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# Dual String Pairs

## With $N = 1$ and $N = 2$ Supersymmetry in Four Dimensions

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Based on a simple adiabatic argument and by considering the heterotic string counterpart of certain symmetries of Type II superstrings such as  $(-1)^{F_L}$  and orientation reversal, we construct orbifold candidates for dual pairs of heterotic and Type II string theories with  $N = 2$  and  $N = 1$  supersymmetry. We also analyze from a similar point of view the K3 fibrations that enter in recently proposed  $N = 2$  candidates and use this structure together with certain orientation-reversing symmetries to construct  $N = 1$  dual pairs. These pairs involve generalizations of Type I vacua which can be equivalent to  $E_8 \times E_8$  heterotic strings, while standard Type I vacua are related to  $SO(32)$ .

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## 1. Introduction

One of the recent lessons in physics is that there is apparently only one string theory.  $E_8 \times E_8$  and  $SO(32)$  heterotic strings and Type IIA, Type IIB, and Type I superstrings are apparently all manifestations of one underlying theory, which also has the well-known wild card, eleven-dimensional supergravity, as one of its limits.

String theory dualities leading to such statements have originally been studied in models with  $N = 4$  and  $N = 8$  supersymmetry [1-3]. In field theory, the physical phenomena controlled by electric-magnetic duality tend to become richer as the amount of supersymmetry is reduced (for a recent review see [4]). Recent developments involving string-string duality applied to models with  $N = 2$  supersymmetry in four dimensions [5,6] have shown that this is true in string theory as well. (For tests of some of the models in [5] see [7-9].)

The duality between string theories has an amazing problem-solving ability: a problem of dynamics defined by writing down one string theory can sometimes be solved by finding a dual theory to which it is equivalent. In that respect, it is fascinating to note that the heterotic string is naturally chiral, with non-abelian gauge groups, like the real world at high energies, while Type II strings are naturally non-chiral, with abelian gauge groups, like the real world at low energies. It is roughly as if, in the real world, the heterotic string should be used to describe grand unification and an equivalent Type II description should be used to understand the physical properties of the low energy vacuum.

The main purpose of the present paper is to construct additional dual pairs in four dimensions, with  $N = 2$  or  $N = 1$  supersymmetry; by a dual pair we mean simply a pair of equivalent Type II and heterotic string vacua. Such dual pairs have particular interest in the  $N = 1$  case, where one might meet models similar to the real world (though the models we will actually explore in this paper will not be terribly realistic) and one might encounter issues of supersymmetry breaking and the cosmological constant. Our models will be constructed by taking certain orbifolds of models – such as those of [5] and [6] – for which dualities have been found. This must be done with some care, because in general orbifolding seems not to commute with string-string duality; that is, it seems that in general the orbifold of a dual pair is not a dual pair.

This paper is organized as follows. In section two, we discuss the inequivalent world-sheet structures by which  $N = 2$  and  $N = 1$  space-time supersymmetry can be realized by Type II superstrings in four dimensions. We illustrate the discussion with a few simple

examples of what appear to be dual pairs with  $N = 1$  or  $N = 2$  supersymmetry. We also make, in a similar vein, some simple remarks relevant to new  $N = 4$  models [10-11] which can also be used as starting points for constructing dual pairs with  $N < 4$  supersymmetry. In section three, we try to deduce the dualities discovered in [5] from six-dimensional string-string duality by using the interpretation of the relevant Calabi-Yau manifolds as fiber bundles with K3 fibers [7], by means of which the  $j$ -functions appearing in the examples of [5] have been related to the results of Lian and Yau [12] on families of K3's. Though we do not succeed in fully explaining the models of [5], we draw a few lessons that we then use to construct additional dual pairs with  $N = 1$  supersymmetry – at least one such model for each  $N = 2$  model in [5]. These models involve a generalization of Type I superstrings, and have the notable property of reproducing certain  $E_8 \times E_8$  heterotic string vacua, while standard Type I models are limited to  $SO(32)$ .

## 2. World-Sheet Structure And Space-Time Supersymmetry

For a heterotic string vacuum in four dimensions to have  $N = 1$  space-time supersymmetry, the world-sheet theory must be a  $(0, 2)$  superconformal field theory with a  $U(1)$   $R$  symmetry with integral charges [13]. Likewise,  $N = 2$  space-time supersymmetry for the heterotic string comes from a  $(0, 4)$  superconformal world-sheet theory with an  $SU(2)$   $R$  symmetry [14]. For Type II superstrings, there are instead several distinct world-sheet structures that lead to a given space-time supersymmetry. For  $N = 2$  there are the following possibilities.

(1)  $N = 2$  space-time supersymmetry in four dimensions can arise from a  $(2, 2)$  world-sheet theory with left and right moving  $U(1)$   $R$  symmetries. This is the most familiar case; it arises in compactification of Type II superstrings on a Calabi-Yau manifold. One space-time supersymmetry is carried by left-moving degrees of freedom on the world-sheet and one is carried by right-moving degrees of freedom.

(2)  $N = 2$  space-time supersymmetry can likewise arise from a  $(1, 4)$  world-sheet theory. In this case, both space-time supersymmetries are carried by right-moving degrees of freedom on the world-sheet.

(3) Finally, one can consider a left-right symmetric world-sheet structure with  $(4, 4)$  world-sheet supersymmetry in which by introducing in some way unorientable world-sheets one projects the massless states onto states that are left-right symmetric. (This can be done by introducing open strings as in the Type I theory, or by constructions – discussed later –

that are similar to the orientifolds of [15].) In this case, both space-time supersymmetries are carried by linear combinations of left and right-moving variables.

Similarly, two distinct world-sheet structures give  $N = 1$  supersymmetry in four dimensions for Type II superstrings. They are analogs of cases (2) and (3) above.

(2')  $N = 1$  supersymmetry can arise from a  $(1, 2)$  superconformal world-sheet theory with a right-moving  $U(1)_R$  symmetry. The supersymmetry is carried by right-movers.

(3')  $N = 1$  supersymmetry can likewise arise from a  $(2, 2)$  world-sheet with a projection on left-right invariant states. The structure arising generalizes that of a Type I theory compactified on a Calabi-Yau manifold: the  $N = 1$  space-time supersymmetry is carried by a combination of left-moving and right-moving degrees of freedom.

In the rest of this section, we first answer a question of basic importance for  $N = 2$  strings in four dimensions: where is the dilaton? For method (1) of realizing space-time supersymmetry, the dilaton is known to be part of a hypermultiplet [16]; in case (2), we will see that the dilaton is part of a vector multiplet while in case (3) it is a linear combination of part of a vector multiplet and part of a hypermultiplet. Because (except for gauge couplings) vector multiplets and hypermultiplets are decoupled at low energies in  $N = 2$  theories in four dimensions [17], it follows from the above that in case (1), there are no quantum corrections to the vector moduli space - a fact that has played an important role recently - while in case (2), there are no corrections to the hypermultiplet moduli space, and in case (3) (as the dilatons can couple at low energy to modes of both kinds) there can be corrections to each moduli space.

After locating the dilaton, we move on to discuss simple illustrative models of types (2) and (3), and also of types (2') and (3'), for which heterotic string duals can apparently be identified. Finally, we discuss in a somewhat similar spirit new  $N = 4$  models [10] that can apparently be modified to give additional dual pairs with  $N < 4$ .

### *2.1. Finding The Dilaton*

First we recall how the dilaton arises in string theory in general. Left-moving massless states have helicity at most one in absolute value; likewise the right-moving states have helicity at most one. By taking the tensor product of left and right-moving massless states, one can make the graviton, of helicity  $\pm 2$ . One can also, by taking the tensor product of a left-moving state of helicity  $-1$  with a right-moving state of helicity  $1$ , construct a scalar; a second scalar comes from the tensor product of left-moving helicity  $1$  and right-moving helicity  $-1$ . The symmetric combination of these states is known as the dilaton. (For

oriented closed strings, the antisymmetric combination of these states is the axion; for unoriented superstrings, the axion is instead a Ramond-Ramond state.)

Irreducible massless  $N = 2$  representations have four states of helicities  $(j, j - 1/2, j - 1/2, j - 1)$  where  $j$  is an integer or half-integer which is the “highest weight” of the representation. Because of constraints such as CPT, pairs of irreducible representations combine into the  $N = 2$  multiplets that actually appear in Lagrangians. For instance, the  $N = 2$  vector multiplet is the direct sum of representations with highest weight  $j = 1$  and  $j = 0$  (which contain respectively the vector states of helicity 1 and  $-1$ ), while the hypermultiplet is the direct sum of two  $N = 2$  multiplets with highest weight  $j = 1/2$ .

Now it is fairly straightforward, by locating the left and right-moving states of helicity  $\pm 1$  in appropriate supermultiplets and taking the tensor products, to see what kind of supermultiplet contains the dilaton. We consider the three cases in turn.

(1) For (2,2) models, the left-moving states and right-moving states each carry  $N = 1$  supersymmetry – leading to  $N = 2$  for the tensor product. Irreducible massless multiplets of  $N = 1$  have precisely two helicity states. The left-moving state of helicity  $-1$  is thus in a multiplet containing two states of helicities  $(-1/2, -1)$ , and the right-moving state of helicity 1 is in a multiplet containing two states of helicities  $(1, 1/2)$ . Taking the tensor product of these  $N = 1$  multiplets gives an  $N = 2$  multiplet  $W$  with four states of helicities  $(1/2, 0, 0, -1/2)$ . which we recognize as part of a hypermultiplet. Therefore, the dilaton is part of a hypermultiplet when  $N = 2$  supersymmetry is realized using a (2,2) world-sheet structure.

(2) If  $N = 2$  supersymmetry is realized in a (1,4) model, then the left-moving states carry Lorentz quantum numbers only and the right-moving states form  $N = 2$  multiplets. To find the dilaton, we consider the tensor product of a left-moving state of helicity  $-1$  with a right-moving  $N = 2$  multiplet of helicities  $(1, 1/2, 1/2, 0)$ . The tensor product gives an irreducible  $N = 2$  representation with states of helicity  $(0, -1/2, -1/2, -1)$ , which we recognize as part of a vector multiplet. Hence in this case the dilaton is in a vector multiplet. (Precisely the same argument shows that for  $N = 2$  heterotic strings, the dilaton is in a vector multiplet; all that matters is that in each case, the supersymmetry is carried entirely by right-movers.)

(3) More complex is the case of a world-sheet (4,4) model with a projection on symmetric combinations of left and right-moving states. In this case, the left and right-moving states are each in  $N = 2$  multiplets. We start with left-moving  $N = 2$  representations  $A_L$  of highest weight  $j = 0$ , that is helicities  $(0, -1/2, -1/2, -1)$  and  $B_L$  of highest weight

$j = 1$ , that is helicities  $(1, 1/2, 1/2, 0)$ , and right-moving  $N = 2$  multiplets  $A_R$  and  $B_R$  also of highest weights 0 and 1, respectively. The dilaton and axion are contained in the tensor product  $A_L \otimes B_R$ . ( $A_R \otimes B_L$  need not be considered separately, since the projection on left-right symmetric combinations effectively identifies  $A_R \otimes B_L$  with  $A_L \otimes B_R$ .) The  $N = 2$  representation  $A_L \otimes B_R$  has sixteen states and can be decomposed as a direct sum of four irreducible four-dimensional representations of  $N = 2$ . Let us find the highest weights of these irreducible representations. One of them is the highest weight state  $|0, 1\rangle$  of  $A_L \otimes B_R$ . Having helicity one, it is the highest weight of a representation that is part of a vector multiplet. To find the other highest weights, we let  $Q'_i$ ,  $i = 1, 2$  be the left-moving helicity lowering operators,  $Q''_i$ ,  $i = 1, 2$  the right-moving helicity lowering operators,  $Q_i = Q'_i + Q''_i$  the diagonal combination that is observed as the physical  $N = 2$  supersymmetry, and  $\tilde{Q}_i = Q'_i - Q''_i$ . Of the states of helicity  $1/2$ , clearly  $Q_i|0, 1\rangle$  are in the vector multiplet headed by  $|0, 1\rangle$ , and  $\tilde{Q}_i|0, 1\rangle$  are highest weights of new  $N = 2$  representations. These states have  $j = 1/2$  and so are part of a hypermultiplet. At helicity zero we have the following states:

$$\begin{aligned}
&\epsilon^{ij} Q_i Q_j |0, 1\rangle \\
&Q_i \tilde{Q}_j |0, 1\rangle \\
&\epsilon^{ij} \tilde{Q}_i \tilde{Q}_j |0, 1\rangle.
\end{aligned} \tag{2.1}$$

The state in the first row, being obtained by lowering of  $|0, 1\rangle$ , is in a vector multiplet; the states in the second row, being obtained by lowering of the  $j = 1/2$  highest weight  $\tilde{Q}_j|0, 1\rangle$ , are in a hypermultiplet; and the state in the last row, being a  $j = 0$  highest weight (of an  $N = 2$  representation that also contains a state of helicity  $-1$ ), is in a vector multiplet again. Now to locate the dilaton, we note simply that the dilaton of this model is the state obtained by tensoring a left-moving state of helicity  $-1$  with a right-moving state of helicity 1 (because of the left-right projection, this is equivalent to the tensor product of helicities 1 and  $-1$ ; the axion is a certain Ramond-Ramond state). The dilaton is therefore  $D = \epsilon^{ij} Q'_i Q'_j |0, 1\rangle$ . Expressing  $D$  as a linear combination of the states given in (2.1), we see that for models of this type, the dilaton is a linear combination of states that belong to a vector multiplet and states that belong to a hypermultiplet. <sup>1</sup>

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<sup>1</sup> In the orbifold examples discussed later in this section, the  $T$  modulus of the heterotic string is in a vector or hypermultiplet or in a mixture depending on whether the  $\mathbf{Z}_2$  acts with two, zero, or one invariant directions on the common  $\mathbf{T}^2$ . This easily verified statement is converted by string-string duality into the above assertions about the dilaton.

We need not repeat this analysis for theories with  $N = 1$  supersymmetry in four dimensions, since for  $N = 1$  there is only one kind of multiplet – the chiral multiplet – containing states of helicity zero. The dilaton is therefore in a chiral multiplet in both cases (2') and (3') of  $N = 1$  supersymmetry. There is, however, an interesting general difference in the way that duality is realized for the two ways of obtaining  $N = 1$  supersymmetry in four dimensions from Type II. The Type IIB theory in ten dimensions has an  $SL(2, \mathbf{Z})$  symmetry mixing the dilaton with a certain Ramond-Ramond scalar. Upon compactification to four dimensions, this turns into a standard  $S$ -duality for  $N = 1$  orientifolds (since the Ramond-Ramond scalar in question becomes the axion). It acts differently for  $N = 1$  models with (1, 2) world-sheet supersymmetry.

## 2.2. A (1,4) Example

Our next goal is to construct a simple example of a Type II vacuum with (1,4) world-sheet supersymmetry and  $N = 2$  spacetime supersymmetry. Then we will propose a candidate for the heterotic string dual of this vacuum. We start with the Type IIA theory on  $\mathbf{R}^6 \times \text{K3}$ . This is a familiar model with (4,4) world-sheet supersymmetry and  $N = 4$  spacetime supersymmetry – two supersymmetries being carried by left-movers and two by right-movers. The supersymmetries carried by left-movers have one chirality in spacetime and the supersymmetries carried by right-movers have the opposite space-time chirality.

We want to modify this model to obtain a model with only  $N = 2$  spacetime supersymmetry, carried entirely by right-movers. To do this, we will make an orbifold projection, dividing by a symmetry that commutes with the spacetime supersymmetries that come from right-movers and anticommutes with the ones that come from left-movers.

The obvious symmetry with that property is  $(-1)^{F_L}$ , the operation that acts as  $-1$  on states the left-moving part of which is fermionic and as  $+1$  on states the left-moving part of which is bosonic. Before proceeding, let us ask what  $(-1)^{F_L}$  transforms into under the conjectured duality between the Type IIA theory on  $\mathbf{R}^6 \times \text{K3}$  and the heterotic string on  $\mathbf{R}^6 \times \mathbf{T}^4$ . Apart from anything else, the ability to find a symmetry of the heterotic string with the properties of  $(-1)^{F_L}$  is a test of string-string duality (assuming that  $(-1)^{F_L}$  is actually a valid symmetry of the Type IIA theory with no nonperturbative anomalies).

Translating  $(-1)^{F_L}$  into heterotic string language can be done easily as follows.  $(-1)^{F_L}$  acts as  $-1$  on all Ramond-Ramond states. All 24  $U(1)$  vector bosons of the model are Ramond-Ramond states, so the symmetry must act as  $-1$  on all 24 of them. This means that on the heterotic string side, the symmetry must be the operator  $W = -1$  acting

on the Narain lattice  $\Gamma^{20,4}$  describing the compactification. Apart from the fact that  $W$  transforms the gauge bosons correctly, the identification of  $(-1)^{F_L}$  with  $W$  can be checked as follows. The inversion of the Narain lattice commutes with the  $T$ -duality group  $SO(20, 4; \mathbf{Z})$ , as expected for  $(-1)^{F_L}$ . Also note that, as expected of  $(-1)^{F_L}$ ,  $W$  acts as  $+1$  on the two supersymmetries of one chirality and as  $-1$  on the other two. Indeed, for the heterotic string on  $\mathbf{R}^6 \times \mathbf{T}^4$ , the inversion  $W$  of the Narain lattice acts in particular as  $-1$  on  $\mathbf{T}^4$ ; as in standard constructions of K3 orbifolds, this operation acts as  $-1$  on the spinors of one chirality and as  $+1$  on the spinors of the other chirality.

Now that we have identified the symmetry, we would like to use it to construct orbifolds with  $N = 2$  supersymmetry. To begin with, one might try to orbifold the Type IIA theory by  $(-1)^{F_L}$ . As a way of obtaining an  $N = 2$  model, this does not succeed; dividing by  $(-1)^{F_L}$  eliminates two supersymmetries from the untwisted sector, but one gets two more supersymmetries from the twisted sector, giving in fact the Type IIB theory on  $\mathbf{R}^6 \times \text{K3}$ . Incidentally, this model gives a simple illustration of the fact that orbifolding does not in general commute with string-string duality. In dividing the heterotic string by  $W$ , one gets a theory whose twisted sector has 20 antiperiodic left-moving oscillators. This gives a  $1/4$  mismatch between left- and right-moving ground state energies, corresponding to an inconsistent theory with sigma model anomalies. (There is no way to restore level matching by shifts because – as all the oscillators are twisted – any shift could be absorbed in adding a constant to the world-sheet bosons.) So the orbifold of the Type IIA theory by  $(-1)^{F_L}$  is not equivalent to the orbifold of the heterotic string by  $W$ .

To modify the construction so that we do get  $N = 2$  supersymmetry on the Type II side, we only need to combine  $(-1)^{F_L}$  with another operation in such a way as to avoid getting extra supersymmetry from the twisted sector. An obvious way to do this is to consider the Type IIA theory on  $\mathbf{R}^5 \times \mathbf{S}^1 \times \text{K3}$ , and to divide by a symmetry  $Y$  that acts as a  $\pi$  rotation of  $\mathbf{S}^1$  together with  $(-1)^{F_L}$ . For generic radius of  $\mathbf{S}^1$ , there are no massless states in the twisted sector, so we do get a model with  $N = 2$  spacetime supersymmetry entirely carried by right-movers on the world-sheet.

### *The Adiabatic Argument And Duality*

We would like to find a heterotic string orbifold that will be equivalent to the Type II model just described. First we should ask: Since orbifolding does not commute with string-string duality in general, why should such a model exist? In the particular case at hand, one can give a physical argument that we consider convincing; we will call it the



adiabatic argument. In this paper we will mostly consider examples to which this argument applies.

The argument uses the fact that the radius  $R$  of  $\mathbf{S}^1$  is arbitrary and that the symmetry  $Y$  acts freely on  $\mathbf{S}^1$ . If  $R$  is very large,<sup>2</sup> the low energy observer in the Type II theory on  $\mathbf{R}^5 \times \mathbf{S}^1 \times \text{K3}$  sees a world which – unless he circumnavigates the  $\mathbf{S}^1$  – is very hard to distinguish from  $\mathbf{R}^6 \times \text{K3}$ . Locally, one can use string-string duality and convert to a heterotic string description on  $\mathbf{R}^6 \times \text{K3}$ . Once the equivalence is established locally, the low energy observer can reasonably expect that it will remain valid globally if he or she suitably redefines all physical variables in circumnavigating the  $\mathbf{S}^1$ .

From this argument alone the low energy observer cannot determine precisely which heterotic string orbifold should correspond to the given Type II orbifold. That is because certain distinctions disappear when  $R$  becomes very large. To see this, consider the Narain lattice  $\Gamma^{1,1}$  appropriate to  $\mathbf{S}^1$ . We can describe it as consisting of pairs of integers  $(m, n)$  with the inner product of  $(m, n)$  and  $(m', n')$  being  $mn' + m'n$ ; this is obviously even and unimodular. A purely right-moving momentum vector is of the form  $(m, m)$  (for which the inner product is positive); pure left-moving momenta are  $(m, -m)$ . A state with ordinary spatial momentum and no winding is given by the null vector  $(m, 0)$  while winding without momentum would be the null vector  $(0, m)$ . Now, let us consider the possible  $\mathbf{Z}_2$  orbifolds of  $\mathbf{S}^1$  (temporarily ignoring questions of level matching which depend on the coupling to the other degrees of freedom). A  $\mathbf{Z}_2$  orbifold is obtained by shifting  $\mathbf{R}^{1,1}$  (in which  $\Gamma^{1,1}$  is embedded) by a vector  $A$  which is one-half of a point in  $\Gamma^{1,1}$ ; moreover, addition to  $A$  of a point in  $\Gamma^{1,1}$  will give an equivalent model. Thus, the non-zero choices of  $A$  are essentially  $A_1 = (1/2, 0)$ ,  $A_2 = (1/2, 1/2)$ , and  $A_3 = (0, 1/2)$ .

Let us think about how the resulting theories look to a low energy observer at large  $R$ . The low energy observer can measure ordinary momenta but not windings,<sup>3</sup> and so can measure inner products of  $A$  only with ordinary momentum vectors  $(m, 0)$ . This means that the shift by  $A_1$  looks trivial to the low energy observer, while the  $A_2$  and  $A_3$  shifts look equivalent for large  $R$ ; each of them acts on a momentum  $m$  by  $(-1)^m$  and so is equivalent on momentum states to a  $\pi$  rotation of the circle. Notice that  $A_3$  is a null

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<sup>2</sup> We want to go to large  $R$  keeping fixed the string coupling constant in the six-dimensional sense – a prescription that is invariant under string-string duality.

<sup>3</sup> The winding states of the heterotic string correspond under string-string duality to soliton states of the Type II theory whose behavior under  $Y$  is not entirely clear.

vector and the shift by  $A_3$  actually is an ordinary rotation, while  $A_2$  obeys  $A_2^2 = 1/2$ , and the shift by  $A_2$  is not a standard rotation.

The low energy observer thus expects that the orbifold of the Type II theory by  $Y$  will be equivalent to the inversion  $W$  of the Narain lattice together with a shift by  $A_2$  or  $A_3$ . Which is correct? This question can be settled by thinking about level matching. The transformation  $W$  acts as  $-1$  on 20 left-moving bosons and so raises the ground state energy of left-movers by  $20/16 = 5/4$ . The total shift of the left-moving ground state energy will then be  $\Delta_L = 5/4 + A^2/2$ . As (by supersymmetry) there is no such shift for right-movers, level-matching requires that  $\Delta_L$  should be an integer multiple of  $1/2$ , so we need  $A^2 = 1/2 \pmod{\mathbf{Z}}$ . This fixes  $A = A_2$ .

A similar effect – a not purely geometrical shift in the internal  $\Gamma^{20,4}$  Narain lattice to preserve level matching for the heterotic string – was found in a different example in [6]. Since in our case the shift is on a geometrical circle that is part of the six dimensions common to type IIA and heterotic strings, we have added a somewhat more detailed explanation of why this does not contradict the experience of the low energy observer.

Let us compactify this model on another circle  $\mathbf{S}^1$  down to four dimensions. This still gives a theory with  $N = 2$ . For later reference, let us summarize the construction of the heterotic string version of this model. We have first considered toroidal compactification down to 6 dimensions, which is characterized by a Narain lattice  $\Gamma^{20,4}$ . We have then compactified on another two dimensional torus characterized by a Narain lattice  $\Gamma^{2,2}$ . Write a general momentum and winding state as

$$|\gamma^{20,4}, \gamma^{2,2}\rangle \in \Gamma^{20,4} \oplus \Gamma^{2,2}.$$

Then the  $\mathbf{Z}_2$  action by which we have modded out is generated by

$$g|\gamma^{20,4}, \gamma^{2,2}\rangle = \exp(2\pi i A_2 \cdot \gamma^{2,2})|-\gamma^{20,4}, \gamma^{2,2}\rangle \quad (2.2)$$

(where  $A_2$  is the vector introduced before but now embedded in  $\Gamma^{2,2}$ ), whereas on the Type IIA side we have modded out by  $(-1)^{F_L} \exp(2\pi i A_3 \cdot \gamma^{2,2})$ .

Let us attempt to check the duality between the Type IIA and heterotic string descriptions in this example. To see that both theories have the same massless spectrum, all we have to check is the untwisted sector because all the twisted states are massive: On the Type II side we have a left-right symmetric shift which thus gives positive mass; on the heterotic side, even though the shift is asymmetric and by itself will not preclude a

massless state at special radius, the fact that we have 20 half-integral oscillators means that the left-moving energy is at least  $1/4 > 0$ .

It is easy to enumerate the massless states we get from the untwisted sector after projecting by the  $\mathbf{Z}_2$  action (2.2). First we consider vector multiplets. Both models have generically a gauge symmetry  $U(1)^4$  coming entirely from the  $\mathbf{T}^2$ , so there are three vector multiplets, with the fourth  $U(1)$  vector field being part of the  $N = 2$  gravity multiplet. The scalar components of the three vector multiplets are the fields usually denoted by  $S, T, U$ , where in going from the heterotic to Type IIA we have to exchange  $S \leftrightarrow T$ . The weak coupling dualities of  $S, T$  and  $U$  will be analyzed at the end of this section. Since the dilaton  $S$  is also a member of a vector multiplet – on both sides, as we have seen – the metric on the vector moduli space is expected to be corrected. Such corrections are certainly present. For example if we choose the second circle (the one which brought us down from five to four dimensions) to be at the self-dual radius, we get in the heterotic string description an  $SU(2)$  gauge symmetry at the classical level, but quantum mechanically the picture is different [18]. Note that if for (1, 4) Type II strings, the dilaton were part of a hypermultiplet (as it is in the (2, 2) case), we would be unable to reproduce these corrections on the Type II side and would meet a contradiction.

There are also 20 hypermultiplets. Since the dilaton is in a vector multiplet in each theory, the hypermultiplet moduli spaces are uncorrected, and for the theories to be equivalent their classical hypermultiplet moduli spaces must coincide. This is so; on the Type IIA side, the moduli space is naturally the moduli space of quantum K3's, and for the heterotic string we get the moduli space of Narain lattices  $\Gamma^{20,4}$ . That these coincide is an essential test of string-string duality in six dimensions.

### *2.3. An Orientifold Example*

Our next goal is to construct a simple  $N = 2$  vacuum of the Type II theory based on unorientable world-sheets together with a candidate for its heterotic string dual.

First of all, recall that in the Type IIA superstring theory, the left-movers on the string world-sheet give spinors of negative chirality in space-time, and the right-movers give spinors of positive chirality. The theory is therefore invariant under reversal of orientation of space-time together with reversal of the orientation of the world-sheet. If the target space is invariant under some orientation-reversing involution, then we get a symmetry of the string theory with that target space; otherwise we get an equivalence between a target space and its orientation-reversed counterpart.

Let us make a general comment about such a symmetry  $X$ . In general, giving a classical orientation-reversing involution of space-time does not quite determine the operation in string theory. Though  $X^2$  is trivial as an operation on space-time, it may act as  $-1$  on spinors; that is, one may have  $X^2 = (-1)^F$ . This situation, however, can always be avoided, as follows. One simply sets  $\tilde{X} = (-1)^{F_L} X$  and observes that (with  $X$  being an operation that exchanges left and right-movers) if  $X^2 = (-1)^F$  then  $\tilde{X}^2 = 1$ . So replacing  $X$  by  $\tilde{X}$  if necessary, we can always achieve  $X^2 = 1$ .<sup>4</sup>

With this in mind, consider the Type IIA theory compactified on  $\mathbf{R}^5 \times \mathbf{S}^1 \times \text{K3}$ . If we take the  $B$ -fields on K3 to vanish, then the sigma model with this target space is left-right symmetric on the world-sheet. This means that the theory is invariant under a symmetry  $X$  that acts by an orientation-reversing diffeomorphism  $x \rightarrow -x$  of  $\mathbf{S}^1$ , together with exchange of left and right-movers.

What does  $X$  correspond to for heterotic strings on  $\mathbf{R}^5 \times \mathbf{S}^1 \times \mathbf{T}^4$ ? Apart from any application we may make of this, the ability to identify  $X$  on the heterotic string side is a test of string-string duality (unless  $X$  is broken explicitly by nonperturbative corrections to the Type IIA theory). Obviously,  $X$  must correspond to an operation that acts as  $x \rightarrow -x$  on  $\mathbf{S}^1$ . But what does it do to  $\mathbf{T}^4$ ?

This question can be answered by using the fact that  $X$  must commute with the appropriate  $T$ -duality group. At first sight one may despair since the center of the usual  $T$ -duality group  $SO(20, 4; \mathbf{Z})$  of K3 has  $-1$  as its only non-trivial element, and we have already used this element in identifying  $(-1)^{F_L}$ !

However, since we have had to set the  $B$ -fields to zero to obtain the symmetry  $X$ , the usual conformal field theory moduli space  $SO(20, 4; \mathbf{Z}) \backslash SO(20, 4; \mathbf{R}) / SO(20) \times SO(4)$  [19] is replaced by a subspace, which is in fact the classical moduli space of Einstein metrics on K3 [20]

$$(SO(1, 1; \mathbf{Z}) \backslash SO(1, 1; \mathbf{R})) \times (SO(19, 3; \mathbf{Z}) \backslash SO(19, 3; \mathbf{R}) / SO(19) \times SO(3)).$$

The first factor parametrizes the volume of K3 and the second factor parametrizes Einstein metrics of unit volume. The decomposition means that the cohomology lattice has an orthogonal decomposition as  $\Gamma^{1,1} \oplus \Gamma^{19,3}$  and that with  $B$ -field zero, the  $T$ -duality group can be reduced to the subgroup  $SO(1, 1; \mathbf{Z}) \times SO(19, 3; \mathbf{Z})$  that respects this decomposition.

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<sup>4</sup> This discussion is still not precise enough to fix the action of  $X$  on certain black holes, as we will explain later.

The center of this smaller group now contains the two elements that act as  $+1$  on  $\Gamma^{1,1}$  and  $-1$  on  $\Gamma^{19,3}$ , or vice-versa. Either of these operations gives a symmetry of the theory (they differ by multiplication by  $(-1)^{FL}$ ), but if we want  $X^2 = 1$  (rather than  $X^2 = (-1)^F$ ), we must choose  $X$  to act as  $+1$  on  $\Gamma^{1,1}$  and  $-1$  on  $\Gamma^{19,3}$ . This can be seen by studying the exchange of left and right-movers in the Ramond-Ramond sector of the Type IIA theory, or more simply in the heterotic string description to which we now turn (where  $X^2 = (-1)^F$  would arise if  $X$  acted with two rather than four eigenvalues  $-1$  on right-moving bosons).

For an interpretation of  $X$  for heterotic strings, it is now clear what we must do. We write the Narain lattice of  $\mathbf{S}^1 \times \mathbf{T}^4$  as  $\Gamma^{1,1} \oplus \Gamma^{20,4} = \Gamma^{1,1} \oplus \Gamma^{1,1} \oplus \Gamma^{19,3}$ .  $X$  acts as  $-1$  on the first  $\Gamma^{1,1}$  (coming from  $\mathbf{S}^1$ ), as  $+1$  on the second  $\Gamma^{1,1}$  (coming from  $\mathbf{T}^4$ , and as  $-1$  on the last factor. Thus we have succeeded in finding a heterotic string counterpart of  $X$ . Actually, the determination of  $X$  was not quite unique: one could add a translation by half a lattice vector in the fixed  $\Gamma^{1,1}$ . (Such a transformation is a symmetry of the Type IIA theory that acts only on Ramond-Ramond black holes, so we have not defined the operation  $X$  precisely enough to be able to say if it should be present here or not.)

It is curious to compare this with the operation  $W$  that represents  $(-1)^{FL}$  for heterotic strings.  $W$  acts on the Narain lattice  $\Gamma^{1,1} \oplus \Gamma^{1,1} \oplus \Gamma^{19,3}$  of  $\mathbf{S}^1 \times \mathbf{T}^4$  as  $+1$  on the first factor and  $-1$  on the last two factors. This differs from  $X$  by exchange of the first two factors. Thus, we get the remarkable result that the symmetries  $(-1)^{FL}$  and  $X$  on  $\mathbf{R}^5 \times \mathbf{S}^1 \times \mathbf{K3}$  are in fact conjugate. One can also easily check, with this description of  $X$  and  $W$ , that  $XW = WX(-1)^F$ , as one can anticipate from the Type II side.

The surprising conjugacy of  $X$  and  $(-1)^{FL}$  can also be understood by going to strong coupling and using the description by eleven-dimensional supergravity. Thus,  $\mathbf{R}^5 \times \mathbf{S}^1 \times \mathbf{K3}$  of the Type IIA description becomes  $\mathbf{R}^5 \times \mathbf{S}^1 \times \mathbf{K3} \times \mathbf{S}^1$ , where the radius of the second  $\mathbf{S}^1$  is related [3] to the Type IIA string coupling constant. In this description,  $(-1)^{FL}$  is the operation that acts as  $-1$  on the second circle and changes the sign of the three-form  $A$  of eleven-dimensional supergravity;  $X$  is the operation that acts as  $-1$  on the first circle and changes the sign of  $A$ . The two operations are conjugate under exchange of the two circles.

### *Orientifolds*

Now we wish, rather as in [15], to discuss orientifolds, by which we mean orbifolds in which one divides by a symmetry such as  $X$  that reverses the orientation of the world-sheet. (These generalize the Type I superstring, which should very plausibly be considered

as such an orbifold of the Type IIB theory – which is invariant under reversal of world-sheet orientation with no action on space-time.)

First let us describe in general what such an orbifold means. We consider a general space-time  $M$  with  $\mathbf{Z}_2$  action that reverses the orientation. The orientifold is then a string propagating on  $M/\mathbf{Z}_2$ , with a not-necessarily-orientable world-sheet  $\Sigma$ , and the following restriction. One considers only Riemann surfaces  $\Sigma$ , and maps  $\Phi : \Sigma \rightarrow M/\mathbf{Z}_2$ , such that the pull-back by  $\Phi$  of  $w_1(M/\mathbf{Z}_2)$  equals  $w_1(\Sigma)$  (where  $w_1$  is the first Steifel-Whitney class, measuring the obstruction to orientability). Or more informally, orientation-reversing loops in  $\Sigma$  are mapped to orientation-reversing loops in space-time. At least when  $\mathbf{Z}_2$  acts freely, it seems fairly clear that this will give a consistent string theory; this is presumably true in much greater generality.

We could simply take the orientation reversing symmetry of  $\mathbf{R}^5 \times \mathbf{S}^1 \times \text{K3}$  discussed above (acting by  $-1$  on  $\mathbf{S}^1$  and trivially on the rest) and attempt to construct the corresponding Type II orientifold. The fixed points in the symmetry action on  $\mathbf{S}^1$  would lead to interesting subtleties.

If we want a model for which the orientifold can be constructed without such subtleties, we can compactify further to  $\mathbf{R}^4 \times \mathbf{S}^1 \times \mathbf{S}^1 \times \text{K3}$ , and consider an involution  $Y$  that acts by a  $\pi$  rotation of the first  $\mathbf{S}^1$ , multiplies the second  $\mathbf{S}^1$  by  $-1$ , and reverses the orientation of the world-sheet. The orientifold obtained by dividing by this symmetry is a simple example of a Type II vacuum with  $N = 2$  space-time supersymmetry realized by a mixture of left-movers and right-movers.

Moreover, since  $Y$  acts freely on  $\mathbf{S}^1 \times \mathbf{S}^1$ , we can – if the adiabatic argument is valid – determine the heterotic string dual of this model. We simply consider the heterotic string on  $\mathbf{R}^4 \times \mathbf{S}^1 \times \mathbf{S}^1 \times \mathbf{T}^4$ , and consider the orbifold by the symmetry  $\tilde{Y}$  that rotates the first circle by  $\pi$ , and acts as  $X$  on  $\mathbf{S}^1 \times \mathbf{T}^4$ . (As we noted when  $X$  was first defined, the criteria leading to its definition do not uniquely determine whether  $X$  should act by a shift on the fixed  $\Gamma^{1,1}$ . If such a shift was omitted in defining  $X$  – natural to make  $(-1)^{FL}$  and  $X$  precisely conjugate – then it must be included here for level matching in the orbifold.)

So far, we have constructed two elementary examples of Type II vacua with  $N = 2$  space-time supersymmetry: one with  $(1,4)$  world-sheet supersymmetry, and one an orientifold. The two models are actually equivalent (after compactification of the first model from  $\mathbf{R}^5$  to  $\mathbf{R}^4 \times \mathbf{S}^1$ ) because the symmetries  $(-1)^{FL}$  and  $X$  are conjugate. The equivalence of the orientifold to models with more familiar world-sheet structures certainly indicates that it is consistent.

#### 2.4. Another $N = 2$ Orientifold Example

In this subsection we will construct another type II  $N = 2$  orientifold whose heterotic dual is the same as the one studied in [6]. This orientifold construction has the advantage that our adiabatic argument applies. (The model of [6], though well-supported by other arguments, cannot be justified by the adiabatic argument because although the orbifolding group acts freely on the total space, it does not act freely when projected to the  $\mathbf{T}^2$  that the heterotic and Type II theories share in common.)

Let us first briefly recall the model of [6]. We have on the Type II side a  $\text{K3} \times \mathbf{T}^2$  compactification, for which there is an involution on K3 giving an Enriques surface. We mod out by this  $\mathbf{Z}_2$  accompanied with a total reflection on  $\mathbf{T}^2$ . On the heterotic side we first have to consider the action of  $\mathbf{Z}_2$  on the cohomology of K3. For K3's which double cover the Enriques surface, it is convenient to write the cohomology lattice as

$$\Gamma^{20,4} = \Gamma_a^{9,1} \oplus \Gamma_b^{9,1} \oplus \Gamma_a^{1,1} \oplus \Gamma_b^{1,1} \quad (2.3)$$

The  $\mathbf{Z}_2$  action which gives the Enriques surface acts on the K3 cohomology by

$$|\gamma_a^{9,1}, \gamma_b^{9,1}, \gamma_a^{1,1}, \gamma_b^{1,1}\rangle \rightarrow |\gamma_b^{9,1}, \gamma_a^{9,1}, -\gamma_a^{1,1}, \gamma_b^{1,1}\rangle \quad (2.4)$$

The  $\mathbf{Z}_2$  action  $h$  considered in [6] combines this involution with the involution on the common  $T^2$ , as well as some phase factors

$$h|\gamma_a^{9,1}, \gamma_b^{9,1}, \gamma_a^{1,1}, \gamma_b^{1,1}, \gamma^{2,2}\rangle = \exp(2\pi i A_2 \cdot \gamma_b^{1,1}) |\gamma_b^{9,1}, \gamma_a^{9,1}, -\gamma_a^{1,1}, \gamma_b^{1,1}, -\gamma^{2,2}\rangle \quad (2.5)$$

where the  $\gamma^{2,2} \in \Gamma^{2,2}$  denotes a vector in the Narain lattice for the common  $\mathbf{T}^2$ . The massless spectrum includes 11 vector multiplets parametrized by the standard  $SO(10, 2) \times SL(2)$  coset, and 12 hypermultiplets parametrized by the standard  $SO(12, 4)$  coset. The dilaton on the heterotic side is part of the  $SL(2)$  coset but on the Type II side is part of a hypermultiplet. It was argued in [6] using duality that the classical geometry of neither multiplet receives quantum corrections.

Now we wish to describe the same model by a Type II orientifold. The model is basically the same as the (1, 4) model discussed in the previous subsection, except that we accompany the orientifold action with the involution on K3 which gives the Enriques surface. To describe this in detail, let us introduce some notation. We consider the common  $\mathbf{T}^2$  as a product  $\mathbf{S}_c^1 \times \mathbf{S}_d^1$ . We consider the orientifold action which acts by a reflection on

$\mathbf{S}_c^1$ , translation by  $\pi$  on  $\mathbf{S}_d^1$ , Enriques involution on K3 and left-right exchange on the world sheet. Note that to write the orientifold action on the K3 cohomology in the decomposition of the lattice given in (2.3), the first three factors combine to give  $\Gamma^{19,3}$  and the last one is  $\Gamma_b^{1,1}$ . As discussed before the orientifold action flips the sign of one of these two sublattices. In this case, unlike the previous case, in order for  $X^2 = 1$  the action is reflection on the  $\Gamma_b^{1,1}$ . Taking all this into account and noting how the Enriques involution acts on the K3 cohomology (2.4) we find on the heterotic side

$$\tilde{h}|\gamma_a^{9,1}, \gamma_b^{9,1}, \gamma_a^{1,1}, \gamma_b^{1,1}, \gamma_c^{1,1}, \gamma_d^{1,1}\rangle = \exp(2\pi i A_2 \cdot \gamma_d^{1,1})|\gamma_b^{9,1}, \gamma_a^{9,1}, -\gamma_a^{1,1}, -\gamma_b^{1,1}, -\gamma_c^{1,1}, \gamma_d^{1,1}\rangle \quad (2.6)$$

where just as in the previous example the level matching forces us to choose  $A_2$  instead of  $A_3$  on the heterotic side. Note that  $\tilde{h}$  is conjugate to  $h$  given in (2.5) and so the resulting heterotic strings are isomorphic. The main difference is that a circle which is part of the cohomology of K3 in the previous case has been exchanged with a circle which is common to Type II and heterotic strings.

Though this orientifold construction has the advantage of satisfying the adiabatic argument, it has the disadvantage compared to the construction of [6] that we cannot determine using it whether there would be any quantum corrections, because as discussed before the dilaton on the type II side is now a linear combination of scalars in a hypermultiplet and a vector multiplet.

### 2.5. $N = 1$ Candidate

In this section, we will propose candidate  $N = 1$  dual pairs. The candidates are obtained by combining the construction of [6] with either orbifolding by  $(-1)^{FL}$  or orientifolding. We give first the former description. We can also choose the orientifold version of that model discussed above, giving a construction much like what follows.

We wish to consider a  $\mathbf{Z}_2 \times \mathbf{Z}_2$  orbifold generated by  $g$  (given in (2.2)) and  $h$  (given in (2.5)). Actually in order to have level matching in the  $gh$  sector for the heterotic string we need to modify the  $g$  action of (2.2) by

$$g \rightarrow g \cdot \exp(2\pi i (A_2^a \cdot \gamma_a^{1,1})) \quad (2.7)$$

with  $A_2$  as defined before. The first point of this formula is that (if one does not consider also  $h$ ) it is equivalent to  $g$  as defined in (2.2). The reason is that  $g$  in (2.2) acts as  $-1$  on  $\Gamma_a^{1,1}$ . A  $-1$  action together with a shift is equivalent to a  $-1$  action, so (2.7) is equivalent



to the earlier definition. But this modification of  $g$  (or an equivalent modification of  $h$ ) is needed to construct the  $\mathbf{Z}_2 \times \mathbf{Z}_2$  orbifold.

The second point is that with this modification of  $g$ , the  $gh$  twisted sector has the same structure as the  $h$  twisted sector with the role of  $\Gamma_a^{1,1}$  and  $\Gamma_b^{1,1}$  exchanged. Note that this modification does not alter the massless spectrum discussed before for that model. To count how many supersymmetries we are left with, note that the action of  $g$  and  $h$  on the right-moving  $T^6$  of heterotic string in a complex basis is of the form  $(-1, -1, 1)$  and  $(-1, 1, -1)$ , which thus gives us  $N = 1$  spacetime supersymmetry in four dimensions. It is easy to see that generically we get no massless states from twisted sectors. To count how many massless fields we have it suffices to consider modding out the action of  $h$  on the massless spectrum of  $(1, 4)$  model discussed above. One easily finds that no gauge fields will survive the projection. At weak coupling, the massless moduli that were previously given locally by the coset of  $SO(4, 20) \times SL(2)^3$  are now given by the coset of  $SO(2, 10) \times SO(2, 10) \times SL(2)^3$ . The factors are respectively the complex structure and Kahler moduli spaces of the Calabi-Yau manifold studied in [6], and the  $S$ ,  $T$ , and  $U$  moduli spaces. In particular, there are  $10 + 10 + 3 = 23$  massless  $N = 1$  chiral multiplets. We will determine later the details of the weak coupling identifications of the moduli spaces on the two sides.

For this to make sense, one would hope that the model in question does not generate a spacetime superpotential and has unbroken supersymmetry on the whole classical moduli space just discussed. A preliminary point in favor of this is that as the gauge group is generically trivial, one does not have gluino condensation generating a superpotential at the field theory level.

For a further test of a similar nature, we should consider the extra massless states that arise at special points on the moduli space. If, for instance, at a special point on the moduli space one would find an  $SU(2)$  gauge theory with a pair of doublet chiral superfields – a model that in field theory generates a nonperturbative superpotential with a pole at the point where naively the  $SU(2)$  is unbroken [21] – then one could very plausibly (though not rigorously) expect the string theory to generate a nonperturbative superpotential with such a pole. However, this does not occur; on the contrary, in the model considered here, the massless spectrum at points of extended gauge symmetry always corresponds to a field theory that is not asymptotically free.

Extra massless states appear when the Narain lattice corresponds in the untwisted heterotic string to a point of enhanced gauge symmetry; modding out by  $g$  and  $h$  then keeps some of the extra massless particles. Let us examine the enhanced gauge symmetry

points considered in [6] and see what happens to them after we project by  $g$ . Two cases were considered in [6]: Level 2 ADE type groups coming from projecting to the invariant subsector of the  $h$  action (which exchanges  $\Gamma_a^{9,1} \leftrightarrow \Gamma_b^{9,1}$ ) with an adjoint hypermultiplet, or  $SU(2)$  with 4 hypermultiplet doublets coming from the twisted sectors. These representations lead to finite (conformally invariant)  $N = 2$  field theories at low energies in four dimensions, and have the rather special further property that in those low energy field theories, there are no quantum corrections to the classical metric. Projecting these enhanced gauge groups by  $h$  to obtain an  $N = 1$  model gives gauge groups with matter in a representation large enough to spoil asymptotic freedom. For example for the points with a level two  $SU(n)$  symmetry in the  $N = 2$  model, after  $g$  projection we get an  $SO(n)$  gauge theory with matter in one copy of the adjoint representation of  $SO(n)$  and two in the symmetric traceless tensor product of two fundamental representations. For an  $SO(2n)$  point of the  $N = 2$  theory, after  $g$  projection we get  $SO(n) \times SO(n)$  gauge group, with one chiral superfield in the adjoint representation and two in the  $(n, n)$  representation. For the point that gives in the  $N = 2$  theory an  $SU(2)$  theory with four pairs of doublets, the gauge symmetry is reduced to  $U(1)$  by the  $g$  projection, and some charged matter survives. In all these cases the spectrum we get is not asymptotically free. This means that for weak coupling the dynamics of the gauge sector at these special points is trivial. These results are compatible with the hypothesis that the theory does not generate a superpotential.

If this is the case, one may ask how a low energy observer in this theory interprets the absence of a superpotential. The reason that this may come as a surprise is that in renormalizable field theory in four dimensions, nonperturbative superpotentials are usually generated unless this is prevented by continuous global symmetries. String theory does not have continuous global symmetries, and discrete global symmetries could generally not forbid a superpotential.

We will now explain how under suitable conditions, a low energy observer who can probe the moduli space  $M$  of vacua of the theory and is familiar with all the dualities could understand the absence of a string-generated superpotential in a theory such as this one. We consider for simplicity in exposition  $S$  and  $T$  only and suppose first that the moduli space is  $M = \mathbf{P}_S \times \mathbf{P}_T$ , where  $\mathbf{P}_S$  and  $\mathbf{P}_T$  are  $S$  and  $T$  moduli spaces, compact except for weak coupling and large volume limits. If the superpotential is non-zero, it must have a pole somewhere on  $M$ .<sup>5</sup> Since the curves  $S = \infty$  and  $T = \infty$  (weak coupling and large

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<sup>5</sup> Thus, in attempts to guess superpotentials invariant under some form of  $S$ -duality, poles were required [22].

volume) generate the Picard group of  $M$ , the pole must intersect either  $S = \infty$  or  $T = \infty$ . Since  $S = \infty$  and  $T = \infty$  are both weak coupling in one theory or the other, the pole would have to result from a physical phenomenon (such as unbroken  $SU(2)$  with two doublets) that is visible in one description or the other. In a model (such as the one that we have discussed) with no such phenomenon, there would be no pole and no superpotential.

### *Critical Look*

Let us look at that argument more critically. The assertion that the moduli space is a product  $\mathbf{P}_S \times \mathbf{P}_T$  has three characteristics: (a) it may not be true even in weak coupling; (b) it is a stronger statement than we need; (c) for the statement to be useful, there should be a practical way of verifying it. We consider the three points in turn.

For (a), we parametrize the weakly coupled theory by the complex variable  $q_S = e^{-S}$ . In string perturbation theory,  $|q_S|$  is visible as the microscopic string coupling constant and is naturally well-defined independent of  $T$  and other moduli. That is not so for the argument of  $q_S$ , whose zero mode is the axion, which is hard to see in string perturbation theory. Around a divisor in  $\mathbf{P}_T$  along which charged fields become massless, the axion jumps by an integer multiple of  $2\pi$ . Geometrically this means that even near  $q_S = 0$ , the moduli space is not a product  $\mathbf{P}_S \times \mathbf{P}_T$  but a complex line bundle over  $\mathbf{P}_T$ . This will commonly occur on the heterotic side at enhanced gauge symmetry points. Conversely, near the zero locus of  $q_T = e^{-T}$ , which can be detected in weak coupling of Type II, one may get a fiber bundle or product structure because of either conifold singularities or monodromy near  $S = \infty$  where the Type II volume goes to infinity and the four-dimensional description breaks down.

Since these phenomena will commonly occur, it is fortunate that as asserted in (b), something much weaker than a product structure will suffice for our analysis. It suffices, in fact, to have a *global* fiber bundle structure over either  $\mathbf{P}_S$  or  $\mathbf{P}_T$ , together with compactness of the total moduli space (the compactness modulo weak coupling and large volume limits by now appears to be a general property of string theory). For such a global fiber bundle, the fiber and base generate the Picard group (one can argue this by using an adiabatic argument or spectral sequence to compute the cohomology of the total space), so that any pole can be seen at weak coupling on one side or the other. This type of argument still works when additional moduli such as  $U$  are included, and it is good enough if the actual moduli space is only birational to a global fiber bundle.

What about (c), the problem of justifying such a *global* fiber bundle structure? We will give an example of a simple criterion that is likely to be useful. Suppose that in weak coupling to first order for  $q_T$  near zero, one has a product structure  $\mathbf{P}_S \times \mathbf{P}_T$ , and that moreover  $H^1(\mathbf{P}_S, \mathcal{O}) = H^1(\mathbf{P}_S, T\mathbf{P}_S) = 0$  ( $\mathcal{O}$  and  $T\mathbf{P}_S$  are the trivial bundle and tangent bundle over  $\mathbf{P}_S$ ). Then the desired structure follows.<sup>6</sup> Some variants of this argument go through, under certain conditions, when the structure near  $q_T = 0$  is a fiber bundle rather than a product.

To see that the criterion just stated is likely to be useful, note that at the end of this section, we will determine  $\mathbf{P}_S$ , for a model such as this one which is constructed on the Type II side by dividing  $\mathbf{T}^2$  by a null vector of order two, to be the quotient of the upper half plane by  $\Gamma_0(2)$ . As this quotient is a copy of  $\mathbf{CP}^1$ , the relevant cohomology groups vanish. So our criterion applies and the space-time superpotential vanishes if the local structure near  $\mathbf{P}_S$  is a product, or a line bundle of negative curvature. This appears plausible, the only issue, which we will not address here, being a possible monodromy of  $q_T$  near  $S = \infty$ .

### *Orientifold Description*

Because of the close relation that we have found between  $(-1)^{FL}$  and orientifold symmetries, we can also give an orientifold description of the same model. We recall the  $N = 2$  orientifold presented at the end of the last subsection. We started with the Type IIA theory compactified on  $\mathbf{R}^4 \times \mathbf{S}^1 \times \mathbf{S}^1 \times \mathbf{K3}$ , and considered the  $\mathbf{Z}_2$  generated by an operation  $Y$  that acts by  $\pi$  rotation of the first circle, multiplication of the second circle by  $-1$ , and reversal of world-sheet orientation. Upon dividing by  $\mathbf{Z}_2 \times \mathbf{Z}_2$ , where the second  $\mathbf{Z}_2$  is the one used in [6] and described in (2.5), we get an orientifold with  $N = 1$  space-time supersymmetry. By simply translating the description of the various symmetries to the heterotic string side, we can describe, as above, a candidate for a heterotic string dual of this  $N = 1$  model. In fact, because of the close relation between  $(-1)^{FL}$  and orientifolding, the  $N = 1$  models obtained this way on the heterotic string side are equivalent – whether the duality is valid or not.

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<sup>6</sup> One proves this by comparing the complex structure of the moduli space to that of  $\mathbf{P}_S \times \mathbf{C}$  ( $\mathbf{C}$  being the  $q_T$  plane), order by order in  $q_T$ , near  $q_T = 0$ . They must coincide order by order if  $\mathbf{P}_S \times \mathbf{C}$  is rigid, that is if its cohomology with values in the tangent bundle vanishes. This is so if  $H^1(\mathbf{P}_S, \mathcal{O}) = H^1(\mathbf{P}_S, T\mathbf{P}_S) = 0$ . Once the equivalence is known to all finite orders, it follows as an exact statement (up to a birational transformation and finite cover) by general facts about compact algebraic manifolds.

## 2.6. Asymmetric $N = 4$ Models

We want to determine the weak coupling dualities of the model above, but first we will describe another interesting model – an  $N = 4$  model studied by Chaudhuri, Hockney, and Lykken [10] – whose dualities can be studied similarly.

According to Chaudhuri and Polchinski [11], one of the models in [10] in nine dimensions can be constructed as follows. After compactifying on  $\mathbf{R}^9 \times \mathbf{S}^1$ , one takes the orbifold by a certain operation  $f$  that consists of a half-lattice shift of  $\mathbf{S}^1$  together with an exchange of the two  $E_8$  lattices. To find a Type II dual of this model after reduction to five dimensions we simply proceed as follows.<sup>7</sup> We replace  $\mathbf{R}^9$  by  $\mathbf{R}^5 \times \mathbf{T}^4$ , and begin on  $\mathbf{R}^5 \times \mathbf{S}^1 \times \mathbf{T}^4$ . Then we apply string-string duality, considering instead a Type IIA theory on  $\mathbf{R}^5 \times \mathbf{S}^1 \times \text{K3}$ . Now we want to find the analog on the Type IIA side of dividing by  $f$ . Since  $f$  acts freely on  $\mathbf{S}^1$ , the adiabatic argument applies and if we can identify the operation  $f'$  that corresponds to  $f$ , the heterotic string orbifold of  $\mathbf{R}^5 \times \mathbf{S}^1 \times \mathbf{T}^4$  by  $f$  should be equivalent to the Type IIA orbifold of  $\mathbf{R}^5 \times \mathbf{S}^1 \times \text{K3}$  by  $f'$ .

What is  $f'$ ? Obviously,  $f'$  acts by a  $\pi$  rotation of  $\mathbf{S}^1$ , but how does it act on K3? It must act in such a way as to exchange the two  $E_8$  lattices in the cohomology of K3 while fixing the rest.

To find an explicit K3 automorphism that does that, it is helpful to first understand explicitly what kind of K3 can exhibit the  $E_8 \times E_8$  gauge symmetry that is familiar in the heterotic string. For this, according to section 4.6 of [3] the K3 must have two  $E_8$  singularities. We recall that an  $E_8$  singularity is the singularity described by the equation

$$w^2 + x^3 + y^5 = 0 \tag{2.8}$$

in  $\mathbf{C}^3$ . To find a K3 with two  $E_8$  singularities, we simply note that a K3 can be described by the equation  $w^2 = P_6(x, y, z)$  where  $P_6$  is a generic homogeneous sixth order polynomial ( $x, y, z$ , and  $w$  are homogeneous coordinates of weights 1, 1, 1, 3 in a weighted projective space). If we pick a particular  $P_6$  giving the equation

$$w^2 + (x - z)^3(x + z)^3 + zy^5 = 0, \tag{2.9}$$

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<sup>7</sup> This has also been analyzed by P. Aspinwall [23]. The model actually has a more challenging Type II dual in six dimensions [24].

we get a K3 that has two  $E_8$  singularities – at  $(x, y, z, w) = (\pm 1, 0, 1, 0)$  – and is otherwise non-singular. So the Type IIA theory compactified on this K3 can have  $E_8 \times E_8$  gauge symmetry.

Now consider the automorphism  $h(x, y, z, w) = (-x, y, z, -w)$  of this K3. This transformation exchanges the two  $E_8$  singularities, but preserves the orientation, complex structure, and holomorphic two-form, and can be seen to leave invariant the polynomial deformations that keep two  $E_8$  singularities. So it exchanges the two  $E_8$  lattices and leaves fixed the rest of the cohomology. The operation  $f'$  that we want is therefore simply  $h$  combined with a  $\pi$  rotation of the circle.

It was noted by Chaudhuri and Polchinski that this model seems to exhibit  $S$ -duality when further compactified to four dimensions. Now we can see why that must be so. The four-dimensional model is an orbifold of  $\mathbf{R}^4 \times \mathbf{S}^1 \times S^1 \times \mathbf{T}^4$ ; its Type II dual is an orbifold of  $\mathbf{R}^4 \times \mathbf{S}^1 \times \mathbf{S}^1 \times \text{K3}$ . The Type II model has a  $T$ -duality acting on  $\mathbf{S}^1 \times \mathbf{S}^1$ , and by a standard argument, this implies  $S$ -duality for the heterotic version of the theory (and vice-versa). The duality groups are proper subgroups of  $SL(2, \mathbf{Z})$  that will be analyzed presently.

## 2.7. Weak Coupling Dualities

If one compactifies ten-dimensional string theory to four dimensions on  $\mathbf{T}^2 \times X$ , with  $X$  any four-manifold, one gets a  $T$ -duality group  $SL(2, \mathbf{Z})_T \times SL(2, \mathbf{Z})_U$ ; string-string duality exchanges  $S$  and  $T$ , adding a third copy of  $SL(2, \mathbf{Z})$ . We want to consider the case in which  $\mathbf{T}^2$  has been divided by a  $\mathbf{Z}_2$  shift of the Narain lattice, perhaps acting also on other degrees of freedom and determine the weak coupling duality group. We first make a general discussion and then apply it to several models constructed above.

First we describe the Narain lattice in a way that exhibits the  $SL(2) \times SL(2)$  symmetry. We consider  $2 \times 2$  integer matrices  $M^{ii'}$ ,  $i, i' = 1, 2$ , with inner product  $(M, N) = \epsilon_{ij}\epsilon_{i'j'} M^{ii'} M^{jj'}$  (so for instance  $(M, M) = 2 \det M$ ).  $SL(2) \times SL(2)$  acts by  $M \rightarrow AMB^{-1}$  with  $(A, B) \in SL(2) \times SL(2)$ . There are  $2^4 - 1 = 15$  non-zero half-lattice vectors, modulo lattice shifts. Write such a vector as  $V = M/2$ , with  $M$  a lattice vector. One can think of  $M$  as a non-zero matrix whose entries are all 0 or 1. Nine such matrices have determinant zero, giving  $V^2 = 0$ . It is easy to check that such matrices form an irreducible orbit of  $SL(2) \times SL(2)$ . A representative such matrix is

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{2.10}$$

and is obviously stabilized by a subgroup conjugate to  $\Gamma_0(2) \times \Gamma_0(2)$  ( $\Gamma_0(2)$  is the index three subgroup of  $SL(2, \mathbf{Z})$  consisting of matrices that stabilize the vector  $(1, 0)$  modulo two). As  $\Gamma_0(2) \times \Gamma_0(2)$  is of index nine in  $SL(2, \mathbf{Z}) \times SL(2, \mathbf{Z})$ , it is the full stabilizer of the half-lattice vector determined by (2.10), and hence is the weak coupling  $T - U$  duality group of a theory constructed by orbifolding by such a shift.

The other case is a matrix of determinant  $\pm 1$ , such as  $M = 1$ . It is again easy to check that the six such matrices are permuted by  $SL(2, \mathbf{Z}) \times SL(2, \mathbf{Z})$ . The stabilizer of  $M = 1$  consists of pairs  $(A, B)$  with  $AMB^{-1} = M$  modulo two, that is,  $A = B$  modulo two. Thus, the  $T - U$  duality group of a model constructed by orbifolding by such a shift is the subgroup  $H$  of  $SL(2, \mathbf{Z}) \times SL(2, \mathbf{Z})$  consisting of pairs  $(A, B)$  with  $A = B$  modulo two. This subgroup is of index six, as expected.

Now we can determine the duality group of the model of the CHL model compactified to four dimensions. This model is constructed on both heterotic and Type II sides by dividing by a null shift vector, so the  $S$ ,  $T$ , and  $U$  duality groups are all  $\Gamma_0(2)$ . This shows, in particular, that just because a model has  $N = 4$  supersymmetry in four dimensions does not mean that the  $S$ -duality group will turn out to be all of  $SL(2, \mathbf{Z})$ . Similarly, many different models with the same low energy physics are likely to have different duality groups at the string level. For instance,  $N = 5$  and  $N = 6$  supersymmetry, which can be constructed in many ways using asymmetric Type II orbifolds, give low energy duality groups with no canonical integral structure, so that different string theory realizations very likely have different duality groups.

Finally, let us consider the  $(1, 4)$  model with  $N = 2$  space-time supersymmetry that was constructed above in several ways. On the heterotic string side, this is constructed with a shift of non-zero length squared, so the heterotic string weak coupling  $T - U$  duality group is the group  $H$  described above. On the Type II side, the shift is by a null vector, so (remembering the exchange of  $T$  and  $S$ ), the  $S - U$  duality group at Type II weak coupling is  $\Gamma_0(2) \times \Gamma_0(2)$ . Note that the two determinations of the  $U$ -duality group appear to disagree. We interpret this as further evidence that for  $N = 2$  there are corrections to the whole vector moduli space. For  $N = 4$  such correction are not expected.

### 3. K3 Fibrations And String-String Duality

In [5], various examples were constructed of apparent dual pairs with  $N = 2$  supersymmetry in four dimensions. The examples involved, on the heterotic string side,

compactification more or less on  $\mathbf{T}^2 \times \text{K3}$  with some vector bundle (in some cases one uses an enhanced gauge symmetry that comes upon taking the  $\mathbf{T}^2$  at a special radius), while on the Type II side they involved compactification on certain Calabi-Yau manifolds. It has been noted [7] that the Calabi-Yau manifolds that arose in these examples have the structure of K3 fibrations, that is, fiber bundles over  $\mathbf{CP}^1$  with K3 fibers.

The question arises of whether the structure of K3 fibrations can actually be used to explain the examples of [5], via the adiabatic argument given in the last section. Going to a region in parameter space in which the  $\mathbf{CP}^1$  has a very large area, while the volume of the K3 fiber is of order one, one has a Calabi-Yau manifold  $X$  which is a family of slowly varying K3's. In the limit that the family varies very slowly, it should be possible to apply string-string duality fiber-wise, replacing the Type II description by a heterotic string description and replacing the K3 fibers by  $\mathbf{T}^4$  fibers. Thus the heterotic string description should involve a family of  $\mathbf{T}^4$ 's fibered over  $\mathbf{CP}^1$ . The  $\mathbf{T}^4$ 's, along with the abelian gauge fields that arise at a generic point in moduli space, are really described by a Narain lattice  $\Gamma^{20,4}$ ; the variation of this Narain lattice over  $\mathbf{CP}^1$  should be determined by the variation of the original family of K3's.

To test this interpretation in a preliminary way, note that the  $\mathbf{T}^2 \times \text{K3}$  used in [5] on the heterotic string side can very well arise as a family of  $\mathbf{T}^4$ 's fibered over  $\mathbf{CP}^1$ . In fact, K3 can be realized as a family of  $\mathbf{T}^2$ 's fibered over  $\mathbf{CP}^1$  (giving an “elliptic surface”—see [25] for the construction of a Ricci-flat metric on this fibered space), so  $\mathbf{T}^2 \times \text{K3}$  can certainly arise as a family of  $\mathbf{T}^4$ 's fibered over K3.

The main obstruction to this attempt at explaining the results of [5] is that there are certain singularities in the K3 fibers, so that the adiabatic approximation is not everywhere valid. In this section, we will examine the K3 fibrations that appear in these particular examples, and show that they have a particularly simple structure which further simplifies in the weak coupling limit (of the heterotic string). The singularities at which the adiabatic argument fails are of a very special nature. Nonetheless, not understanding them, we will not be able to explain the results of [5]. We will, however, be able to deduce from the structure of K3 fibrations a simple “orientifolding” operation which – if the adiabatic argument is valid – should give for every  $N = 2$  dual pair of this type an  $N = 1$  dual pair (or in general several of them).



### 3.1. The K3 Fibrations

String-string duality naturally maps the heterotic string to a Type IIA string. If one then uses mirror symmetry to convert to a Type IIB description, then the vector moduli space of the heterotic string is mapped to the complex structure moduli space of a Type IIB model, which has no world-sheet or space-time corrections and can be conveniently studied classically.

We will study two Type II examples from [5] for which the evidence for a heterotic string description was particularly strong. The first example, which we will call  $X$ , is defined by a hypersurface of degree twelve in a weighted projective space with weights 1, 1, 2, 2, 6:

$$z_1^{12} + z_2^{12} + z_3^6 + z_4^6 + z_5^2 - 12\psi z_1 z_2 z_3 z_4 z_5 - 2\phi z_1^6 z_2^6 = 0. \quad (3.1)$$

We recall that a weighted projective space is defined by identifying the variables under

$$z_i \rightarrow t^{d_i} z_i \quad (3.2)$$

with  $d_i$  the weights and  $t \in \mathbf{C}^*$ . The second example, which we will call  $Y$ , is defined by a hypersurface of degree 24 in a weighted projective space with weights 1, 1, 2, 8, 12:

$$z_1^{24} + z_2^{24} + z_3^{12} + z_4^3 + z_5^2 - 12\alpha z_1 z_2 z_3 z_4 z_5 - 2\beta z_1^6 z_2^6 z_3^6 - \gamma z_1^{12} z_2^{12} = 0. \quad (3.3)$$

Actually, in each case (for Type IIB) one is really interested in certain orbifolds of these spaces, dividing by the group of diagonal phase rotations of the variables that preserve the hypersurfaces and the holomorphic three-forms. The polynomials written are the most general ones of the correct degrees compatible with that symmetry (and these particular models have no non-polynomial deformations).

To see the structure of K3 fibration, we follow the first few pages of [26]. We consider the submanifold  $H_\lambda$  of  $X$  defined by  $z_1 = \lambda z_2$  with some fixed complex  $\lambda$ . Once  $z_1$  is eliminated in this way,  $z_2$  is the only remaining variable of odd weight, and therefore if we set  $t = -1$  in (3.2), we get the transformation  $z_2 \rightarrow -z_2$ , with other variables invariant. We can divide out this equivalence by setting  $y = z_2^2$ , and after doing this, the equation for  $H_\lambda$  becomes

$$(1 + \lambda^{12} - 2\phi\lambda^6)y^6 + z_3^6 + z_4^6 + z_5^2 - 12\lambda\psi y z_3 z_4 z_5 = 0. \quad (3.4)$$

We recognize this as an equation describing a K3 manifold in a weighted  $\mathbf{CP}^3$ . So the  $H_\lambda$ 's are copies of K3, and  $X$  can be regarded as a family of K3's parametrized by  $\lambda$ , that is,

$X$  is fibered over  $\mathbf{CP}^1$  (the Riemann sphere built from the  $\lambda$  plane) with the fibers being K3's. Similarly, upon setting  $z_1 = \lambda z_2$  and  $y = z_2^2$ , (3.3) becomes

$$(1 + \lambda^{24} - \gamma\lambda^{12})y^{12} + z_3^{12} + z_4^3 + z_5^2 - 12\alpha\lambda y z_3 z_4 z_5 - 2\beta\lambda^6 y^6 z_3^6 = 0 \quad (3.5)$$

which again defines a K3 manifold, so we again get a family of K3's fibered over  $\mathbf{CP}^1$ .

Since our plan is to carry out string-string duality fiber-wise, we need to understand the monodromy of these families of K3's. Since  $\mathbf{CP}^1$  is simply-connected, such monodromy is only possible because at some values of  $\lambda$  the K3's develop singularities; the singularities are the reason that fiber-wise application of string-string duality may have difficulties. We have to determine the monodromy representation and translate it to the heterotic string side. One might worry that we will meet very complicated  $SO(20, 4; \mathbf{Z})$ -valued monodromies, but this is not so. The monodromy representations for the above families of K3's are as simple as one could hope for.

To see this, set  $\psi = 0$  in (3.4) (or  $\alpha = \beta = 0$  in (3.5)). Then one finds that, for generic  $\lambda$ , the K3's in (3.4) and (3.5) have a structure that is *independent of*  $\lambda$ , since  $\lambda$  can be eliminated by rescaling  $y$ . This fails (and the K3 is singular) precisely at zeroes of the function  $F(\lambda) = 1 + \lambda^{12} - 2\phi\lambda^6$  (or  $G(\lambda) = 1 + \lambda^{24} - \gamma\lambda^{12}$  in the second example). The monodromy of the K3 around zeroes of  $F$  or  $G$  is very simple. Near a value of  $\lambda$  at which  $F$  or  $G$  has a zero of order  $n$ , the monodromy is given by the orbifold operation  $y \rightarrow \zeta^n y$ , where  $\zeta = e^{2\pi i/6}$  in the first example, and  $\zeta = e^{2\pi i/12}$  in the second example.

Let us compute how  $S : y \rightarrow \zeta y$  acts on the cohomology of these K3's. Obviously,  $H^{0,0}$  and  $H^{2,2}$  are invariant.  $H^{2,0}$  is generated by the holomorphic two-form  $\omega = (y dz_3 dz_4 + \dots)/(\partial P/\partial z_5)$ ,<sup>8</sup> so  $S$  acts as  $\zeta$  on  $H^{2,0}$  and as  $\zeta^{-1}$  on  $H^{0,2}$ . Now consider the action on  $H^{1,1}$ . The Kahler form is obviously invariant under  $S$ . The action of  $S$  on the rest of  $H^{1,1}$  can be conveniently found by considering  $H^1(T)$ , the space of deformations of complex structure. Given a deformation of complex structure represented by a  $T$ -valued  $(0, 1)$  form  $\alpha_{\bar{j}}^i$ , one maps to a  $(1, 1)$  form by multiplying by  $\omega$ :  $\beta_{i\bar{i}} = \omega_{ij} \alpha_{\bar{j}}^i$ . Therefore the transformation law of  $\beta$  has an extra  $\zeta$  relative to that of  $\alpha$ .

Since  $H^{1,1}$  is twenty-dimensional for a K3 surface, and we already know about the Kahler form, we are looking for 19 more modes. For the first example, these come from

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<sup>8</sup> In this and similar formulas below, the  $\dots$  refers to a addition of terms obtained by cyclic permutation of the variables; in the particular case here,  $y dz_3 dz_4 + \dots = y dz_3 dz_4 + z_3 dz_4 dy + z_4 dy dz_3$ .  $P$  is the defining polynomial of the hypersurface.

19 polynomial deformations in  $H^1(T)$ : modulo the derivatives of the defining equation of the hypersurface, there are three polynomials of the form  $y^4 P_2(z_3, z_4)$ , four of the form  $y^3 P_3(z_3, z_4)$ , five of the form  $y^2 P_4(z_3, z_4)$ , four of the form  $y P_5(z_3, z_4)$ , and three of the form  $P_6(z_3, z_4)$ . ( $P_k$  is a homogeneous polynomial of degree given by the subscript.) Since  $y^n P_{6-n}$  transforms under  $S$  as  $\zeta^n$ , the corresponding  $(1, 1)$  form transforms as  $\zeta^{n+1}$ . Hence for these 19 modes,  $\zeta$  and  $\zeta^5$  appear three times,  $\zeta^2$  and  $\zeta^4$  appear four times, and  $\zeta^5$  appears five times.

Now consider the second example. In this case there are only 18 polynomial deformations, corresponding to the monomials  $y^n z_3^{12-n}$ ,  $2 \leq n \leq 10$ , and  $y^n z_3^{8-n} z_4$ ,  $0 \leq n \leq 8$ .  $S$  acts on the corresponding  $(1, 1)$  forms as  $\zeta^n$  with  $3 \leq n \leq 11$  for the first series and  $1 \leq n \leq 9$  for the second series. One more mode is needed; where is it? In fact, the K3 defined in (3.5) has an  $A_1$  singularity at  $y = z_3 = 0$ . There is one collapsed two-cycle sitting in this singularity, and it is invariant under  $S$ .

We would now like to see what the heterotic string dual of this model should look like, assuming that it can be constructed by applying string-string duality fiber-wise. Fiber-wise application of string-string duality will give a family of  $\mathbf{T}^4$ 's parametrized by  $\mathbf{CP}^1$ ; the  $\mathbf{T}^4$ 's, with their Wilson lines, are described by a family of Narain lattices. Right-movers and left-movers of the heterotic string come respectively from the self-dual and anti-self-dual part of the cohomology of K3. For the simplest statement, we look at the invariant part of the Narain lattice, which comes from the monodromy-invariant part of the cohomology of K3.

We consider first the second example  $Y$ . The monodromy-invariant part of the cohomology is four-dimensional, spanned by  $H^{0,0}$ ,  $H^{2,2}$ , the Kahler class and the class associated with the  $A_1$  singularity. The signature of this four-dimensional invariant subspace is  $(2, 2)$ : the self-dual part is the Kahler class and a linear combination of  $H^{0,0}$  and  $H^{2,2}$ . Modulo questions of torsion, this suggests that the Narain lattice has an invariant sublattice  $\Gamma^{2,2}$ . This would correspond to a very special family of  $\mathbf{T}^4$ 's over  $\mathbf{CP}^1$ , consisting of the sum of a *fixed*  $\mathbf{T}^2$ , without Wilson lines, represented by the fixed  $\Gamma^{2,2}$ , and a variable  $\mathbf{T}^2$ , with Wilson lines, represented by a variable  $\Gamma^{18,2}$ . This is exactly the structure proposed for this example in [5]: the heterotic string dual of the second example was proposed to be a product  $\text{K3} \times \mathbf{T}^2$ , with the gauge bundle living entirely on the K3. The K3 can take the form of a  $\mathbf{T}^2$  bundle over  $\mathbf{CP}^1$ , so this  $\text{K3} \times \mathbf{T}^2$  has the structure of a  $\mathbf{T}^2 \times \mathbf{T}^2$  bundle over  $\mathbf{CP}^1$  in which only the first factor varies.

Now let us consider the first example,  $X$ . In this case, the invariant part of the cohomology is only three-dimensional; we can only split off a lattice of signature  $(1, 2)$ , with one left-moving and two right-moving moduli. This again has a natural interpretation in relation to [5], according to which  $X$  corresponds again to a heterotic string on  $K3 \times \mathbf{T}^2$ , but this time the  $\mathbf{T}^2$  is fixed at a point in moduli space at which one of the left-moving  $U(1)$ 's is extended to  $SU(2)$ , and that  $SU(2)$  is then broken in the construction of the gauge bundle. So the  $\mathbf{T}^2$  carries only one free left-moving mode and two free right-moving modes, in agreement with what we found for the monodromy-invariant part of the cohomology of  $X$ .

Our discussion has helped to give a more vivid picture of how the heterotic string description of [5] is related to the classical geometry of  $X$  or  $Y$  via string-string duality. Our discussion, however, can be interpreted purely in classical terms as a computation of the cohomology of  $X$  or  $Y$  in terms of the monodromy action on the cohomology of the fiber, and the cohomology of  $X$  and  $Y$  were already matched in [5] with the moduli of the heterotic string vacua. So it is not clear that the agreement we found between the two sides is really independent evidence for string-string duality.

### 3.2. The Metric, Level Matching, and Weak Coupling

Next let us figure out what the metric of  $X$  and  $Y$  look like in the adiabatic limit in which the area of the  $\lambda$  plane is scaled up, keeping fixed the restriction of the Kahler class to the fiber. The discussion and result are quite similar to the discussion of stringy cosmic strings in [25].

In the case of  $X$ , for instance (the other example is quite similar for the present purposes), let us consider the structure near an isolated simple zero of the function  $F(\lambda)$ , which we take to be at  $\lambda = 0$ . The equation for  $X$  looks near  $\lambda = 0$  like  $p = 0$  with

$$p = \lambda y^6 + z_3^6 + z_4^6 + z_5^2. \quad (3.6)$$

In the adiabatic limit, with the metric on the  $\lambda$  plane scaled up, the form (3.6) for the equation becomes valid throughout the  $\lambda$  plane. Recalling that the holomorphic three-form in the original variables was  $\omega = (z_1 dz_2 dz_3 dz_4 \pm \dots)/(\partial p/\partial z_5)$ , we see that in this description the holomorphic three-form looks like

$$\omega = d\lambda \cdot \frac{y dz_2 dz_3 \pm \dots}{\partial p/\partial z_5}. \quad (3.7)$$

One can eliminate  $\lambda$  from (3.6) by setting  $\tilde{y} = \lambda^{1/6}y$ , whereupon  $\omega$  can be written  $\omega = (d\lambda/\lambda^{1/6}) \cdot \Omega$ , where

$$\Omega = \frac{\tilde{y} dz_3 dz_4 + \dots}{\partial p / \partial z_5} \quad (3.8)$$

is a fixed two-form on a fixed K3.

If then one has a Ricci-flat Kahler metric on the total space of (3.6) which at large  $\lambda$  looks like the product of a metric on the  $\lambda$  plane with a fixed metric on K3, then the metric on the  $\lambda$  plane will be  $ds^2 = |d\lambda/\lambda^{1/6}|^2$ . This is a flat metric (away from the origin) with a deficit angle at infinity of  $2\pi/6$ . Since the polynomial  $F(\lambda)$  is of twelfth order, there are twelve of these singularities. With twelve singularities, the total deficit angle is  $4\pi$ . The interpretation is clear. According to the Gauss-Bonnet theorem, the integrated curvature of any metric on the  $\lambda$  plane is  $4\pi$ . In the adiabatic limit, the  $\lambda$  plane looks flat except for twelve delta functions, of strength  $2\pi/6$ , at zeroes of  $f$ ; each delta function arises at a conical singularity with a deficit angle of  $2\pi/6$ .

### *Orbifold and Level Matching*

It may appear that we can now test string-string duality in the following way. In the special case in which five zeroes of  $\lambda$  coincide (so  $\lambda$  in (3.6) is replaced by  $\lambda^5$ ), the deficit angle becomes  $2\pi \cdot 5/6$ , which is what we would get if we divide the  $\lambda$  plane by  $\lambda \rightarrow \zeta\lambda$ . So our Calabi-Yau becomes a  $\mathbf{Z}_6$  orbifold of  $\mathbf{C} \times \text{K3}$ , with  $\mathbf{C}$  the complex plane and  $\mathbf{Z}_6$  acting by  $\lambda \rightarrow \zeta\lambda$ ,  $y \rightarrow \zeta^5 y = \zeta^{-1}y$ . It would appear that this should correspond under string-string duality to a heterotic string orbifold of  $\mathbf{C} \times \mathbf{T}^2$  by  $\mathbf{Z}_6$ .

To see if this makes sense, we look at the action of  $\mathbf{Z}_6$  on the cohomology of K3, which will determine the behavior of the left-movers of the heterotic string. The left-movers with non-trivial monodromy come from modes of  $H^{1,1}$  derived from polynomials and from the  $\lambda$  plane. The eigenvalue  $\zeta^s$  arises for left-moving bosons with multiplicity four for  $s = 1, 2, 4, 5$  and five for  $s = 3$ . The left-moving ground state energy is thus shifted by  $\sum_i s_i(6 - s_i)/4 \cdot 6^2 = 149/144$ . This is not a multiple of  $1/6$ , so level matching does not hold. Moreover, the discrepancy cannot be corrected by combining the  $\mathbf{Z}_6$  shift with a shift by  $1/6$  of a Narain lattice vector, which adds to the discrepancy in level-matching a multiple of  $1/6^2$ .

So we get another example of the fact that orbifolding does not commute with string-string duality: the Type IIA orbifold of  $\mathbf{C} \times \text{K3}$  by  $\mathbf{Z}_6$  does not correspond to a heterotic string orbifold of  $\mathbf{C} \times \mathbf{T}^4$ . Moreover, it might appear that we face a near-paradox in

reconciling the duality proposed in [5] with what we have learned from the adiabatic argument.

### *Weak Coupling*

The paradox can be overcome if we note that heterotic string level matching is a weak coupling concept so that we must compare heterotic string level matching conditions to the monodromies that arise where the heterotic string is weakly coupled.

According to [5], the region of moduli space of  $X$  in which the heterotic string is weakly coupled is  $\phi \rightarrow \infty$ . In that limit, of the twelve zeroes of  $F$ , six are at the origin and six are at infinity. With suitable rescaling of  $\lambda$ ,  $F$  reduces in the limit to  $F(\lambda) = \lambda^6$ . The mapping from the Type IIA model to the heterotic string is such that the splittings among the groups of six zeroes are non-perturbatively small from the standpoint of the heterotic string. Thus, in comparing to heterotic string perturbation theory, we have simply two groups of six zeroes.

For a group of six zeroes, the monodromy disappears! Our problem with level matching disappears with it.

We can also now see something about what the metric looks like in the limit in which the heterotic string is weakly coupled. Since each zero of  $F$  produces a deficit angle of  $2\pi/6$ , when six of them coincide one gets a deficit angle of  $2\pi$ . What a deficit angle of  $2\pi$  means is that in the limit that the heterotic string is weakly coupled, the  $\mathbf{CP}^1$  degenerates to a semi-infinite cigar with the six zeroes of  $F$  coalescing at the tip of the cigar and the other end being an infinite flat cylinder. When the heterotic string coupling is very small but not zero, one has a very long but not infinite cigar with six zeroes of  $F$  at one end and six at the other.

Presumably, in perturbation theory of the heterotic string, one sees one end of the cigar or the other, but not both. World-sheet instantons of the Type II theory, in which the world-sheet wraps all the way around the cigar, would see both ends of the cigar. In the mapping between the Type II and heterotic string descriptions, these correspond to space-time instantons of the heterotic string. It would be quite interesting to construct for each such world-sheet instanton of type IIA, a spacetime instanton of the heterotic string, ideally in the form of a  $(0, 4)$  superconformal field theory.

### *A Note On The Example of [6]*

We have been discussing the examples of [5] as K3 fibrations. It is interesting to note that the example of [6] also has the structure of a K3 fibration over  $\mathbf{CP}^1$ . This example

was constructed by starting with  $K3 \times \mathbf{T}^2$  and acting by a  $\mathbf{Z}_2$  that acts on both factors. By forgetting the first factor,  $(K3 \times \mathbf{T}^2)/\mathbf{Z}_2$  maps to  $\mathbf{T}^2/\mathbf{Z}_2 = \mathbf{CP}^1$ , and the fibers are K3's. In this case, the monodromies of the Narain lattice come from the  $\mathbf{Z}_2$  action on K3, and in this particular case, there is no difficulty with level matching in the weakly coupled limit of the heterotic string.

### 3.3. $N = 1$ Orientifolds

One of the most interesting consequences of the structure of K3 fibrations of the examples considered in [5] is that this makes it possible to identify  $N = 1$  orientifolds for which a heterotic string dual can be found in a way that should be reliable.

First of all, given a Calabi-Yau compactification of the Type IIA superstring, to construct an orientifold we need to divide by an isometry that reverses the orientation. The most obvious orientation-reversing isometries of Calabi-Yau threefolds are symmetries that reverse the complex structure.

To begin with, note that  $\mathbf{CP}^1$  has an antiholomorphic symmetry that acts without fixed points. In terms of homogeneous coordinates  $(u, v)$ , the transformation is  $\tau(u, v) = (\bar{v}, -\bar{u})$ . (The quotient of  $\mathbf{CP}^1$  by this transformation is  $\mathbf{RP}^2$ .) Suppose that we find an antiholomorphic involution  $w$  of one of the above examples that preserves the structure of K3 fibration and acts as  $\tau$  on the base. Then – upon taking the area of the base to be very large – the adiabatic argument should apply and it should be possible to find a heterotic string dual by fiber-wise application of string-string duality.

Actually, all of these models have antiholomorphic involutions with the right properties. For example, consider the  $X$  manifold, defined by the equation

$$z_1^{12} + z_2^{12} + z_3^6 + z_4^6 + z_5^2 - 12\psi z_1 z_2 z_3 z_4 z_5 - 2\phi z_1^6 z_2^6 + \dots = 0. \quad (3.9)$$

For constructing an orientifold and applying string-string duality, it is most natural to consider the Type IIA rather than Type IIB superstring, so we are interested in the  $X$  manifold rather than an orbifold of it, and many terms can be added to (3.9). This manifold has the antiholomorphic symmetry

$$w(z_1, z_2, z_3, z_4, z_5) = (\bar{z}_2, -\bar{z}_1, \bar{z}_3, \bar{z}_4, \bar{z}_5) \quad (3.10)$$

provided that  $\phi$  is real and  $\psi$  imaginary (and similar conditions are put on the other complex parameters). This acts as  $\tau$  on the base since it reduces to  $\tau$  if one forgets  $z_3, z_4,$

and  $z_5$ . There also are some variants of this; one can exchange  $z_3$  and  $z_4$ , with or without a change of sign.

Let us determine the massless spectrum of the orientifold obtained by dividing by  $w$ . An important question is how  $w$  acts on  $H^{1,1}$ . In this particular example,  $w$  acts as  $-1$  on  $H^{1,1}$ .<sup>9</sup> Now, each mode  $\beta$  in  $H^{1,1}$  determines in the Type IIA theory a vector multiplet consisting of a Kahler mode, a mode of the  $B$  field, and an abelian gauge field. The Kahler mode is made by multiplying  $\beta$  by the complex structure, which is odd under  $w$ , so the Kahler mode is even and survives in the orientifold. The  $B$  field mode is odd under  $w$ , but also odd under exchanging left and right-movers on the world-sheet, so it also survives in the orientifold. As for the gauge field in the vector multiplet, it is projected out in forming the orientifold. So  $N = 2$  vector multiplets associated with modes in  $H^{1,1}$  odd under  $w$  are truncated to  $N = 1$  chiral multiplets in the orientifold.

In a more general model of this type,  $H^{1,1}$  might contain a subspace even under  $w$ . From the corresponding scalar multiplets, the Kahler deformation and  $B$  field would be projected out, but the gauge field would survive. So  $N = 2$  vector multiplets associated with modes in  $H^{1,1}$  even under  $w$  are truncated to  $N = 1$  vector multiplets in the orientifold.

A hypermultiplet, on the other hand, contains four states, two from the NS-NS sector and two from the Ramond-Ramond sector. In the orientifold, one NS-NS state and one Ramond-Ramond state survive, making an  $N = 1$  chiral multiplet. For example, the fact that (3.9) is  $w$ -invariant only for  $\phi$  real and  $\psi$  imaginary means that of the complex scalars NS-NS scalars associated with  $\phi$  and  $\psi$ , only one real component survives in the orientifold. Its  $N = 1$  chiral partner survives in the orientifold.

### 3.4. Concrete Description On Heterotic String Side

Now we will describe in a specific case what the heterotic string equivalent to one of these orientifolds looks like. We consider on the Type II side the  $Y$  manifold. We want to translate the orientifold symmetry of the  $Y$  manifold to the heterotic string side and then divide by it.

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<sup>9</sup>  $w$  acts as  $+1$  on  $H^{2,2}$  and therefore, by Poincaré duality (given that  $w$  reverses the orientation) it acts as  $-1$  on  $H^{1,1}$ . To see that  $w$  acts as  $+1$  on  $H^{2,2}$ , note that  $H^{2,2}$  is generated according to [26] by the divisor  $z_1 = 0$  (which can be deformed into its image under  $w$ ) and the  $w$ -invariant divisor obtained by blowing up the curve  $z_1 = z_2 = 0$ .



On the heterotic string side, according to [5], the  $Y$  manifold corresponds to compactification on  $\mathbf{T}^2 \times \text{K3}$ , with the following gauge bundle. One takes an  $E_8 \times E_8$  bundle over  $\text{K3}$  with second Chern class twelve on each factor. The gauge bundle on  $\mathbf{T}^2 \times \text{K3}$  is thus derived purely from a bundle on  $\text{K3}$ . The model has four vectors coming from the moduli of  $\mathbf{T}^2$ .

Incidentally, the alternative case of an  $SO(32)$  bundle over  $\mathbf{T}^2 \times \text{K3}$  which is derived from a bundle over  $\text{K3}$  with second Chern class 24 – an example also mentioned in [5] – can be identified in the following way with one of the  $\text{K3}$  fibrations listed in [7]. With standard embedding of the gauge bundle in the tangent bundle, this model has gauge group  $SO(28) \times SU(2)$  with ten massless hypermultiplets in the  $(\mathbf{28}, \mathbf{2})$ . Expectation values of these generically break  $SO(28)$  to  $SO(8)$  (the complete Higgsing assumed in [5] to relate this model to the  $Y$  manifold cannot occur). Going to the Coulomb phase of the  $SO(8)$  gives a model with eight vectors (four from  $\mathbf{T}^2$  and four from the Cartan subalgebra of  $SO(8)$ ) and 272 hypermultiplets, corresponding quite likely to a  $\text{K3}$  fibration with  $h^{1,1} = 7$ ,  $h^{2,1} = 271$  that is listed in [7]. Similarly, the heterotic string model in section (4.2) of [5] can be taken with gauge group  $E_8 \times E_8 \times SU(2)$  or  $SO(32) \times SU(2)$ . In the former case, after complete Higgsing one gets the  $X$  manifold with three vectors, as described in [5], while in the latter case the hypermultiplets can break the gauge symmetry only to  $SO(12)$ , and one gets presumably the model with  $h^{1,1} = 8$ ,  $h^{2,1} = 194$  given in [7]. One should aim to similarly reproduce from the heterotic string many examples in [7], and their toric generalizations.

The heterotic string analog of the orientifold symmetry  $w$  of the  $Y$  manifold can be studied as follows. First of all,  $w$  must act as  $-1$  on  $\mathbf{T}^2$  so that the four vectors coming from  $\mathbf{T}^2$  are all odd under  $\tau$ . To find the action on  $\text{K3}$ , we use the fact that the  $\text{K3}$  in question is fibered over  $\mathbf{CP}^1$  and that the  $w$  action, according to the adiabatic philosophy, preserves this fibration and acts as  $\tau$  on  $\mathbf{CP}^1$ . In particular, since  $\tau$  acts freely on  $\mathbf{CP}^1$ ,  $w$  acts freely on  $\text{K3}$ . Moreover, since the action of  $w$  commutes with half the supersymmetries,  $w$  preserves one of the complex structures of  $\text{K3}$  (though not the one in which the fibration  $\text{K3} \rightarrow \mathbf{CP}^1$  is holomorphic). A  $\mathbf{Z}_2$  symmetry of  $\text{K3}$  that preserves a complex structure and acts freely is equivalent to the Enriques involution, so we can identify  $w$  with that involution, and we have determined how  $w$  acts on  $\mathbf{T}^2 \times \text{K3}$ .

Notice that the involution  $w$  of  $\mathbf{T}^2 \times \text{K3}$  is the same one used on the Type II side in [6] to obtain a model with  $N = 2$  supersymmetry, and therefore the  $N = 1$  model we are

constructing uses on the heterotic string side the same Calabi-Yau manifold used in [6] on the Type II side.

To complete the description of the model on the heterotic string side, we have to lift the action of  $w$  to the  $E_8 \times E_8$  gauge bundle. We cannot be as rigorous in describing this, but the fact that in the construction of [5] the second Chern class was equally divided between the two  $E_8$ 's (otherwise complete  $E_8 \times E_8$  breaking was not possible) creates an elegant possibility that would not exist otherwise:  $w$  can be the automorphism that exchanges the two  $E_8$ 's. The same possibility exists in the example of [5] related to the  $X$  manifold since again the second Chern class is divided equally.

What happens to the hypermultiplets on the heterotic string side when one divides by  $w$ ? On general grounds, from each  $N = 2$  hypermultiplet one  $N = 1$  chiral multiplet survives. From the geometrical moduli of K3, the survivors are the moduli of the Enriques surface (twenty chiral multiplets). From the moduli of the  $E_8 \times E_8$  bundle, assuming that  $w$  exchanges the two  $E_8$ 's, what survive are the moduli associated with one  $E_8$  (224 chiral multiplets). The other surviving chiral multiplets are  $S$ ,  $T$ , and  $U$ .

### 3.5. Conifold Singularities And Transitions

One might wonder what becomes in such  $N = 1$  orientifolds of some of the interesting physics associated with conifold singularities and their generalizations [27-28].

In  $N = 2$  models, Type IIA vector multiplets do not couple to elementary charged string states, but they do couple to charged black holes. At the mirror of a conifold singularity, such a charged black hole hypermultiplet goes to zero mass. Let  $M$  and  $\widetilde{M}$  be the  $N = 1$  chiral superfields in such a hypermultiplet. Let  $A$  be a gauge field with respect to which  $M$  and  $\widetilde{M}$  are charged, and let  $a$  be the  $N = 1$  chiral superfield related to  $A$  by  $N = 2$ . The couplings important at the conifold singularity are a superpotential term  $\Delta W = aM\widetilde{M}$  and the couplings of the gauge field to  $M$  and  $\widetilde{M}$ . In conformal field theory,  $M$  and  $\widetilde{M}$  have, in effect, been integrated out, but two traces of this show up in couplings of  $a$  and  $A$  computed in conformal field theory. (1) There is a logarithmic singularity in the Kahler metric on the  $a$  plane. (2) The effective coupling of the  $A$  field (after performing a duality transformation to the "right" photon) goes to zero at the conifold point, while there is a  $2\pi$  monodromy in the effective  $\theta$  angle as one winds around the conifold point.

Now if one goes to an orientifold  $N = 1$  model, what happens in the weak coupling, conformal field theory limit is that either  $a$  or  $A$  is projected out, but the couplings of the field that does survive are unchanged. If it is the chiral multiplet  $a$  that survives, then

one has the logarithmic singularity in the metric on the  $a$  plane in the projected conformal field theory. This is interpreted as resulting from integrating out a chiral superfield  $M'$  with a superpotential  $\Delta W = a(M')^2$ . Presumably  $M'$  is a linear combination of  $M$  and  $\widetilde{M}$ .<sup>10</sup> In the other case, where  $A$  survives, the vanishing effective gauge coupling and monodromy in the  $\theta$  angle that one sees in the conformal field theory limit are signals that  $A$  is coupled to a charged multiplet that is becoming massless at the conifold point.

Perhaps it should be stressed that the occurrence of these phenomena – though not the precise point in moduli space at which they occur – is stable under quantum corrections to the conformal field theory analysis. That is because these effects are associated with certain topological invariants which cannot just disappear when space-time quantum corrections are turned on. In the case that  $A$  survives in the orientifold, the topological invariant that makes the effect stable is the  $2\pi$  shift of the  $\theta$  angle in a circuit around the locus at which  $M$  and  $\widetilde{M}$  are massless; when  $a$  survives, the relevant effect is a  $2\pi$  shift in the  $(M')^2$  (or  $M\widetilde{M}$ ) term in the superpotential in a circuit around this locus; this shift determines a  $2\pi$  shift in the phase of the mass of the fermion field in  $M'$ . Note that topological invariance of these  $2\pi$  shifts in  $\theta$  angles and phases of fermion masses does not depend on supersymmetry, so massless charged fermions associated with conifolds also survive in non-supersymmetric orbifolds.

### *Alternative Class Of $N = 1$ Models*

The role of the conifold can be similarly discussed in the alternative class of  $N = 1$  models derived from Calabi-Yau manifolds, in which one finds an involution  $g$  that preserves the complex structure and holomorphic three-form, and divides by  $g(-1)^{F_L}$ . From an  $N = 2$  vector multiplet  $V = (a, A)$ , the  $N = 1$  chiral multiplet  $a$  survives in the projected theory if  $V$  is even under  $g$ , while the  $N = 1$  vector multiplet containing  $A$  survives if  $V$  is odd under  $g$ . The conformal field theory couplings of the surviving modes are the couplings of the  $N = 2$  model, and the rest of the discussion proceeds as above. For this class of  $N = 1$  models, however, we cannot reliably exhibit heterotic string duals.

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<sup>10</sup> It may be that both  $M$  and  $\widetilde{M}$  survive. In an  $N = 1$  theory, given a logarithmic singularity  $ds^2 = |da|^2 \ln |a|^2$  in the Kahler metric, there is no way to infer how many chiral superfields with  $a(M')^2$  couplings were integrated out to produce the singularity. This is because the coefficient of the logarithm can be absorbed in the normalization of  $a$ . For  $N = 2$ ,  $a$  is in a supermultiplet with a gauge field that has a natural normalization, and this gives a natural normalization for  $a$ .

### 3.6. Relation To Type I Models

In this discussion, we have emphasized orientifolds of the Type IIA model in which one divides by a symmetry that reverses orientation of both the world-sheet and the target space-time. The analog for Type IIB would be to divide by a symmetry that reverses orientation of the world-sheet while preserving the orientation of the target. The two classes of models are closely related and are very plausibly exchanged by mirror symmetry.

A special case of a Type IIB orientifold is the one obtained by dividing by orientation reversal on the world-sheet together with trivial action on space-time. The  $SO(32)$  Type I superstring should presumably be interpreted as this orbifold of Type IIB. As this example shows, in constructing orientifolds, non-abelian gauge groups in space-time will naturally arise from fixed points of the involution in space-time.

It appears that many  $N = 1$  vacua of the heterotic string are equivalent to Type II orientifolds. For instance, it now seems [3,29-30] that the  $SO(32)$  heterotic string in ten dimensions is equivalent to the Type I superstring in ten dimensions. So any heterotic string vacuum that can be understood as a compactification of the  $SO(32)$  model in ten dimensions will be equivalent to an analogous Type I compactification. But our results indicate that many other heterotic string vacua are equivalent to orientifolds, including  $E_8 \times E_8$  models as well as the  $SO(32)$  models that can be related directly to Type I.

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