Does one need the anomaly to describe the polarized structure functions?

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Abstract

The SLAC data on the p, d and n polarized structure functions are fairly well reproduced with and without the contribution of the anomaly. The results are compared with a previous study based mainly on SMC data. The implications on the solution of the spin-crisis are discussed.

The EMC experiment [1] on polarized deep inelastic scattering muon-proton at $<Q^2>= 10.7$ GeV^2 led to the result

$$I_p = \int_0^1 g_1^p(x)dx = 0.126 \pm 0.010 \pm 0.015 \qquad (EMC/SLAC), \qquad (1)$$

smaller than the value predicted by the Ellis and Jaffe sum rule [2]

$$\frac{F}{2} - \frac{D}{18} = 0.185 \pm 0.001, \qquad (2)$$

and gave rise to the "spin crisis". An explanation of this result has been proposed in terms of the anomaly of the axial vector current related to the gluon polarization ΔG [3], but, to recover the EMC result, the rather high value of $\Delta G \sim 5$ at $<Q^2>= 10\, GeV^2$ was necessary, as pointed out in Ref. [4], to be compensated by a large negative orbital angular momentum $<L_z>$. This large component of the proton spin carried by the gluon and this compensating mechanism is far from being natural, but the Bjorken sum rule [5] including first order QCD corrections,

$$I_p - I_n = \frac{1}{6}(F+D)\left(1 - \frac{\alpha_s}{\pi}\right) = \frac{1}{6}\frac{g_A}{g_V}\left(1 - \frac{\alpha_s}{\pi}\right), \qquad (3)$$

would be obeyed and the generally accepted framework to describe deep inelastic phenomena would not be spoiled.

The EMC result, assuming the validity of the Bjorken sum rule, would imply a large negative value for the neutron Ellis and Jaffe sum rule

$$I_n = \int_0^1 g_1^n(x)dx, \qquad (4)$$

while the measured value at SLAC by E142 at $<Q^2>= 2\, GeV^2$ [6] with a polarized He^3 target, is

$$I_n = -0.022 \pm 0.011, \qquad (5)$$

in good agreement with the prediction of Ellis and Jaffe in the absence of the anomaly contribution:

$$\frac{F}{3} - \frac{2}{9}D = -0.022 \pm 0.004. \qquad (6)$$

Higher twist operators have been advocated [7] to play an important role in the interpretation of the E142 data, which has been taken at the rather small $<Q^2>= 2\, GeV^2$.

More recently, at CERN [8] and SLAC [9] new deep inelastic scattering experiments have been performed with polarized proton and deuteron targets. An analysis of the available data has been achieved by Gehrmann and Stirling [10] to describe the $g_1(x)$ distributions. In the framework of the explanation of the defect in the Ellis and Jaffe sum rule for I_p by means of the gluon anomaly, they try to describe $g_1^p(x)$ and $g_1^n(x)$ in terms of the contributions of the valence quarks, $\Delta u_v(x)$ and $\Delta d_v(x)$, and of $\Delta G(x)$:

$$g_1^p(x, Q^2) = \frac{2}{9}\Delta u_v(x, Q^2) + \frac{1}{18}\Delta d_v(x, Q^2) - \frac{1}{3}\frac{\alpha_s(Q^2)}{2\pi}\Delta G(x, Q^2), \qquad (7)$$

$$g_1^n(x, Q^2) = \frac{1}{18}\Delta u_v(x, Q^2) + \frac{2}{9}\Delta d_v(x, Q^2) - \frac{1}{3}\frac{\alpha_s(Q^2)}{2\pi}\Delta G(x, Q^2), \qquad (8)$$

with

$$
\begin{aligned}
x\Delta u_v &= \eta_u A_u x^{a_u}(1-x)^{b_u}(1+\gamma_u x) \\
x\Delta d_v &= \eta_d A_d x^{a_d}(1-x)^{b_d}(1+\gamma_d x) \\
x\Delta G &= \eta_G A_G x^{a_G}(1-x)^{b_G}(1+\gamma_G x)
\end{aligned}
\tag{9}
$$

and $A_q = A_q(a_q, b_q, \gamma_q)$ (q = u,d,G) in such a way that

$$
\int_0^1 \Delta q(x)dx = \eta_q.
\tag{10}
$$

At $Q_0^2 = 4\,GeV^2$, lower limit for the data considered in order to avoid the uncertainty on the higher twist contributions, the $\eta's$ are fixed by [10] $(\alpha_s(4\,GeV^2) = 0.2879)$:

$$
\eta_u(Q_0^2) = 2\tilde{F}(Q_0^2) = 2\left[\left(1 - \frac{3\alpha_s(Q_0^2)}{5\pi}\right)F - \frac{2\alpha_s(Q_0^2)}{15\pi}D\right] = 0.848 \pm 0.016
\tag{11}
$$

$$
\begin{aligned}
\eta_d(Q_0^2) &= \tilde{F}(Q_0^2) - \tilde{D}(Q_0^2) = \left(1 - \frac{\alpha_s(Q_0^2)}{5\pi}\right)F - \left(1 - \frac{11\alpha_s(Q_0^2)}{15\pi}\right)D \\
&= -0.294 \pm 0.011
\end{aligned}
\tag{12}
$$

$$
\eta_G(Q_0^2) = 1.971 \pm 0.929.
\tag{13}
$$

With theoretical and phenomenological motivations they require

$$
\begin{aligned}
a_u &= a_d \quad a_G = 1 \\
b_d &= b_u + 1 = 4.64 \\
\gamma_u &= \gamma_d
\end{aligned}
\tag{14}
$$

and they obtain a very good fit to the data, $\chi^2/N_{DF} = 0.63$, corresponding to the parameters

$$
\begin{aligned}
a_u &= 0.46 \quad \pm \quad 0.15, \\
\gamma_u &= 18.36 \quad \pm \quad 14.49, \\
b_G &= 7.44 \quad \pm \quad 3.52,
\end{aligned}
\tag{15}
$$

with the choice $\gamma_G = 0$.

As the authors stress, the bulk of the SLAC data falls outside their fit region, excluding almost all the E142 neutron data. Given the large uncertainty on the SMC deuteron data, one can conclude that the constraint coming from $g_1^n(x)$ is not very strong and the fair description obtained for $g_1^p(x)$ is not surprising, once η_u, η_d, and η_G are chosen consistently with the experimental value found for I_p.

In order to test the Bjorken sum rule, which is the most appealing target of experiments on polarized structure functions, the value of I_n should be measured and one should try to reproduce $g_1^n(x)$.

Concerning the exclusion of data below $Q^2 = 4\,GeV^2$, because of the poor theoretical knowledge on the higher twist contributions, it is worth observing that the values of I_p measured at $<Q^2>= 3\,GeV^2$ at SLAC [9], $0.127 \pm 0.004 \pm 0.010$, and at $<Q^2>= 10\,GeV^2$ at CERN [8], $0.136 \pm 0.011 \pm 0.011$, are consistent, showing that for the proton one may safely neglect higher twist contributions.

Here we try to describe the SLAC data on proton and deuteron at $<Q^2>= 3\,GeV^2$ and on neutron at $<Q^2>= 2\,GeV^2$ in terms of Δu_v, Δd_v, and ΔG, given by Eq. (9), with the values of η_u and η_d scaled down to $Q_0^2 = 3\,GeV^2$. We are using the values of F and D of Ref. [11] and $\alpha_s(3\,GeV^2) = 0.35 \pm 0.05$,

$$\tilde{\eta}_u = 2\tilde{F} = 0.835 \pm 0.022\,, \tag{16}$$

$$\tilde{\eta}_d = \tilde{F} - \tilde{D} = -0.276 \pm 0.020\,, \tag{17}$$

and

$$\tilde{\eta}_G = \frac{6\pi}{\alpha_s}\left(\frac{2}{9}\eta_u + \frac{1}{18}\eta_d - I_p\right) = 2.3 \pm 0.7 \tag{18}$$

to recover the measured value for $I_p = 0.127 \pm 0.004 \pm 0.010$ [9].

As in Ref. [10] we fix $a_u = a_d$, $\gamma_u = \gamma_d$, $a_G = 1$, and further, we require

$$b_u > 1, \quad b_d > 3, \quad b_G > 5, \quad \gamma_u = \gamma_d > 0, \quad \text{and} \quad \gamma_G > 0. \tag{19}$$

The lower limits for b_u and b_d are smaller than the values chosen for these parameters in Ref. [10], as shown in our equation (14): the smaller one for b_u is taken since $u^\uparrow$ dominates at high x. The lower limit on b_G, of the order of the corresponding exponent for G in Eq. (15), has also the purpose to prevent $\Delta G(x)$ from carrying too much proton momentum (the momentum carried by ΔG should of course be less than the momentum carried by G). In order to have a definite sign for each spin asymmetry we do not allow values of $\gamma < -1$. In the range $(-1, 0)$ there is a strong correlation between the values of γ and b (in particular $\gamma = -1$ and b give the same function than $\gamma = 0$ and $b+1$). The large error in the determination of γ_u in Ref. [10] shows that this parameter does not play a crucial role and therefore we assume all the $\gamma's$ to be positive.

To keep into account the D-state admixture for deuterium, we take

$$g_1^d(x) = \frac{1}{2}\left(1 - \frac{3}{2}\omega_D\right)\left(g_1^p(x) + g_1^n(x)\right)\,, \tag{20}$$

where $\omega_D = 0.058$ [12].

The best fit to the SLAC polarized structure functions of the theoretical expressions given by Eqs. (7)-(10) and (16)-(20) is found, with a $\chi^2/N_{DF} = 1.8$, corresponding to the following values

for the parameters ($a_G = 1$):

$$
\begin{aligned}
a_u = a_d &= 0.57 \pm 0.03 \\
b_u &= 2.0 \pm 0.3 \\
b_d &= 3.0 \pm 0.1 \\
b_G &= 20 \pm 1 \\
\gamma_u = \gamma_d &= 1.0 \pm 0.8 \\
\gamma_G &= 0.0 \pm 1.0
\end{aligned}
\tag{21}
$$

The resulting predictions are compared with SLAC data in Figs. 1, 2, 3, showing a tendency to get large negative values at small x for g_1^d, g_1^n, just in the region where the negative contributions of ΔG are more important. The behaviour of $x\Delta u(x)$, $x\Delta d(x)$, and $x\Delta G(x)$ is described in Fig. 4.

If one leaves η_G free, one gets a better fit, $\chi^2/N_{DF} = 1.0$, with ($a_G = 1$)

$$
\begin{aligned}
a_u = a_d &= 0.40 \pm 0.08 \\
b_u &= 1.8 \pm 0.2 \\
b_d &= 3.0 \pm 0.5 \\
b_G &= 20 \pm 15 \\
\gamma_u = \gamma_d &= 5.3 \quad {}^{+5.4}_{-2.7} \\
\gamma_G &= 0 \pm 18 \\
\eta_G/\tilde{\eta}_G &= 0.48 \pm 0.09
\end{aligned}
\tag{22}
$$

Finally, with $\eta_G = 0$ and η_u and η_d free, the best fit, with $\chi^2/N_{DF} = 1.0$, is obtained with

$$
\begin{aligned}
a_u = a_d &= 0.98 \pm 0.09 \\
b_u &= 1.8 \quad {}^{+0.5}_{-0.2} \\
b_d &= 4.9 \pm 0.8 \\
\gamma_u = \gamma_d &= 0 \pm 3 \\
\eta_u/\tilde{\eta}_u &= 0.76 \pm 0.03 \\
\eta_d/\tilde{\eta}_d &= 0.93 \pm 0.08
\end{aligned}
\tag{23}
$$

The predictions corresponding to the values in Eqs. (22) and (23) are also compared with the SLAC data in Figs. 1, 2, 3. The parameters given in Eqs. (21) and (23) are reported in Table 1 together with the parameters of reference [10], hereafter referred as *Fit A*, *Fit B*, and *Fit C* respectively. It is worth stressing that b_u and b_d in *Fit A* come out smaller than the values fixed in Ref. [10], while b_G comes out larger than the corresponding value found in Ref. [10] and the total proton momentum carried by ΔG in *Fit A* is about twice smaller than in *Fit C*.

The values of η_u and η_d of *Fit B* support a different interpretation for the defect in the Ellis and Jaffe sum rule for the proton, which has been related to the defect in the Gottfried sum rule and to the role of Pauli principle for parton distributions [13]. It amounts to say that the $u^\uparrow$ parton,

largely the most abundant in the proton, receives less contribution from the sea because its energy levels are almost completely occupied. Indeed, the role of Pauli principle in parton distributions explains, in a natural way, the dominance of $u^\uparrow$ at high x, shown by the increase towards 1 of the asymmetry parameter $A_1^p(x)$ at high x and also the fast decrease in the same limit of the ratio $F_2^n(x)/F_2^p(x)$. This was known since the time of the phenomenological analysis performed by Field and Feynman [14], who observed that Pauli principle may be responsible for less $u\bar{u}$ pairs than $d\bar{d}$ in the proton, just due to the presence of two valence u quarks and only one d quark. The defect in the Gottfried sum rule confirms their conjecture and it is natural to assume that the $u^\uparrow$ quark rather than the $u^\downarrow$ quark receives less contribution from the sea. The fact that η_u, reported in Eq. (23), is smaller by 25% than the expected value, while the reduction of η_d is less relevant, complies with this picture.

The importance of the role of Pauli principle in the parton distributions therefore suggests to describe them as Fermi-Dirac functions [15, 16]:

$$p(x) = f(x) \left[exp \left\{ \frac{x - \tilde{x}(p)}{\bar{x}} \right\} + 1 \right]^{-1} \tag{24}$$

where $\bar{x}$, $\tilde{x}(p)$, and $f(x)$ play the role of the "temperature", "thermodynamical potential", depending on the flavour and on the spin of the parton, and "weight function" for the density of levels in the x variable.

Two models have been proposed with the common feature of relating the shape of the distributions of the different partons to their first moments (broader shapes for larger first moments). In one of them [15] the polarized distributions are found, without additional parameters, from the unpolarized distributions with the assumptions:

$$\Delta u(x) + \Delta \bar{u}(x) = u(x) + \bar{u}(x) - d(x) - \bar{d}(x) \tag{25}$$

$$\Delta d(x) = (F - D)\left(d(x) - \bar{d}(x)\right) \tag{26}$$

$$\Delta \bar{d}(x) = 0 \tag{27}$$

In the second one [16] the potentials of u and d of both helicities are put as parameters in order to describe unpolarized and polarized available distributions.

Eq. (25) would imply, with only u, d and their antiparticles contributing to the polarized structure functions,

$$x \left[g_1^p(x) - \frac{1}{4} g_1^n(x) \right] = \frac{5}{8} \left[F_2^p(x) - F_2^n(x) \right] . \tag{28}$$

The distributions found in [16], where the nucleon unpolarized structure functions $F_2^p(x)$, $F_2^n(x)$,

6

$x\bar{q}(x)$, and $xF_3(x)$ were also described, give the following results

$$\Delta u(x) \;\; = \;\; 2.66\,x^{-0.203}\,(1-x)^{2.34}\left(\left[exp\left\{\frac{x-1.00}{0.235}\right\}+1\right]^{-1}-\left[exp\left\{\frac{x-0.123}{0.235}\right\}+1\right]^{-1}\right)$$

$$\Delta d(x) \;\; = \;\; 2.66\,x^{-0.203}\,(1-x)^{2.34}\left(\left[exp\left\{\frac{x+0.068}{0.235}\right\}+1\right]^{-1}-\left[exp\left\{\frac{x-0.200}{0.235}\right\}+1\right]^{-1}\right),$$

$$(29)$$

in good agreement with the Eq. (28) and with *Fit B*, as shown in Fig. 5. [1]

We conclude that the SLAC data might be fairly described in terms of $\Delta u(x)$ and $\Delta d(x)$, with a smaller first moment for $\Delta u(x)$ than expected. We cannot, however, conclude that the SLAC data give evidence against the Bjorken sum rule. In fact, with η_u and η_d given in Eqs. (16) and (17), it is possible to find a good fit to the distributions by leaving η_G free with a value $1/2$ of the rhs of Eq. (18).

Concerning the very important issue of the validity of the Bjorken sum rule, it is worth stressing that the SMC data on $g_1^d(x)$ imply a value for I_n more negative than the SLAC data with polarized He^3, expecially from the contributions of $g_1^n(x)$ in the small x region not yet measured by SLAC. More precise measurements in this region would be very crucial to clarify this point: indeed, two new experiments at SLAC, E154 and E155, will be running this year and next year with an electron beam energy of 50 GeV/c, and we hope their results will help solving this important problem. Precise measurements in the small x region are in fact very relevant also since the negative contribution from the gluons advocated to account for the defect in the Ellis and Jaffe sum rule for the proton is expected to dominate the small x region.

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[1] The values for b_u in Eqs. (21), (22) and (23) are smaller than the exponents fixed in Ref. [10] by the positivity requirement that $\Delta u(x)$ should be smaller than parameterization assumed there for $u(x)$.
We think there is no problem of positivity since $\Delta u(x)$ as obtained by Eqs. (9) and (23) is in good agreement, as shown by Fig. 5, with $\Delta u(x) = u^{\uparrow}(x) - u^{\downarrow}(x)$ of Ref. [16] shown in Eq. (29), with $u(x) = u^{\uparrow}(x) + u^{\downarrow}(x)$ in good agreement with the experimental unpolarized distributions.

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Figure Captions

Figure 1. The predictions about $x\, g_1^p(x)$ corresponding to *Fit A* (continuous line), *Fit B* (dashed line) and to the values of the parameters reported in Eq. (22) (dotted line) are compared with SLAC data.

Figure 2. The predictions about $x\, g_1^n(x)$ corresponding to *Fit A* (continuous line), *Fit B* (dashed line) and to the values of the parameters reported in Eq. (22) (dotted line) are compared with SLAC data.

Figure 3. The predictions about $x\, g_1^d(x)$ corresponding to *Fit A* (continuous line), *Fit B* (dashed line) and to the values of the parameters reported in Eq. (22) (dotted line) are compared with SLAC data.

Figure 4. The quantities $x\, \Delta u(x)$ (continuous line), $x\, \Delta d(x)$ (dashed line) and $x\, \Delta G(x)$ (dotted line) corresponding to the *Fit A* are shown.

Figure 5. The predictions of $x\, \Delta u(x)$ (continuous line) and $x\, \Delta d(x)$ (dashed line) obtained from Ref. [16] are compared with $x\, \Delta u(x)$ (dotted line) and $x\, \Delta d(x)$ (dashed-dotted line) corresponding to *Fit B*.

TABLE I

	Fit A	*Fit B*	*Fit C*
χ^2/N_{DF}	1.8	1.0	0.63
$a_u = a_d$	0.57 ± 0.03	0.98 ± 0.09	0.46 ± 0.15
b_u	2.0 ± 0.3	$1.8 \ {}^{+0.5}_{-0.2}$	3.64 (fixed)
b_d	3.0 ± 0.1	4.9 ± 0.8	4.64 (fixed)
$\gamma_u = \gamma_d$	1.0 ± 0.8	0 ± 3	18.36 ± 14.49
a_G	1 (fixed)	-	1 (fixed)
b_G	20 ± 1	-	7.44 ± 3.52
γ_G	0 ± 1	-	0 (fixed)
$\eta_u/\tilde{\eta}_u$	1 (fixed)	0.76 ± 0.03	1 (fixed)
$\eta_d/\tilde{\eta}_d$	1 (fixed)	0.93 ± 0.08	1 (fixed)
$\eta_G/\tilde{\eta}_G$	1 (fixed)	0 (fixed)	1 (fixed)
$\Delta u^{(2)}$	0.16	0.17	0.15
$\Delta d^{(2)}$	-0.039	-0.037	-0.042
$\Delta G^{(2)}$	0.10	0	0.24

Table 1: We report, for a comparison, the fitted values for the parameters of the polarized parton distributions and their first and second momenta for the cases discussed in the paper. *Fit C* is the one of Ref. [10] based mainly on SMC data. *Fit A* concerns SLAC data with mainly the same ansatz of Ref. [10], while *Fit B* concerns the same data assuming no gluon anomaly contribution.