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Variational solution of the Gross-Neveu model at finite temperature in the large N limit.

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Abstract

In this work we investigate the restoration of chiral symmetry in the Gross-Neveu model at finite temperature using a nonperturbative variational method which is compatible with the usual renormalization program of the theory. It is shown, in this first letter, that the variational procedure can be generalized to the finite temperature case. The large N result for the phase transition is correctly reproduced.

I. INTRODUCTION

The study of chiral symmetry breaking/restoration in QCD requires the use of nonperturbative techniques like numerical simulations or analytical methods such as the $1/N$ expansion, the Hartree-Fock approximation and variational methods [1-3]. In general, most approximations designed to tackle the nonperturbative QCD regime are tested in effective models which share some of QCD's characteristics. The renormalizable and asymptotically free Gross-Neveu model (GN) [4] is particularly useful in the study of chiral symmetry breaking/restoration. The exact solution for the chiral symmetry related mass gap equation, which is known in the large N limit, turns the model into an ideal laboratory for testing newly developed or improved existing nonperturbative approximations.

In this letter, we will be concerned with a variational method which was originally proposed in Ref. [1] and which is related to the optimized δ expansion [2,3]. It has been recently improved, in order to cope with the renormalization program of quantum field theories, and successfully tested in the GN model at zero temperature both in the large [5] and finite [6] N limits. Our purpose is to extend the method to the finite temperature domain where the related optimized δ expansion has already been successfully applied, without addressing the renormalization problem, to the study of chiral symmetry restoration in the GN model [7] as well as in Nambu-Jona-Lasinio model [8]. Most applications performed with the optimized δ expansion start from an interpolated renormalized lagrangian written in terms of non original arbitrary parameters around which perturbative calculations are performed. The perturbative result is then extremized with respect to arbitrary mass parameters. This optimization procedure known as the Principle of Minimal Sensitivity (PMS) [9] gives the calculation a nonperturbative character. However, this whole program makes the introduction of counterterms rather confusing, mixing perturbative orders and rendering the renormalization process unclear particularly away from the well behaved large N limit where the renormalizability of the linear δ expansion has already been investigated [10].

On the other hand, the method proposed in Ref. [5] suggests that one should start with a *bare* lagrangian density and that the evaluation of physical quantities should be accomplished with bare parameters which, at the end, are related to the renormalized ones via the Renormalization Group (RG). The calculation is improved by a simple but powerful analytical method which allows to attain arbitrarily large and even infinite order of perturbation in the parameter δ. This procedure reconciles the variational method with the renormalization program of the theory for *any* N [6].

In analysing the large N finite temperature behaviour of the GN model one observes a second order phase transition which leads to chiral symmetry restoration at the critical temperature

$$T_c = 0.57 \times M_F(0) \quad , \tag{1}$$

where $M_F(0)$ is the renormalized fermion mass, generated through the breaking of chiral symmetry at $T = 0$ [11,12]. However, one should note that this result is only valid in the large N limit where some important kink effects are missed [13]. That is due to the fact that in 1+1 dimensions it is energetically favorable to have kink configurations, which implies that the sign of the order parameter alternates over small regions of space. The average over space of these kink configurations then makes the order parameter zero. Thus, there is no symmetry breaking in one space dimension at finite temperatures (Landau's theorem [14]) and one expects calculations performed at finite N to take this fact into account. Indeed, it has been explicitly demonstrated , in the context of the effective potential (free energy), that the optimized δ expansion predicts a smaller critical temperature for the phase transition at finite N in agreement with Landau's theorem [7]. Although reassuring, in what concerns convergence, this result is not completely satisfactory since renormalization has not been dealt with.

It is our goal to show that the renormalization friendly variational method developed in Ref. [5] generalizes to finite temperatures. In this first investigation we restrict ourselves to the large N limit GN model where the calculational scheme can be set up more clearly. Then, in a subsequent work [15] we shall treat the technically more complex finite N limit which is interesting due to Landau's theorem.

In the next section we review the usual large N result for the temperature dependent fermionic mass and perform the variational calculation to lowest orders. Because we are in the large N limit the renormalization problems discussed above will not show up explicitly although we will be able to see another unwanted problem arising during the optimization process. In Section III we follow Ref. [5] to rectify the situation by performing the variational calculation to all orders. When this is done the variational method reproduces the large N result for chiral symmetry restoration in the GN model. The conclusions and future perspectives are presented in section IV.

II. THE VARIATIONAL CALCULATION TO LOWEST ORDERS

The variational calculation starts with the addition of an arbitrary bare mass (m_0) to the original massless Gross-Neveu [4]

$$\mathcal{L} = i \sum_{i=1}^{N} \bar{\psi}_i \not{\partial} \psi_i + m_0 \sum_{i=1}^{N} \bar{\psi}_i \psi_i + \frac{g_0^2}{2} \left(\sum_{i=1}^{N} \bar{\psi}_i \psi_i \right)^2 \tag{2}$$

where g_0 is the bare coupling constant (in the following we shall suppress the summation over the index i). The relation to the linear δ expansion and other variational methods becomes clear by performing the substitutions

$$m_0 \to m_0(1-\delta) \quad , \tag{3}$$

$$g_0^2 \to \delta g_0^2 \; . \tag{4}$$

These will be done at a later stage in order to avoid the explicit evaluation of Feynman graphs which differ only by δm_0 insertions. To perform finite temperature calculations in the imaginary time formalism one does the following substitutions [16]

$$\int \frac{dp_0}{2\pi} \to iT \sum_n \quad , \tag{5}$$

$$p_0 \to i\omega_n \quad , \tag{6}$$

where for fermions

$$\omega_n = T(2n+1)\pi \quad . \tag{7}$$

The sum over Matsubara's frequencies can be performed with

$$T\sum_n \ln(\omega_n^2 + m_0^2) = E + 2T\ln[1+\exp(-E/T)] + c \quad , \tag{8}$$

where c is a E-independent constant [16]. In 1+1 dimensions

$$E = (p_1^2 + m_0^2)^{\frac{1}{2}} \quad . \tag{9}$$

The remaining space integral is evaluated in $1-\epsilon$ dimensions using conventional dimensional regularization techniques.

In the large N limit a perturbative calculation of the fermionic mass (M_F) to $\mathrm{O}(g_0^4)$ yields

$$M_F^{(2)} = m_F^{(0)} + g_0^2 m_F^{(1)} + g_0^4 m_F^{(2)} + \mathrm{O}(g_0^6) \quad , \tag{10}$$

where $m_F^{(0)} = m_0$,

$$m_F^{(1)} = \frac{N}{2\pi} m_0^{1-\epsilon}\left[(4\pi)^{\frac{\epsilon}{2}}\Gamma(\epsilon/2) - 4I_1^T(y_0)\right] \quad , \tag{11}$$

and

$$\begin{aligned} m_F^{(2)} = \frac{N^2}{4\pi^2} m_0^{1-2\epsilon} \Big\{ & (4\pi)^{\epsilon}\Gamma^2(\epsilon/2)(1-\epsilon) \\ & + (4\pi)^{\frac{\epsilon}{2}}\Gamma(\epsilon/2)\left[(\epsilon-2)I_1^T(y_0) + (m_0/T)^2(I_2^T(y_0) + I_3^T(y_0))\right] \\ & + I_1^T(y_0)\left[I_1^T(y_0) - (m_0/T)^2(I_2^T(y_0) + I_3^T(y_0))\right]\Big\} \quad . \end{aligned} \tag{12}$$

The temperature dependent integrals are

$$I_1^T(y_0) = \frac{\pi^{\frac{1-\epsilon}{2}}}{\Gamma(1/2-\epsilon/2)}(2\pi y_0)^{\epsilon}\int_0^{\infty} dx \frac{x^{-\epsilon}}{(x^2+y_0^2)^{\frac{1}{2}}[1+\exp(x^2+y_0^2)^{\frac{1}{2}}]} \quad , \tag{13}$$

$$I_2^T(y_0) = \frac{\pi^{\frac{1-\epsilon}{2}}}{\Gamma(1/2-\epsilon/2)}(2\pi y_0)^\epsilon \int_0^\infty dx \frac{x^{-\epsilon}}{(x^2+y_0^2)^{\frac{3}{2}}[1+\exp(x^2+y_0^2)^{\frac{1}{2}}]} \quad , \tag{14}$$

and

$$I_3^T(y_0) = \frac{\pi^{\frac{1-\epsilon}{2}}}{\Gamma(1/2-\epsilon/2)}(2\pi y_0)^\epsilon \int_0^\infty dx \frac{x^{-\epsilon}\exp(x^2+y_0^2)^{\frac{1}{2}}}{(x^2+y_0^2)[1+\exp(x^2+y_0^2)^{\frac{1}{2}}]^2} \quad , \tag{15}$$

with $y_0 = m_0/T$ and $x = p_1/T$. These integrals are related to each other via

$$---2y_0^\epsilon \frac{\partial(y_0^{-\epsilon} I_1^T)}{\partial y_0^2} = I_2^T + I_3^T \quad . \tag{16}$$

The nonperturbative large N calculation consists in the evaluation of all cactus diagrams which can be summed up as

$$M_F = m_0 \left\{ 1 - \frac{N}{2\pi} g_0^2 M_F^{-\epsilon} \left[(4\pi)^{\frac{\epsilon}{2}} \Gamma(\epsilon/2) - 4 I_1^T(y_F) \right] \right\}^{-1} \quad , \tag{17}$$

where $y_F = M_F/T$. The bare and renormalized parameters are related via

$$m_0 = Z_m m \quad , \tag{18}$$

and

$$g_0^2 = Z_g g^2 \mu^\epsilon \quad , \tag{19}$$

where μ is the arbitrary scale introduced by dimensional regularization. The renormalization constants are

$$Z_m = Z_g = \left[1 + \frac{g^2 N}{\pi \epsilon} \right]^{-1} \quad . \tag{20}$$

Substituting Eqs. (18) and (19) into Eq. (17) yields the finite expression for the dimensionless quantity $M_F/\Lambda_{\overline{MS}}$

$$\frac{M_F}{\Lambda_{\overline{MS}}} = \frac{m}{\Lambda_{\overline{MS}}} \left\{ 1 + \frac{N}{\pi} g^2 \left[\ln\left(\frac{M_F}{\bar{\mu}} \right) + 2 I_1^T(y_F) \right] \right\}^{-1} \quad , \tag{21}$$

where $\Lambda_{\overline{MS}} = \bar{\mu}\exp[-\pi/(Ng^2)]$ and $\bar{\mu} = \mu(4\pi)^{\frac{1}{2}}\exp(-\gamma_E/2)$ with $\gamma_E = 0.577215$. Equation (21) satisfies the RG equation

$$\mu \frac{d}{d\mu} M_F = \left(\mu \partial y_0^2 = I_2^T + I_3^T \frac{\partial}{\partial \mu} + g\beta(g) \frac{\partial}{\partial g} - m\gamma_m(g) \frac{\partial}{\partial m} \right) M_F = 0 \quad , \tag{22}$$

where

$$\beta(g) = -\frac{N}{2\pi} g^2 \quad \text{and} \quad \gamma_m(g) = \frac{N}{\pi} g^2 \quad . \tag{23}$$

Then, in the chiral limit ($m = 0$), the finite temperature mass gap equation is given by

$$M_F(T) = M_F(0) \exp[-2I_1^T(y_F)] \ , \tag{24}$$

where $M_F(0) = \Lambda_{\overline{MS}}$. This is just the large N result which reproduces the phase diagram for the fermion mass [11].

The variational calculation starts with the substitutions Eqs. (3) and (4) . With this procedure Eq. (2) interpolates between the original massless GN model (at $\delta = 1$) and a massive free theory (at $\delta = 0$). The perturbative calculation is now done in powers of the bookkeeping parameter δ and extremized with respect to m_0 at $\delta = 1$ (PMS). Starting with O(δ) one has at $\delta = 1$

$$M_F^{(1)}(m_0) = m_F^{(0)} + g_0^2 m_F^{(1)} - m_0 \frac{\partial m_F^{(0)}}{\partial m_0} = g_0^2 m_F^{(1)} \quad , \tag{25}$$

which has no nontrivial extremum in m_0 at $T = 0$ nor at finite temperatures. At $\delta = 1$ the O(δ^2) fermionic mass is given by

$$M_F^{(2)}(m_0) = g_0^2 m_F^{(1)} - g_0^2 m_0 \frac{\partial m_F^{(1)}}{\partial m_0} + g_0^4 m_F^{(2)} \quad . \tag{26}$$

The extremization with respect to m_0 in the limit $\epsilon \to 0$ does not give any useful information since the term $g_0^4 m_F^{(2)}$, which has no nontrivial extremum, dominates. This behaviour, which persists to higher orders, has also been noted at zero temperature [5] and we find that the situation does not change at finite temperatures.

III. THE VARIATIONAL CALCULATION TO ALL ORDERS.

In this section we shall follow Ref. [5] to perform an all order variational calculation in the large N limit eliminating the optimization problem encountered in the previous section. The general philosophy within variational methods is to start with a trial value which is expected to be reasonably close to the true value of the physical parameters. In our case this means that we can start by formulating a nonperturbative ansatz which already resums a good part of the RG behaviour of the fermionic mass before launching into the actual variational calculation. Of course this is a rather easy task within the large N limit, where the exact answer Eq. (17) constitutes the natural choice. Performing the substitutions Eqs. (3) and (4) in Eq. (17) we get

$$M_F(f) = \frac{m_0(1-\delta)}{f(\delta)} \quad , \tag{27}$$

where we have defined

$$f(\delta) = 1 - \frac{N}{2\pi}\delta g_0^2 m_0^{-\epsilon}(1-\delta)^{-\epsilon} f^{\epsilon}\left[(4\pi)^{\frac{\epsilon}{2}}\Gamma(\epsilon/2) - 4I_1^T(y_F')\right] \quad , \tag{28}$$

with $y_F' = M_F(f)/T$. It is now possible to perform an expansion in powers of δ to order-n around the free theory ($\delta = 0$). Using contour integration one obtains M_F to $n^{\rm th}$ order of perturbation theory

$$M_F^{(n)}(m_0) = \frac{1}{2\pi i}\oint dz (\frac{1}{z} + \frac{\delta}{z^2} + \ldots + \frac{\delta^n}{z^{n+1}})\frac{m_0(1-z)}{f(z)} \quad , \tag{29}$$

which, at $\delta = 1$, gives

$$M_F^{(n)}(m_0) = \frac{1}{2\pi i}\oint \frac{dz}{z^{n+1}}\frac{m_0}{f(z)} \quad . \tag{30}$$

Once Z_m and Z_g are applied to the bare m_0 and g_0^2 one gets the finite expression for the dimensionless quantity $M_F/\Lambda_{\overline{MS}}$

$$\frac{M_F^{(n)}(m)}{\Lambda_{\overline{MS}}} = \frac{1}{2\pi i}\oint \frac{dz}{z^{n+1}}\frac{m}{\Lambda_{\overline{MS}}}\left\{1 + \frac{N}{\pi} g^2 z \left[\ln\left(\frac{M_F(f)}{\bar{\mu}}\right) + 2I_1^T(y_F')\right]\right\}^{-1} \quad . \tag{31}$$

As noted in the case of the anharmonic oscillator [17], it is possible to extract more structure from the limit of infinite order by rescalling m with the order n. After distortion of the contour it is clear that only the vicinity of $z = 1$ survives in the limit $n \to \infty$, which can be analyzed by changing variables

$$1 - z = \frac{v}{n} \quad . \tag{32}$$

Rescalling m by introducing $m' = m/n$ we get, in the $n \to \infty$ limit

$$\frac{M_F(m'')}{\Lambda_{\overline{MS}}} = \frac{1}{2\pi i} \oint \frac{dv e^v m''}{K(v)} \quad , \tag{33}$$

where the integration runs counterclockwise around the negative real axis. The function $K(v)$ is given by

$$K(v) = \ln\left(\frac{m'' v}{K(v)}\right) + 2 I_1^T \left(\frac{m'' v}{t K(v)}\right) \quad , \tag{34}$$

where

$$m'' = \frac{m'}{\Lambda_{\overline{MS}}} \left(\frac{N g^2}{\pi}\right)^{-1} \quad , \tag{35}$$

with $t = T/\Lambda_{\overline{MS}}$. Equations (33) and (34), which summarize our variational approach, should be understood as follows: for a given variational parameter m'', and a given temperature T, Eq. (34) enables one to determine $K(v)$ self consistently. The variational result is then given in an explicitly RG invariant way by extremizing Eq. (33) with respect to m''. In general this program has to be achieved numerically. However, before doing that one can use the fact that the large N limit is free from infra red divergences to perform an analytical exploitation of the $m'' \to 0$ limit where the integral is dominated by the $v \sim 0$ region. Simple considerations show that for $T < T_c$

$$K(v) \underset{m'' \to 0}{\sim} m'' v \frac{\Lambda_{\overline{MS}}}{M_F(T)} \quad , \tag{36}$$

where $M_F(T)$ is given in Eq. (24). Moreover, $K(v)$ has a cut starting at a negative value of v and lying along the negative real axis. Hence, the integral Eq. (33) converges exponentially to the expected result as $m'' \to 0$. As $T \to T_c$, the branching point approaches the value $v = 0$ merging to it at $T = T_c$. At this point the integral becomes divergent at $m'' = 0$ and does not allow more extrema for $T > T_c$. Numerical results obtained at different temperatures (see Fig. 1) indicate that $m'' = 0$ is in fact the only real extremum. The standard large N structure of the phase transition is then clearly reproduced .

IV. CONCLUSIONS

In this letter we have shown that a recently proposed variational method can be successfully generalized to the finite temperature domain. This nonperturbative scheme respects the renormalization program of the theory, avoiding potential problems which may arise in the application of variational methods to quantum field theories. For comparison purposes, and to introduce the method in the study of finite temperature chiral symmetry restoration we have chosen to start with the large N limit of the Gross-Neveu model. We have seen that optimization problems encountered in the zero temperature low order variational calculation persist at finite temperatures. Then by applying the all order variational calculation scheme developed in Ref. [5] we were able to recover exactly the usual large N result for chiral symmetry restoration in the GN model. The finite N case, which is also interesting due to Landau's theorem, is more complex and will be discussed in a forthcoming work.

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FIGURES

FIG. 1. $M_F(T)/M_F(0)$ as a function of the arbitrary parameter m'' for different temperatures. From top to bottom the curves represent $t = 0.1$, $t = 0.5$ and $t = 0.55$ respectively.