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Incompressibility of nuclear matter, and Coulomb and volume-symmetry coefficients of nucleus incompressibility in the relativistic mean field theory

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ABSTRACT

The volume coefficient K (=incompressibility of the nuclear matter), the Coulomb coefficient K_c , and the volume-symmetry coefficient K_{vs} of the nucleus incompressibility are studied in the framework of the relativistic mean field theory, with aid of the scaling model. It is found that $K = 300 \pm 50\text{MeV}$ is necessary to account for the empirical values of K_v , K_c , and K_{vs} , simultaneously. The result is independent on the detail descriptions of the potential of the σ -meson self-interaction and is almost independent of the strength of the ω -meson self-interaction.

One way to determine the incompressibility K of nuclear matter from the giant monopole resonance (GMR) data is using the leptodermous expansion[1] of nucleus incompressibility $K(A, Z)$ as follows.

$$K(A, Z) = K + K_{sf}A^{-1/3} + K_{vs}I^2 + K_cZ^2A^{-4/3} + \dots \quad ; \quad I = 1 - 2Z/A, \quad (1)$$

where the coefficients, K , K_{sf} , K_{vs} and K_c are volume coefficient (incompressibility of nuclear matter), surface term coefficient, volume-symmetry coefficient and Coulomb coefficient, respectively. We have omitted higher terms in eq. (1). Although there is uncertainty in the determination of these coefficients by using the present data, Pearson [2] pointed out that there is a strong correlation between K and K_c . (See table I.) Similar observations are done by Shlomo and Youngblood [3].

Table I

According to this context, Rudaz et al. [4] studied the relation between incompressibility and the skewness coefficient by using the generalized version of the relativistic Hartree approximation [5]. The compressional and the surface properties are studied by Von-Eiff et al. [6-8] in the framework of the relativistic mean field approximation of the σ - ω - ρ model with the nonlinear σ terms. They found that low incompressibility ($K \approx 200\text{MeV}$) and a large effective nucleon mass M^* at the normal density ($0.70 \leq M^*/M \leq 0.75$) are favorable for the nuclear surface properties [8]. On the other hand, using the same model, Bodmer and Price [9] found that the experimental spin-orbit splitting in light nuclei supports $M^* \approx 0.60M$. The result of the generator coordinate calculations for breathing-mode GMR by Stoitsov, Ring and Sharma [10] suggests $K \approx 300\text{MeV}$.

In previous papers[11,12], we have studied the relation between K and K_c

in detail, using the relativistic mean field theory with the nonlinear σ terms [13] and the one with the nonlinear σ and ω terms [14]. We found that, under the assumption of the scaling model [1], $K = 300 \pm 50\text{MeV}$ is favorable to account for K , K_c and K_{vs} , simultaneously. It seems that this conclusion is not drastically changed in the use of the relativistic mean field theory and the scaling model. In this paper, we examine the conclusion in more general way, in which the result does not depend on the detail descriptions of the σ -meson self-interaction. The reason why we restrict our discussions to K , K_c and K_{vs} is that the general discussions, which are independent of the detail of the model (e.g., types of the interactions, values of the parameters in the Lagrangian, etc.) are possible to a considerable extent, since, as is shown below, these quantities are almost analytically estimated by using the result for the nuclear matter, if we assume the scaling model [1].

We use the relativistic mean field theory based on the σ - ω - ρ model with the nonlinear σ terms. The Lagrangian density consists of four fields, the nucleon ψ , the scalar σ -meson ϕ , the vector ω -meson V_μ , and the vector-isovector ρ meson \mathbf{b}_μ , i.e.,

$$\begin{aligned}
L_{N\sigma\omega\rho} = & \bar{\psi}(i\gamma_\mu\partial^\mu - M)\psi \\
& + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m_s^2\phi^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_v^2V_\mu V^\mu - \frac{1}{4}\mathbf{B}_{\mu\nu} \cdot \mathbf{B}^{\mu\nu} + \frac{1}{2}m_\rho^2\mathbf{b}_\mu \cdot \mathbf{b}^\mu \\
& + g_s\bar{\psi}\psi\phi - g_v\bar{\psi}\gamma_\mu\psi V^\mu - g_\rho\bar{\psi}\gamma_\mu\frac{\boldsymbol{\tau}}{2} \cdot \mathbf{b}^\mu\psi - U(\phi) \quad ; \\
F_{\mu\nu} = & \partial_\mu V_\nu - \partial_\nu V_\mu, \quad \mathbf{B}_{\mu\nu} = \partial_\mu\mathbf{b}_\nu - \partial_\nu\mathbf{b}_\mu - g_\rho\mathbf{b}_\mu \times \mathbf{b}_\nu, \quad (2)
\end{aligned}$$

where m_s , m_v , m_ρ , g_s , g_v and g_ρ are σ -meson mass, ω -meson mass, ρ -meson mass, σ -nucleon coupling, ω -nucleon coupling, and ρ -nucleon coupling, respectively. The $U(\phi)$ is a nonlinear self-interaction potential of σ meson field ϕ . For example, in ref. [11], we have used the quartic-cubic terms of ϕ as in ref. [13],

i.e.,

$$U(\phi) = \frac{1}{3}b\phi^3 + \frac{1}{4}c\phi^4, \quad (3)$$

where b and c are the constant parameters which are determined phenomenologically. However, in this paper, we do not give an explicit expression of $U(\phi)$ and discuss the problem in more general way, without any assumption of $U(\phi)$.

In the scaling model [1], K , K_c and K_{vs} in eq. (1), are given by

$$K = 9\rho_0^2 \frac{\partial^2 E_b}{\partial \rho^2} \Big|_{\rho=\rho_0}, \quad (4)$$

$$K_c = -\frac{3q_{el}^2}{5R_0} \left(\frac{9K'}{K} + 8 \right), \quad (5)$$

$$K_{vs} = K_{sym} - L \left(9\frac{K'}{K} + 6 \right), \quad (6)$$

where ρ , ρ_0 , E_b and q_{el} are the baryon density, the normal baryon density, the binding energy per nucleon, and the electric charge of proton, respectively, and $R_0 = [3/(4\pi\rho_0)]^{1/3}$,

$$K' = 3\rho_0^3 \frac{d^3 E_b}{d\rho^3} \Big|_{\rho=\rho_0}, \quad (7)$$

$$L = 3\rho_0 \frac{dJ}{d\rho} \Big|_{\rho=\rho_0}, \quad K_{sym} = 9\rho_0^2 \frac{d^2 J}{d\rho^2} \Big|_{\rho=\rho_0} : \quad J = \frac{1}{2}\rho^2 \frac{\partial^2 E_b}{\partial \rho_3^2} \Big|_{\rho_3=0}. \quad (8)$$

The quantity such as K' is sometimes called "skewness".

In the mean field theory with the Lagrangian (2), L and K_{sym} are given by [7]

$$L = \frac{3\rho}{8M^2} C_\rho^2 + \frac{\rho}{2} \left(\frac{2k_F k'_F}{E_F^*} - \frac{k_F^2 E_F^{*'}}{E_F^{*2}} \right) \quad (\rho = \rho_0), \quad (9)$$

and

$$K_{sym} = \frac{3}{2}\rho^2 \left(\frac{2k_F'^2}{E_F^*} + \frac{2k_F k_F''}{E_F^*} - \frac{4k_F k'_F E_F^{*'}}{E_F^{*2}} + \frac{2k_F^2 E_F^{*'}^2}{E_F^{*3}} - \frac{k_F^2 E_F^{*''}}{E_F^{*2}} \right) \quad (\rho = \rho_0), \quad (10)$$

where $k_F (= [3\pi^2\rho/2]^{1/3})$ is the Fermi momentum, and $C_\rho = g_\rho M/m_\rho$,

$$k'_F = \frac{dk_F}{d\rho} = \frac{k_F}{3\rho}, \quad k''_F = \frac{d^2 k_F}{d\rho^2} = \frac{-2k_F}{9\rho^2}, \quad (11)$$

$$E_F^* = \sqrt{k_F^2 + M^{*2}}, \quad E_F^{*'} = \frac{dE_F^*}{d\rho}, \quad E_F^{*''} = \frac{d^2 E_F^*}{d\rho^2}. \quad (12)$$

M^* in eq. (12) is the effective nucleon mass. Furthermore, at $\rho = \rho_0$, $E_F^{*'}$ and $E_F^{*''}$ are related to K and K' in the following relations, respectively [7,11].

$$E_F^{*'} = \frac{K}{9\rho_0} - \frac{C_v^2}{M^2}; \quad C_v = \frac{g_v M}{m_v}. \quad (13)$$

$$E_F^{*''} = \frac{K + K'}{3\rho_0^2}. \quad (14)$$

At $\rho = \rho_0$, C_v and C_ρ are also related to M^* as follows.

$$C_v^2 = \frac{M^2}{\rho_0} \left(M - a_1 - \sqrt{k_F^2 + M^{*2}} \right), \quad (15)$$

[13] and

$$C_\rho^2 = \frac{8M^2}{\rho_0} \left(a_4 - \frac{k_F^2}{6E_F^*} \right), \quad (16)$$

[15] where a_1 and a_4 are the binding energy and the symmetry energy at $\rho = \rho_0$, respectively. We remark that eqs. (5), (6), and (9)~(16) have no explicit dependence on $U(\phi)$.

From eq. (5), K' are determined, if ρ_0 , K and K_c are given. Therefore, from the eqs. (5), (6), and (9)~(16), it is seen that K_{vs} is determined, if ρ_0 , M , a_1 , a_4 , K , K_c , and M^* are given, without giving the detail descriptions for $U(\phi)$. Using these equations, we calculate K_{vs} . In the calculations, we put $\rho_0 = 0.16\text{fm}^{-3}$, $M = 939\text{MeV}$, $a_1 = 15.75\text{MeV}$ and $a_4 = 30.0\text{MeV}$. For K and K_c , we use the values in table I. We assume that $M^* = 0.5M \sim 0.94M$, the phenomenologically acceptable values. (We remark the upper bound for M^* is gotten, if we put $C_v = 0$ in eq. (15).) In fig. 1 (solid lines), we show K_{vs} as a function of M^* for two sets of K and K_c in table I. In the figures, K_{vs} decreases as M^* increases. This is understood as follows. As M^* increases, L and K_{sym} decreases, because of large E_F^* in the denominators in eqs. (9) and (10). Small K_{sym} makes K_{vs} smaller (more negative) and, on the contrary, small L makes K_{vs} larger (less

negative), since $(9K'/K + 6) > 0$ in eq. (6). In the cases of fig. 1, the effect of the small K_{sym} overcomes that of the small L , therefore, K_{vs} decreases. In the case of $K = 300\text{MeV}$ and $K_c = -3.990\text{MeV}$, $K_{vs} = -289 \sim -98\text{MeV}$. These values are in good agreement with the corresponding empirical values in table I. We also remark that the uncertainty of K_{vs} in changing M^* is comparable to the empirical error bar of K_{vs} in table I. In the case of $K = 250\text{MeV}$ and $K_c = -0.7065\text{MeV}$, $K_{vs} = 50 \sim 410\text{MeV}$. These values are somewhat larger than the empirical ones. In table II(a), we summarize the range of the calculated K_{vs} for each set of K and K_c in table I. Comparing the table I and table II(a), we see that $K = 300 \pm 50\text{MeV}$ is necessary to account for K , K_c and K_{vs} , simultaneously.

Fig. 1(a),(b), Table II(a),(b)

We remark the following three points.

(1) The results are independent of the form of $U(\phi)$, since eqs. (5), (6), and (9)~(16) are required for any type of $U(\phi)$.

(2) The question, whether there are coupling parameters, which reproduce the set of K and K_c in table I, or not, is still open, and the answer for the question depends on the detail descriptions of $U(\phi)$. For example, if we use the quartic-cubic potential (3), we could not find the coupling parameters (i.e., g_s , g_v , b , and c), which reproduce $K = 200\text{MeV}$ and $K_c = 2.577\text{MeV}$ [11]. (We remark that K and K_c are independent on g_ρ in the mean field theory.) Also, using eq. (3), we get the parameter set for $K = 300\text{MeV}$ and $K_c = -3.990\text{MeV}$, only in the case of $M^* = 0.83M$ [11]. In those cases, parameter set is uniquely determined or could not be found, since the number of the parameters is not

larger than the number of the inputs, i.e., K , K_c , and two conditions for the saturation. If the higher terms of ϕ are added to (3), the wider range of M^* may be available. However, $K = 300 \pm 50\text{MeV}$ is *necessary* to reproduce the empirical values of K , K_c , and K_{vs} simultaneously, for any type of $U(\phi)$.

(3) The results do not have a strong-dependence on ρ_0 , a_1 and a_4 , since the calculated K_{vs} is much more sensitive to the ratio K'/K than to those quantities.

Next we add the following attractive term of vector self-interaction (VSI) [14,12] to the Lagrangian (2).

$$L_{VSI} = \frac{g_v^2 Y^2}{4} m_v^2 (V_\mu V^\mu)^2, \quad (17)$$

, where Y is a positive constant which determine the strength of VSI. Although, according to this modification, the formalism for calculating K_{vs} , which is described above, is slightly modified, it is still possible to study M^* - K_{vs} relation without giving the detail descriptions of $U(\phi)$, as in the case of no VSI, i.e., $Y = 0$. In fig. 1, we also show K_{vs} as a function of M^* , in the cases of $y = 1$ and $y = 5$, where $y = g_v^2 \rho_0 Y / m_v^2$ [12]. In those figures, it is seen that the VSI makes K_{vs} smaller in the small M^* region. K_{vs} is almost independent on y in the large M^* region, where the value of M^* dominates the features of the equations of state. As a result, the uncertainty of K_{vs} in changing M^* becomes smaller. In the case of $K = 300\text{MeV}$ and $K_c = -3.990\text{MeV}$, $K_{vs} = -298 \sim -282(-303 \sim -289)\text{MeV}$ with $y = 1.0(5.0)$. In the case of $K = 250\text{MeV}$ and $K_c = -0.7065\text{MeV}$, $K_{vs} = 50 \sim 147(49 \sim 86)\text{MeV}$ with $y = 1.0(5.0)$. In each case, K_{vs} with $y = 5.0$ is not much different from that with $y = 1.0$. K_{vs} is hardly changed, if y increases much more. In table II(b), we summarize the range of the calculated K_{vs} in the cases of $y = 5.0$. Comparing this table with table I, we see that $K = 300 \pm 50\text{MeV}$ is necessary to

account for the empirical values of K , K_c , and K_{vs} at the same time, as is in the case of no VSI. Although the VSI makes K_{vs} smaller (more negative), the change of K_{vs} is comparable to the magnitude of the empirical error bars at the empirical available value of K ($\sim 300\text{MeV}$), and, therefore, the conclusion is hardly changed. The result is also independent on the detail of $U(\phi)$ as in the case of no VSI.

In summary, we have studied K , K_c , and K_{vs} by using the relativistic mean field theories with the nonlinear σ term and with the nonlinear σ and ω terms, with aid of the scaling model. It is found that, in both cases, $K = 300 \pm 50\text{MeV}$ is *necessary* to account for the empirical values of K , K_c , and K_{vs} at the same time. The result is independent on the detail descriptions of $U(\phi)$ and is almost independent on the strength of VSI. It seems that this conclusion is not drastically changed, if we use any type of the relativistic mean-field theory and the scaling model, since the calculated K_{vs} is most sensitive to the ratio K'/K , which is adjusted to the empirical values.

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Table and Figure Captions

Table I

The sets of the empirical values of K , K_c and K_{vs} in the table 3 in ref. [2]. (According to the conclusion in ref. [2], we only show the data in the cases of $K = 150 \sim 350\text{MeV}$.) All quantities in the table are shown in MeV.

Table II

Range of the calculated K_{vs} using the sets of K and K_c in table I as inputs. (a) The result in the the mean-field theory only with σ meson self-interaction. (b) The result in the mean-field theory with σ and ω mesons self-interactions with $y = 5.0$. : In each table, "upper bound", "mean value", and "lower bound" mean that the results are obtained by using the upper bound, the mean value, and the lower bound of K_c in table I, respectively. All quantities in the table are shown in MeV.

Fig. 1 K_{vs} as a function of M^* . (a) The cases of $K = 300\text{MeV}$ and $K_c = -3.990\text{MeV}$. (b) The cases of $K = 250\text{MeV}$ and $K_c = -0.7065\text{MeV}$: In each figure, the solid line is the result in the the mean-field theory only with σ meson self-interaction, and the dotted and the dashed lines are the results in the mean-field theory with σ and ω mesons self-interactions with $y = 1.0$ and $y = 5.0$, respectively.

	Set 1	Set 2	Set 3	Set 4	Set 5
K	150.0	200.0	250.0	300.0	350.0
K_c	5.861 ± 2.06	2.577 ± 2.06	-0.7065 ± 2.06	-3.990 ± 2.06	-7.274 ± 2.06
K_{vs}	66.83 ± 101	-46.94 ± 101	-160.7 ± 101	-274.5 ± 101	-388.3 ± 101

Table I

	upper bound	mean value	lower bound
$K = 150$	940~1738	728~1426	517~1113
$K = 200$	601~1230	389~ 918	178~ 605
$K = 250$	262~ 722	50~ 410	-161~ 97
$K = 300$	-77~ 214	-289~ -98	-506~-411
$K = 350$	-418~-294	-651~-606	-921~-840

Table II(a)

	upper bound	mean value	lower bound
$K = 150$	939~1103	728~ 863	516~ 623
$K = 200$	600~ 714	389~ 474	177~ 235
$K = 250$	261~ 325	49~ 86	-165~-154
$K = 300$	-80~ -63	-303~-289	-542~-500
$K = 350$	-451~-416	-691~-628	-931~-839

Table II(b)