (in cgs units and logarithmic scale) for a fixed temperature $T_9 = 1$ and four values of the electron density $\rho Y_e = 10^9, 10^{11}, 10^{13}, 10^{14} g/cm^3$, as a function of B_{13} (B_{13} ranging from unity up to 10^3), as obtained numerically. For these high magnetic fields, the number of populated Landau levels (n_{max} , defined above) can be of order unity, and we enter into the quantum regime. We have used, in these cases, the expression of A as given by Eqs. (4) and (5) directly, instead of making the approximations shown in section 3.

As can be seen from Fig. 3, for a given density and temperature, the emissivity first increases and, after reaching a maximum value, will fall to zero for large values of the magnetic field. This can be understood since the number of possible $n \rightarrow n'$ transitions decreases as $\frac{eB}{\mu T} \gg 1$. In fact, a rough analytic expression in this region can be obtained by putting $s = 1$ in Eq. (14) and $\omega = \frac{eB}{\mu}$. Therefore one has :

$$\begin{aligned}\Psi &\simeq 1 \\ \Phi &\simeq 0\end{aligned}\tag{25}$$

The integrals in Eq. (14) can be done analytically and one obtains the following expression :

$$\varepsilon_\nu = 6.6 \cdot 10^{12} B_{13}^2 T_9^5 x^{-5} e^{-1/x} \text{ erg/cm}^3/\text{s}\tag{26}$$

We have verified that the latter expression gives values which are in agreement with our numerical results around the maximum of the emissivity. This is shown in Fig. 4,

where we have compared our results for $\rho Y_e = 10^{11} g/cm^3$ (solid line) with the prediction of Eq. (26) (dotted line). We have also plotted (dashed line) the corresponding analytical approximation given by (Kaminker et al. 1991) for this case. As can be seen from this figure, Eq. (26) provides a reasonable approximation to the emissivity, which works better than the formula of Kaminker et al.

5. Comparison with other processes

In order to investigate the relevance of the process studied here for neutron star cooling, we have made the comparison with the emissivities corresponding to other neutrino processes which are competitive with the synchrotron emission. We have considered pair production $e^+e^- \rightarrow \nu\bar{\nu}$, plasmon decay $\Gamma \rightarrow \nu\bar{\nu}$, bremsstrahlung $e^-(Z, A) \rightarrow e^-(Z, A)\nu\bar{\nu}$ and photoproduction $\gamma e^- \rightarrow e^-\nu\bar{\nu}$. For these processes, we have made the assumption that they do not vary significantly with the magnetic field. The numerical fit to these emissivities have been taken from Munakata et al. 1985. Although these rather simple formulae are not the most up-to-date available fits to the above processes, they serve to our purpose as a first approximation (see, for example, Itoh et al. 1989, for a more elaborated fit). The bremsstrahlung process is taken from Maxwell 1979. A more complete calculation, as in Itoh et al. (Itoh et al. 1989), gives the same order of magnitude in the region where this process dominates.

In Fig. 5 we present the result of comparing all these processes for a temperature $T = 10^8 K$ as the product ρY_e varies from zero up to $10^{12} g/cm^3$. The bremsstrahlung energy emission was calculated assuming that the dominant nucleus is ^{56}Fe . The synchrotron emissivity is plotted (solid lines) for two values of the magnetic field : $B_{13} = 10$ and $B_{13} = 100$. Other relevant processes are : bremsstrahlung (dashed-dotted line), plasma (long dash), and photoneutrino emission (short dash). As can be seen from this

figure, the synchrotron emission is competitive with the above processes within the range $\rho Y_e \sim 10^9 - 10^{12}$. Moreover, as pointed out by Pethick and Thorsson (Pethick and Thorsson 1993), band-structure effects can suppress bremsstrahlung by a factor of 10 or more for temperatures less than about $10^9 K$. In this case, the synchrotron emission would be the dominant process in the above electron density range, if the magnetic field reaches values of the order $\sim 10^{14} G$.

For higher temperatures, the synchrotron emission corresponding to a given value of B 'switches on' at lower electron densities, as can be inferred from Eq. (26). However, other processes have a faster increase with temperature and, therefore, the dominance of synchrotron emission reduces to a narrow interval of densities, although it effectively competes for high densities. This is shown in Fig. 6, where we have made the above comparison for a temperature $T = 10^9 K$ (pair emission is represented by the dotted line).

6. CONCLUSIONS

We have performed numerical calculations of the synchrotron emissivity from relativistic degenerate electrons. This calculations allow us to present the results in terms of an universal function $f(x)$ which can be used in astrophysical codes. For two different regimes of the electron gas we have derived analytical formulae, in a simpler way than previous references. These formulae have been tested, and we have found a reasonable agreement with our numerical calculations. We also have compared our results to the analytical formulae derived recently by Kaminker et al. Agreement is found for the classical region, where many Landau levels contribute to the emissivity, but some differences arise in the quantum regime, for the values of the magnetic field recently suggested in new-born neutron stars.

We have shown that neutrino-pair synchrotron radiation for moderately high magnetic

fields $B \geq 10^{14}G$ is an efficient cooling mechanism for temperatures not larger than 10^9K , and can compete effectively (or even dominate) with other processes.

We claim that the influence of the neutrino-pair synchrotron radiation in presence of strong magnetic fields ($\sim 10^{15}G$) merits to be examined in different astrophysical scenarios, such as the delayed mechanism of type II Supernovae, neutrino cooling of proto-neutron stars during the Kelvin epoch or the secular cooling of the neutron star. Furthermore, these neutrinos originated from the combined effect of the dynamo action with the neutrino-pair synchrotron radiation process could be envisaged as a signature of this mechanism. This will be the subject of future investigations.

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Figure Captions

Figure 1.- The function $f(x)$ appearing in Eq. (12). See the text for the definition of the dimensionless variable x and comments about its behaviour.

Figure 2.- The function $\Theta(a)$ as obtained from the approximation Eq.(20) (dotted line) compared to Eq.(17) (solid line) for $s = 200$.

Figure 3.- Neutrino synchrotron emissivity as a function of the magnetic field B_{13} for different values of the electron density. The temperature is the same ($T = 10^9 K$) in all cases.

Figure 4.- Comparison of our numerical results for $\rho Y_e = 10^{11} g/cm^3$ (solid line) with the analytical approximation Eq. (26) (dotted line). We have also plotted (dashed line) the approximation given by (Kaminker et al. 1991) for this case.

Figure 5.- Competition of synchrotron neutrino emission (solid lines) with other processes, as a function of the electron density, for a temperature $T = 10^8 K$. Two different values of the magnetic field ($B_{13} = 10$ and $B_{13} = 100$ have been considered. The bremsstrahlung emissivity (dash-dotted line) has been calculated for ^{56}Fe . Plasma process is represented by long-dashed line, and photoneutrino by short-dashed line.

Figure 6.- Same as figure 5 for $T = 10^9 K$. Dotted line corresponds to pair emission.