# Can the Fundamental Theory of Everything be Renormalizable?

**J. Gegelia and N. Kiknadze**

*High Energy Physics Institute and Dept. of General Physics, Tbilisi State University, Chavchavadze ave. 1, Tbilisi 380028, Georgia.*

**Abstract**

A simple argument showing that renormalizable theories with consistent perturbative series can not be nonperturbatively finite (in terms of bare parameters) is provided. Accordingly any fundamental unified theory has to be either nonrenormalizable or order by order finite.

For several dozens of years renormalization procedure serves well for extracting physical information from renormalizable theories. Wilson's renormalization group approach [1] has contributed much to deeper understanding of renormalizability. However, despite the considerable success of renormalizable quantum field theories a lot of physicists feel uneasy by necessity to deal with divergent expressions. Some even declare that such QFT-s are completely inconsistent [2]. It is understood that any self-contained theory, and hence fundamental unified theory of all interactions too, must be nonperturbatively finite in terms of bare parameters. In this light, efforts to find order by order finite theories (main hopes are relied on supersymmetric theories [3]) seem quite natural. There is a general feeling that divergences in e.g. QED appear because QED itself is a low energy limit of unified theory, and that correct treatment of gravity would allow us to find self-consistent unified theory (see e.g. [4]). However, one may believe that divergences in renormalizable theories are just artefact of perturbative approach and the exact (nonperturbative) renormalization constants are finite in terms of bare parameters. Below we are going to demonstrate that this kind of viewpoint is not realistic. Although this result is not a surprise, it was firmly established only for superrenormalizable theories [5].

Let us consider some renormalizable theory and assume for a moment that divergences are due only to perturbation theory, i.e. exact expressions of physical quantities in terms of bare parameters are finite. Let us employ dimensional regularization [6]. Relation between bare ($g_0$) and renormalized ($g_\Lambda$) coupling constants has the form:

$$g_0 = \sum_{i=1}^{\infty} a_i g_\Lambda^i \ . \tag{1}$$

Here $\Lambda$ is the normalization point and divergences inhabit coefficients $a_i$. Our assumption of the finiteness of exact solutions implies that (1) is a formal expansion of some finite (in $\epsilon \to 0$ limit) relation:

$$g_0 = f(g_\Lambda, \epsilon) \stackrel{\epsilon\to 0}{\longrightarrow} g_\Lambda Z_{exact}(g_\Lambda) \ , \tag{2}$$

with $Z_{exact}$ being finite. Of course existence of the zero $\epsilon$ limit in (2) does not imply that this limit necessarily exists for the coefficients of its expansion in powers of $g_\Lambda$.

Alternately one could use the MS scheme [7]. Then expression (1) takes the form:

$$g_0 = \mu^\epsilon \left( g_{MS} + \sum_{i=1} b_i(g_{MS})\epsilon^{-i} \right) \ . \tag{3}$$

The important point here is that coefficients $b_i$ are independent of 't Hooft's unit mass $\mu$ as well as of other dimensional parameters. Relation between $g_\Lambda$ and $g_{MS}$ is given by series with some finite coefficients:

$$g_{MS} = g_\Lambda + \sum_{i=3}^{\infty} c_i g_\Lambda^i \ . \tag{4}$$

Suppose (3) is a formal expansion of some finite (in the $\epsilon \to 0$ limit) function (evidently, it is impossible in superrenormalizable theory, where there are only finite number of diverging terms in (3)):

$$g_0 = \mu^\epsilon \phi^*(g_{MS}, \epsilon) \ . \tag{5}$$

Taking limit $\epsilon \to 0$ in (5) we get

$$g_0 = \phi(g_{MS}) \tag{6}$$

and hence $g_{MS}$ does not depend on $\mu$ (here $\phi$, like $Z_{exact}$ in (2), is defined up to a function with zero asymptotic expansion). It implies vanishing of $\beta_{MS}$ function. On the other hand, in perturbation theory for renormalizable models $\beta_{MS} \neq 0$ and certain relations between $b_i$ in (3) guarantee order by order finiteness of it [7]. As far as there exist no other asymptotic expansion of zero then with coefficients identically equal to zero, we see that the asymptotic character of the series in minimal schemes is not compatible with the finiteness of the exact solutions of the theory. One may claim that the minimal schemes are inconsistent (i.e. (4) is not asymptotic even if the renormalized series in $g_\Lambda$ are). However, the same kind of analysis holds in any particular scheme. Consider, for example, regularization $\Lambda$, which was absent initially. Differentiation of the expression $g_0 = Z(\Lambda, g(\Lambda))\, g(\Lambda)$ with respect to $\Lambda$ can be employed in derivation of series for $\beta$-function (example of such calculation can be found e.g. in [8]). Order by order finiteness of resulting series for $\beta$-function in the limit when regularization is removed is related to the renormalizability of the theory. If the theory were nonperturbatively finite, the perturbative series for $Z$ would represent expansion of some $Z_{exact}$, independent from $\Lambda$ in the removed regularization limit and hence leading to zero exact $\beta$-function.

Note that our argument holds only if differentiation with respect to mass scale commutes with "summing" of perturbation series. But if it were not the case, then perturbation series would have nothing to do with the exact expressions. Although there is no theoretical proof of the asymptotic character of perturbation series for physically interesting renormalizable theories (QED, for example), it is anticipated due to the success of them in describing experimental data.

So we have demonstrated that if renormalized perturbative series in renormalizable theory have any status (i.e. are asymptotic), then nonperturbative relations between bare and renormalized quantities are necessarily divergent. Hence, it is clear that any candidate for fundamental unified theory must be either order by order finite or perturbatively nonrenormalizable by standard approach. If taking (supposedly) low-energy limit in such theory leaves us with the standard model, it means that relations between bare and renormalized quantities diverge in that limit, while relations between renormalized quantities remain finite and they can be extracted by renormalization procedure.

Acknowledgements: We would like to thank Prof. A. Khelashvili for stimulating interest to this work.

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