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BRST Analysis of QCD_2 as a Perturbed WZW Theory

D. C. Cabra[1]
Departamento de Física, Universidad Nacional de La Plata,
C.C. 67, (1900) La Plata, Argentina.

K. D. Rothe
Institut für Theoretische Physik Universität Heidelberg
Philosophenweg 16, D-69120 Heidelberg, Germany

F.A. Schaposnik[2]
Departamento de Física, Universidad Nacional de La Plata
C.C. 67, (1900) La Plata, Argentina

Abstract

Integrability of Quantum Chromodynamics in 1+1 dimensions has recently been suggested by formulating it as a perturbed conformal Wess-Zumino-Witten theory. The present paper further elucidates this formulation, by presenting a detailed BRST analysis.

[1]CONICET, Argentina
[2]Investigador CICBA

1 Introduction

As the non-Abelian extension of the exactly soluble Schwinger model [1], Quantum Chromodynamics in $1+1$ dimensions (QCD_2) has received much attention in the past 25 years. Its equivalent bosonic description was, however, fully understood only

as late as 1984, with the work of Polyakov-Wiegmann [2] and Witten [3].

The basic idea for arriving at an equivalent bosonic description of QCD_2 is to perform a change of variable which decouples the fermions from the gauge field (decoupled picture). This idea, explored by [4] in the bosonization of the Schwinger model [1], has been successfully applied in a number of papers on QCD_2 [5], [6]. For a general review the reader is referred to [7]. Following this procedure one arrives at an effective action involving a conformally invariant Wess-Zumino-Witten [WZW] functional plus the Yang-Mills action.

By regarding the Yang-Mills action of QCD_2 as a perturbation of a conformally invariant WZW theory of positive and negative

level WZW fields, as well as ghosts, in the decoupled picture, Abdalla and Abdalla [8] obtained an infinite set of conservation laws. In their description a number of first and second class constraints emerged the significance and role of which, however, remained unclear.

It is the objective of this note to clarify this and related aspects of their approach, by performing a detailed BRST analysis of QCD_2 following the path-integral formulation described above.

2 Local, decoupled formulation of QCD_2

The partition function of QCD_2 is given by

$$Z = \int [\mathcal{D}A_\mu] \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp[i(S_{YM} + S_F)] \tag{2.1}$$

where S_{YM} and S_F are the Yang-Mills and fermionic action, respectively[3]

$$S_{YM} = -\frac{1}{4} \int d^2x tr(F_{\mu\nu}F^{\mu\nu}) \tag{2.2}$$

$$\begin{aligned} S_F &= \int d^2x \bar{\psi}(i\partial\!\!\!/ + eA\!\!\!/)\psi \\ &= \int d^2x \left\{ \psi_1^\dagger (i\partial_+ + eA_+)\psi_1 + \psi_2^\dagger (i\partial_- + eA_-)\psi_2 \right\} \end{aligned} \tag{2.3}$$

[3]Our conventions are $\partial_\pm = \partial_0 \pm \partial_1, A_\pm = A_0 \pm A_1$, $F_{\mu\nu} = t^a F^a_{\mu\nu}$, etc., with the normalization $trt^a t^b = \delta^{ab}$ for the hermitian generators in the fundamental representation, and the commutation relation $[t^a, t^b] = ifabc\ t^c$,
with $fabcfabd = \frac{C_V}{2}\delta cd$. We follow in general the notation and conventions of ref. [8].

and $[\mathcal{D}A_\mu]$ stands for the measure including gauge fixing. It will be convenient to work in the light cone gauge $A_+ = (A_0 + A_1) = 0$. We implement this gauge in terms of a Lautrup-Nakanishi-Lagrange multiplier field B,

$$Z = \int \mathcal{D}A_\mu \int \mathcal{D}\psi\mathcal{D}\bar{\psi} \int \mathcal{D}B \int \mathcal{D}b_-\mathcal{D}c_- \exp(iS_{GF}) \tag{2.4}$$

with

$$S_{GF} = S_{YM} + S_F + S_{gh} + \int d^2x tr(BA_+) \tag{2.5}$$

where S_{gh} is the ghost action

$$S_{gh} = \int d^2x tr(b_- i\mathcal{D}_+ c_-) \tag{2.6}$$

with

$$\mathcal{D}_\pm = \partial_\pm - ie[A_\pm, \quad] \tag{2.7}$$

the covariant derivative in the adjoint representation.

The gauge-fixed action (2.5) is invariant under the BRST tranformation

$$\begin{aligned}
\delta A_\mu &= \epsilon\frac{1}{e}\mathcal{D}_\mu c_- \\
\delta\psi_\alpha &= \epsilon c_-\psi_\alpha \\
\delta c_- &= \epsilon\frac{1}{2}\{c_-, c_-\} \\
\delta b_- &= \epsilon\frac{1}{e}B \\
\delta B &= 0
\end{aligned} \tag{2.8}$$

with ϵ a Grassman-valued infinitesimal parameter.

Performing the integration over B and A_+, we obtain the following partition function

$$Z = \int \mathcal{D}A_- \int \mathcal{D}\psi_1^{(0)} D\psi_1^{\dagger(0)} \int \mathcal{D}\psi_2\mathcal{D}\psi_2^\dagger \int \mathcal{D}b_-^{(0)}\mathcal{D}c_-^{(0)} \exp\left(iS_{GF}\right) \tag{2.9}$$

with the corresponding gauge-fixed Lagrangian

$$\begin{aligned}
\mathcal{L}_{GF} &= tr\frac{1}{8}(\partial_+ A_-)^2 + \psi_1^{(0)\dagger} i\partial_+\psi_1^{(0)} \\
&\quad + \psi_2^\dagger i D_-\psi_2 + tr\left(b_-^{(0)} i\partial_+ c_-^{(0)}\right).
\end{aligned} \tag{2.10}$$

We have denoted the fields obbeying a free-field dynamics by a superscript "(0)", and D_- is the covariant derivative

$$D_\pm = \partial_\pm - ieA_\pm \tag{2.11}$$

with $A_\pm$ in the fundamental representation.

The classical action S_{GF} is invariant under the BRST transformation

$$\begin{aligned}
\delta A_- &= -i\epsilon\frac{1}{e}\mathcal{D}_- c_-^{(0)} \\
\delta\psi_1^{(0)} &= \epsilon c_-^{(0)}\psi_1^{(0)} \\
\delta\psi_2 &= \epsilon c_-^{(0)}\psi_2 \\
\delta c_-^{(0)} &= \epsilon\frac{1}{2}\{c_-^{(0)}, c_-^{(0)}\}
\end{aligned} \tag{2.12}$$

$$\begin{aligned}
\delta b_-^{(0)} &= \epsilon\frac{1}{e^2}\mathcal{D}_-(\partial_+ A_-) + \epsilon\psi_1^{(0)}\psi_1^{(0)\dagger} \\
&\quad +\epsilon\{b_-^{(0)}, c_-^{(0)}\}
\end{aligned} \tag{2.13}$$

These transformation laws can be obtained from (2.8) by eliminating A_+ and B using the equations of motion

$$\begin{aligned}
A_+ &= 0 \\
B^a &= -\frac{\delta S_{YM}}{\delta A_+^a} - e\psi_1^\dagger t^a\psi_1 + ief_{abc}b_-^b c_-^c
\end{aligned} \tag{2.14}$$

Regarding (2.12) as a classical transformation, it is a symmetry of the gauge-fixed Lagrangian (2.10). In order to analyse the corresponding symmetry at the partition function level, one has however to take into account the following facts:

i) The fermionic path-integral in (2.4) (i.e., before fixing the gauge to $A_+ = 0$) leads to

$$Z_{fer} = det(i\partial_+ + eA_+)det(i\partial_- + eA_-) \times \exp(-i\frac{e^2}{4\pi}\int d^2x A_+A_-) \tag{2.15}$$

The last term in the r.h.s. of (2.15) has been added, exploiting regularization ambiguities, so as to ensure gauge invariance [2].

ii) Analogously, the determinant arising from integration of ghosts with action (2.6) has to be adjusted with the same type of A_+A_- counterterm,

$$Z_{ghosts} = det^{Adj}(i\partial_+ + eA_+) \times \exp(-i\alpha\int d^2x A_+A_-) \tag{2.16}$$

Since, as it is well-known [2], the determinant in the adjoint

representation is related with that in the fundamental through the Casimir C_V (see [6] for details), consistency of the regularization implies that α should be chosen as $\alpha = (e^2/4\pi)C_V$.

Taking into account the i) and ii), the transformation law for the ghost field $b_-^{(0)}$ takes the form

$$\delta b_-^{(0)} = -\frac{\epsilon}{e}\mathcal{D}_-(\partial_+ A_-) + \epsilon\psi_1^{(0)}\psi_1^{(0)\dagger} + \epsilon\{b_-^{(0)}, c_-^{(0)}\} - \epsilon e\frac{(1+C_V)}{4\pi}A_- \quad (2.17)$$

The last term in the r.h.s. of (2.17) arises from the A_+A_- counterterms as discussed above. The transformations (2.12) and (2.17) now represent a symmetry transformation of the partition function associated with (2.10).

We now go to a new set of variables such that the partition function factorizes in terms of decoupled fields, by writing

$$\begin{aligned} A_- &= \frac{i}{e}V\partial_- V^{-1} \\ \psi_2 &= V\psi_2^{(0)} \end{aligned} \quad (2.18)$$

For the corresponding transformation of the integration measure one has

$$\begin{aligned} \mathcal{D}A_- &= \int \mathcal{D}b_+\mathcal{D}c_+ \exp\left(i\int d^2x\ tr\ b_+ i\mathcal{D}_- c_+\right) \times \mathcal{D}V \\ &= \exp(-iC_V\Gamma[V]) \int \mathcal{D}b_+^{(0)}\mathcal{D}c_+^{(0)} \\ &\quad \exp(i\int d^2x\ tr\ b_+^{(0)} i\partial_- c_+^{(0)}) \times \mathcal{D}V \end{aligned} \quad (2.19)$$

$$\mathcal{D}\psi_2\mathcal{D}\psi_2^\dagger = \mathcal{D}\psi_2^{(0)}\mathcal{D}\psi_2^{(0)\dagger} \exp(-i\Gamma[V]) \quad (2.20)$$

where $\Gamma[g]$ is the Wess-Zumino-Witten functional [3]

$$\Gamma[g] = \frac{1}{8\pi}\int d^2x\ tr\ \partial_\mu g^{-1}\partial^\mu g + \frac{1}{12\pi}\int d^3y\epsilon^{\alpha\beta\gamma} tr[g^{-1}\partial_\alpha g g^{-1}\partial_\beta g^{-1}\partial_\gamma g] \quad (2.21)$$

with the remarkable property (see [7] for details)

$$\begin{aligned} \delta\Gamma[g] &= \frac{1}{4\pi}\int d^2x\ trg\delta g^{-1}\partial_+(g\partial_- g^{-1}) \\ &= \frac{1}{4\pi}\int d^2x\ trg^{-1}\delta g\partial_-(g^{-1}\partial_+ g) \end{aligned} \quad (2.22)$$

In terms of the new variables, the partition function reads

$$Z = Z_F^{(0)} Z_{gh}^{(0)} Z_V \quad (2.23)$$

where

$$Z_V = \int \mathcal{D}V \exp\left\{-i(1+C_V)\Gamma[V] + \frac{i}{8e^2}\int d^2x tr[\partial_+(Vi\partial_- V^{-1})]^2\right\} \quad (2.24)$$

and

$$Z_F^{(0)} = \int \mathcal{D}\psi^{(0)}\mathcal{D}\bar{\psi}^{(0)} \exp\left(i\int d^2x\bar{\psi}^{(0)} i\partial\!\!\!/\psi^{(0)}\right) \tag{2.25}$$

$$Z_{gh}^{(0)} = \int \mathcal{D}b_{\pm}^{(0)} Dc_{\pm}^{(0)} \exp[i\int d^2x\ tr\left(b_+^{(0)} i\partial_- c_+^{(0)} + b_-^{(0)} i\partial_+ c_-^{(0)}\right)] \tag{2.26}$$

Notice that the WZW action enters in (2.24) with negative level $-(1+C_V)$.

It is interesting at this stage to compare our results, summarized in eqs.(2.23)-(2.26), with those presented in [8]. Eq.(2.23) shows that the QCD_2 partition function factorizes into the partition functions for free fermions, ghosts and perturbed WZW fields. This factorization (including the remanence of free fermions) is characteristic of path-integral bosonization which is always based in the decoupling of the interacting fermions, which thus become free [4]-[6] (Of course, these free fermions can in turn be bosonized in terms of Wess-Zumino fields $-\tilde{g}$ in ref. [8]).

In terms of the new variables the BRST symmetry transformation (2.12), (2.17) reads

$$\begin{aligned}
V\delta V^{-1} &= -\epsilon c_-^{(0)}, \\
\delta\psi_1^{(0)} &= \epsilon c_-^{(0)}\psi_1^{(0)}, \quad \delta\psi_2^{(0)} = 0 \\
\delta c_-^{(0)} &= \frac{\epsilon}{2}\{c_-^{(0)}, c_-^{(0)}\}, \quad \delta c_+^{(0)} = 0 \\
\delta b_-^{(0)} &= \epsilon B_-^{(0)} + \epsilon\Delta_-(V), \quad \delta b_+^{(0)} = 0
\end{aligned} \tag{2.27}$$

where

$$\begin{aligned}
&B_-^{(0)} = \psi_1^{(0)}\psi_1^{(0)\dagger} + \{b_-^{(0)}, c_-^{(0)}\} \\
&\Delta_-(V) = -\frac{1}{4e^2}\mathcal{D}_-(V)(\partial_+(Vi\partial_- V^{-1})) - \left(\frac{1+C_V}{4\pi}\right)Vi\partial_- V^{-1}
\end{aligned} \tag{2.28}$$

and

$$\mathcal{D}_-(V) = \partial_- + [V\partial_- V^{-1}, \quad] \tag{2.29}$$

Using (2.22) one readily checks that the partition function (2.23) is invariant under the above transformation. The corresponding BRST current, as obtained via the usual Noether construction, is found to be (the superscript "(B)" stands for "BRST")

$$\begin{aligned}
J_-^{(B)} = tr c_-^{(0)} \Big[&-\frac{1}{4e^2}D_-(V)\partial_+\left(Vi\partial_- V^{-1}\right) - \left(\frac{1+C_V}{4\pi}\right)Vi\partial_- V^{-1} \\
&+\psi_1^{(0)}\psi_1^{(0)\dagger} + \frac{1}{2}\{b_-^{(0)}, c_-^{(0)}\}\Big]
\end{aligned} \tag{2.30}$$

with

$$\partial_+ J_-^{(B)} = 0 \tag{2.31}$$

Remarkably enough, BRST symmetry leads to a current which only depends on the variable x^-.

It is desirable to rewrite (2.30) in standard form, exhibiting explicitly its BRST character. To this end we observe that

$$\Omega_- := -\frac{1}{4e^2}\mathcal{D}_-(V)\partial_+(Vi\partial_- V^{-1}) - \left(\frac{1+C_V}{4\pi}\right) Vi\partial_- V^{-1} + j_- \approx 0 \tag{2.32}$$

with

$$\begin{aligned} j_- &= \psi_1^{(0)}\psi_1^{(0)\dagger} + \{b_-^{(0)}, c_-^{(0)}\} \\ &\partial_+ j_- = 0 \end{aligned} \tag{2.33}$$

is a constraint of the theory. To see this, we follow the general ideas outlined in ref. [15], "and gauge" the partition function (2.23) with an external field $W_+ = i\omega^{-1}\partial_+\omega$, by making the substitutions

$$\begin{aligned} \partial_+ &\to D_+(\omega) = \partial_+ - iW_+ \\ \partial_+ &\to \mathcal{D}_+(\omega) = \partial_+ - i\,[W_+, \quad] \end{aligned} \tag{2.34}$$

in the right-hand sector of (2.25) and (2.26), as well as the substitution

$$\Gamma[V] \to \Gamma[V] - \frac{1}{4\pi}\int trW_+ Vi\partial_- V^{-1} \tag{2.35}$$

$$tr\left[\partial_+(V\partial_- V^{-1})\right]^2 \to tr\left[\mathcal{D}_+(\omega)V\partial_- V^{-1} + i\partial_- W_+\right]^2 \tag{2.36}$$

in (2.24). Noting that

$$tr\left[\mathcal{D}_+(\omega)V\partial_- V^{-1} + i\partial_- W_+\right]^2 = tr\left[\partial_+(\omega V)\partial_-(\omega V^{-1})\right]^2 \tag{2.37}$$

and making use of the Polyakov-Wiegmann identity [2]

$$\Gamma[gh] = \Gamma[g] + \Gamma[h] + \frac{1}{4\pi}\int d^2x tr\left(g^{-1}\partial_+ g h\partial_- h^{-1}\right) \tag{2.38}$$

one has after a change of integration variable $V \to \omega V$,

$$\begin{aligned} &Z_F^{(0)} \to Z_F^{(0)} e^{-i\Gamma[\omega]} \\ &Z_{gh}^{(0)} \to Z_{gh}^{(0)} e^{-iC_V\Gamma[\omega]} \\ &Z_V \to Z_V e^{i(1+C_V)\Gamma[\omega]} \end{aligned} \tag{2.39}$$

This shows that the partition function (2.23) is unchanged by the transformations (2.34) to (2.36). From here we derive the constraint (2.32) by taking the functional derivative of the gauged partition function with respect to W_+, and setting $W_+ = 0$. This constraint (Gauss' law) can be shown to satisfy a Kac-Moody algebra[4] with vanishing central charge; hence, in the terminology of Dirac [10], it is first class.

In terms of the constraint (2.32), the current (2.30) takes the standard form expected from general considerations [11], [12]:

$$J_-^{(B)} = tr\left[c_-^{(0)}\Omega_- - \frac{1}{2}b_-^{(0)}\left\{c_-^{(0)}, c_-^{(0)}\right\}\right] \tag{2.40}$$

Since Ω_- is first class, the corresponding charge $Q_-^{(B)}$ is nilpotent.

A second BRST symmetry

As is well known [13], [14], one expects a further BRST current associated with the change of variables (2.18). In fact, it is easy to see that the partition function (2.23) is also invariant under the transformation

$$\begin{aligned} &V^{-1}\delta V = -\epsilon c_+^{(0)} \\ &\delta\psi_1^{(0)} = 0, \quad \delta\psi_2^{(0)} = \epsilon c_+^{(0)}\psi_2^{(0)} \\ &\delta c_-^{(0)} = 0, \quad \delta c_+^{(0)} = \frac{\epsilon}{2}\left\{c_+^{(0)}, c_+^{(0)}\right\} \\ &\delta b_-^{(0)} = 0, \quad \delta b_+^{(0)} = \epsilon B_+^{(0)} + \epsilon\Delta_+(V) \end{aligned} \tag{2.41}$$

with $B_+^{(0)}$ and $\Delta_+(V)$ given by

$$\begin{aligned} B_+^{(0)} &= \psi_2^{(0)}\psi_2^{(0)\dagger} + \{b_+^{(0)}, c_+^{(0)}\} \\ \Delta_+(V) &= \frac{1}{e^2}V^{-1}\left(\partial_+^2(Vi\partial_-V^{-1})\right)V - \frac{1+C_V}{4\pi}V^{-1}i\partial_+V \end{aligned} \tag{2.42}$$

This transformation law should be compared with the one in (2.27). The corresponding BRST current obtained via the standard Noether construction is found to be

$$\begin{aligned} J_+^{(B)} = \; & trc_+^{(0)}\left[\frac{1}{4e^2}V^{-1}(\partial_+^2(Vi\partial_-V^{-1}))V - \frac{1+C_V}{4\pi}V^{-1}i\partial_+V\right. \\ & \left. +\psi_2^{(0)}\psi_2^{(0)\dagger} + \frac{1}{2}\left\{b_+^{(0)}, c_+^{(0)}\right\}\right] \end{aligned} \tag{2.43}$$

[4] In ref. [9] the terminology "affine Lie algebra" is preferred.

To put this expression into standard form we observe, following again the method of [15], that gauging with the external field,

$$W_- = \frac{i}{e}\omega\partial_-\omega^{-1} \tag{2.44}$$

by making the substitutions

$$\begin{aligned} \partial_- &\to \partial_- - iW_- \\ \partial_- &\to \partial_- - i[W_-, \quad] \end{aligned} \tag{2.45}$$

in the left-hand sector of (2.25) and (2.26), we have in analogy to (2.39),

$$\begin{aligned} &Z_F^{(0)} \to Z_F^{(0)} e^{-i\Gamma[\omega]} \\ &Z_{gh}^{(0)} \to Z_{gh}^{(0)} e^{-iC_V\Gamma[\omega]} \\ &e^{-i(1+C_V)\Gamma[V]} \to e^{-i(1+C_V)(\Gamma[V\omega]-\Gamma[\omega])} \\ &Vi\partial_-V^{-1} \to V(i\partial_- + W_-)V^{-1} = (V\omega)i\partial_-(V\omega)^{-1} \end{aligned} \tag{2.46}$$

Hence the partition function (2.23) is left invariant by this transformation. From this, we can derive the constraint

$$\Omega_+ \equiv \frac{1}{4e}V^{-1}\left[\partial_+^2(Vi\partial_-V^{-1})\right]V - \frac{(1+C_V)}{4\pi}V^{-1}i\partial_+V + j_+ \approx 0 \tag{2.47}$$

where

$$\begin{aligned} j_+ &= \psi_2^{(0)}\psi_2^{(0)\dagger} + \left\{b_+^{(0)}, c_+^{(0)}\right\} \\ &\partial_- j_+ = 0 \end{aligned} \tag{2.48}$$

One readily checks that this constraint is first class (vanishing central charge).

In terms of Ω_+, the BRST current (2.43) is seen to take the standard form expected from general considerations [11]

$$\begin{aligned} J_+^{(B)} &= tr\left[c_+^{(0)}\Omega_+ - \frac{1}{2}\left\{b_+^{(0)}, c_+^{(0)}\right\}\right] \\ &\partial_- J_+^{(B)} = 0. \end{aligned} \tag{2.49}$$

and hence the associated charge is nilpotent.

3 Non-local decoupled formulation of QCD_2

The partition function (2.23) is particularly useful in the strong coupling regime. Following the ideas of ref. [8], we now obtain an alternative, nonlocal representation useful in the weak coupling regime. To this end we make use of the identity

$$
\begin{aligned}
&\exp[\frac{i}{4e^2}\int tr\frac{1}{2}\left[\partial_+(Vi\partial_-V^{-1})\right]^2] \\
&= \int \mathcal{D}E \exp[-i\int tr\left[\frac{1}{2}E^2 + \frac{E}{2e}\partial_+(Vi\partial_-V^{-1})\right]]
\end{aligned} \tag{3.1}
$$

and make the change of variable

$$
\partial_+ E = \left(\frac{1+C_V}{2\pi}\right)\beta^{-1}i\partial_+\beta \tag{3.2}
$$

or

$$
E = \left(\frac{1+C_V}{2\pi}\right)(\partial_+)^{-1}\beta^{-1}i\partial_+\beta \tag{3.3}
$$

The Jacobian associated with this change of variables is

$$
\mathcal{D}E = \exp(-iC_V\Gamma[\beta])\mathcal{D}\beta \tag{3.4}
$$

Making use of the above results, the partition function (2.23) reads

$$
\begin{aligned}
Z &= Z_F^{(0)}Z_{gh}^{(0)}\int\mathcal{D}V\int\mathcal{D}\beta\exp\{-i(1+C_V)[\Gamma[V]+\Gamma[\beta] \\
&\quad -\frac{1}{4\pi}\int tr(\beta^{-1}\partial_+\beta V\partial_-V^{-1})]\} \\
&\quad \times\exp(i\Gamma[\beta])\exp\left\{i\left(\frac{1+C_V}{2\pi}\right)^2 e^2\int\frac{1}{2}tr\left[\partial_+^{-1}(\beta^{-1}\partial_+\beta\right]^2\right\}
\end{aligned} \tag{3.5}
$$

Now, using the Polyakov-Wiegmann identity (2.38) and making the change of variable $V \to \beta V = \tilde{V}$, we are left with

$$
Z = Z_F^{(0)}Z_{gh}^{(0)}Z_{\tilde{V}}Z_\beta \tag{3.6}
$$

where

$$
Z_\beta = \int\mathcal{D}\beta\exp\left\{i\Gamma[\beta] + i\left(\frac{1+C_V}{2\pi}\right)^2 e^2\int\frac{1}{2}tr\left[\partial_+^{-1}(\beta^{-1}\partial_+\beta)\right]^2\right\} \tag{3.7}
$$

$$
Z_{\tilde{V}} = \int\mathcal{D}\tilde{V}\exp[-i(1+C_V)\Gamma[\tilde{V}]] \tag{3.8}
$$

Expression (3.6) agrees with that of ref. [8].

BRST invariance of the $(F - gh - \tilde{V})$ sector

The product $Z_F^{(0)} Z_{gh}^{(0)} Z_{\tilde{V}}$ is invariant under the BRST transformations (2.27) and (2.41) with the substitution $V \to \tilde{V}$, and $\Delta_{\mp}(\tilde{V})$ now given by

$$\Delta_-(\tilde{V}) = -\left(\frac{1+C_V}{4\pi}\right) \tilde{V} i \partial_- \tilde{V}^{-1} \tag{3.9}$$

and

$$\Delta_+(\tilde{V}) = -\left(\frac{1+C_V}{4\pi}\right) \tilde{V}^{-1} i \partial_+ \tilde{V} \tag{3.10}$$

implying the conservation of the corresponding Noether currents

$$\tilde{J}_-^{(B)} = tr\ c_-^{(0)} \left[\psi_1^{(0)\dagger} \psi_1^{(0)} + \frac{1}{2}\left\{b_-^{(0)}, c_-^{(0)}\right\} - \left(\frac{1+C_V}{4\pi}\right) \tilde{V} \partial_- \tilde{V}^{-1}\right] \tag{3.11}$$

$$\tilde{J}_+^{(B)} = tr\ c_+^{(0)} \left[\psi_2^{(0)\dagger} \psi_2^{(0)} + \frac{1}{2}\left\{b_+^{(0)}, c_+^{(0)}\right\} - \left(\frac{1+C_V}{4\pi}\right) \tilde{V}^{-1} \partial_+ \tilde{V}\right] \tag{3.12}$$

One easily verifies that the corresponding charges are nilpotent. Indeed, following again the procedure of ref.[15], one gauges the left- and right-handed sector as in the conformal sector ($e \to \infty$) of the local formulation, and shows (with V replaced by $\tilde{V}$) that the partition function of the $F - gh - \tilde{V}$ sector remains invariant under this gauging.

This implies the existence of the two first-class constraints

$$\begin{aligned} \tilde{\Omega}_- &\equiv \psi_1^{(0)\dagger} \psi_1^{(0)} + \{b_-^{(0)}, c_-^{(0)}\} - \frac{1+C_V}{4\pi} \tilde{V} i \partial_- \tilde{V}^{-1} \approx 0 \\ \tilde{\Omega}_+ &\equiv \psi_2^{(0)\dagger} \psi_2^{(0)} + \{b_+^{(0)}, c_+^{(0)}\} - \frac{1+C_V}{4\pi} \tilde{V}^{-1} i \partial_+ \tilde{V} \approx 0 \end{aligned} \tag{3.13}$$

In terms of these constraints, the currents (3.11) and (3.12) take the standard form of BRST currents associated with a first-class constraint algebra [11]:

$$\tilde{J}_{\pm}^{(B)} = tr\left[c_{\pm} \tilde{\Omega}_{\pm}^{(0)} - \frac{1}{2} b_{\pm}^{(0)} \left\{c_{\pm}^{(0)}, c_{\pm}^{(0)}\right\}\right] \tag{3.14}$$

It is important to note that these BRST currents

correspond in the non-local formulation to the currents (2.40) and (2.49) were in the local one.

To conclude this section let us remark that there exist further BRST-like symmetries, which, however, are not generated by nilpotent charges. To take an example, consider the $gh - \tilde{V} - \beta$ sector. The partition function $Z_{gh} Z_{\tilde{V}} Z_{\beta}$ is readily seen to be invariant under the symmetry transformation

$$
\begin{aligned}
\tilde{V}\delta\tilde{V}^{-1} &= -\epsilon c_-^{(0)} \\
\beta\delta\beta^{-1} &= -\epsilon c_-^{(0)} \\
\delta c_-^{(0)} &= \frac{\epsilon}{2}\{c_-^{(0)}, c_-^{(0)}\}, \quad \delta c_+^{(0)} = 0 \\
\delta b_-^{(0)} &= -\epsilon\frac{1+C_V}{4\pi}\tilde{V}i\partial_-\tilde{V}^{-1} + \epsilon\{b_-^{(0)}, c_-^{(0)}\} + \epsilon\Delta_-[\beta]
\end{aligned}
\tag{3.15}
$$

where $\Delta_-[\beta]$ is given by

$$
\Delta_-(\beta) = \left(\frac{1+C_V}{2\pi}\right)^2 e^2\left[\beta\partial_+^{-2}(\beta^{-1}i\partial_+\beta)\beta^{-1}\right] + \frac{1}{4\pi}\beta i\partial_-\beta^{-1}
\tag{3.16}
$$

The corresponding Noether current is found to be

$$
\tilde{J}_- = tr\left[c_-^{(0)}\Omega - \frac{1}{2}b_-^{(0)}\left\{c_-^{(0)}, c_-^{(0)}\right\}\right].
\tag{3.17}
$$

where

$$
\begin{aligned}
\Omega \;\equiv\; & -\left(\frac{1+C_V}{2\pi}\right)^2 e^2\beta\left(\partial_+^{-2}(\beta^{-1}i\partial_+\beta)\right)\beta^{-1} + \frac{1}{4\pi}\beta i\partial_-\beta^{-1} \\
& - \left(\frac{1+C_V}{4\pi}\right)\tilde{V}i\partial_-\tilde{V}^{-1} + \left\{b_-^{(0)}, c_-^{(0)}\right\}.
\end{aligned}
\tag{3.18}
$$

By gauging the right-handed ghost, and $\tilde{V}$
sector as in (2.39), (with $V \to \tilde{V}$), supplemented by the transformations

$$
\begin{aligned}
\Gamma[\beta] &\to \Gamma[\beta] - \frac{i}{4\pi}\int trW_+\beta\partial_-\beta^{-1} = \Gamma[\omega\beta] - \Gamma[\beta] \\
\beta^{-1}\partial_+\beta &\to \beta^{-1}(\partial_+ - iW_+)\beta = (\omega\beta)^{-1}\partial_+(\omega\beta)
\end{aligned}
\tag{3.19}
$$

in the β-sector, one finds that expression (3.18) is constrained to vanish:

$$
\Omega \approx 0
\tag{3.20}
$$

As pointed out in [8], this constraint is, however, not first-class with respect to the constraints (3.13), and as a consequence the BRST charges associated with the currents (3.14) already represent a complete set.

The role of the constraint (3.20) is best appreciated, by rewriting the partition function Z_β in (3.7) with the aid of an auxiliary field C_- as [8]

$$
Z_\beta = \int \mathcal{D}C_-\mathcal{D}\beta e^{iS[\beta,C_-]}
\tag{3.21}
$$

where

$$S[\beta, C_-] = \Gamma[\beta] + \int tr\left[\frac{1}{2}(\partial_+ C_-)^2 + \left(\frac{1+C_V}{2\pi}\right) e(C_-\beta^{-1} i\partial_+\beta)\right] \qquad (3.22)$$

Gauging $Z_{gh} Z_{\tilde{V}} Z_\beta$ as described before ($\partial_+ C_-$ and C_- remain unchanged), one arrives at the

constraint:

$$\left(\frac{1+C_V}{2\pi}\right) e\beta i C_- \beta^{-1} + \left(\frac{1+C_V}{4\pi}\right) \tilde{V} i\partial_- V^{-1} - \frac{1}{4\pi}\beta i\partial_-\beta^{-1} + \{b_-^{(0)}, c_-^{(0)}\} = 0 \qquad (3.23)$$

This constraint determines C_- as a function of the other fields. Using the equation of motion for C_-,

$$\partial_+^2 C_- + \left(\frac{2+C_V}{2\pi}\right) e\beta^{-1}\partial_+\beta = 0$$

one then formally arrives at constraint (3.20).

4 Discussion

The main objective of this paper was to

further elucidate the interesting analysis of ref. [8], by supplementing it with an analysis of the BRST symmetries as well as the construction of the corresponding BRST currents.

The study of BRST currents is important if one wants to obtain a complete characterization of the physical Hilbert space. The formulation of ref. [8] appears to be particularly suited for this purpose. Indeed, we have seen that the conformal invariance of the pure gauged fermionic partition function, described by a WZW-type theory [3], is broken by the presence of the Yang-Mills action, which implies the presence of

a coupling constant carrying dimensions. We have nevertheless seen the BRST currents to be either left- or right-moving as in a conformally invariant model. This is remarkable, and is at the heart of the claims in ref. [8] on the exact integrability of QCD_2. The structure of the physical Hilbert space

as determined by the BRST conditions is presently under investigation [16].

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