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Approximate Flavour Symmetries and See-Saw Mechanism

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Abstract

We study the approximate flavour symmetries imposed on the lepton sector assuming see-saw mechanism as the neutrino mass structure. We apply the symmetry to various neutrino phenomenologies and obtain constraints on neutrino masses and mixings.

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I. INTRODUCTION

In the standard model (SM), there exists one Higgs doublet as the only source of the electroweak symmetry breaking. However many extensions of the SM need more complex Higgs structure. When we suggest additional Higgs bosons, we face some constraints. One of the major issues with respect to multi-Higgs models is suppressions of flavour-changing neutral currents (FCNC) mediated by the exchange of neutral Higgs bosons. Strong limits on FCNC have been given from several experiments such as low rates of $K_L \to \mu^+\mu^-$, limits for $K^+ \to \pi^+\nu\bar{\nu}$, $D^0 \to \mu^+\mu^-$, $B^0 \to \mu^+\mu^-$ processes and $K^0 - \bar{K}^0$, $B^0 - \bar{B}^0$ mixings.

Following Glashaw-Weinberg criteria one usually propose to impose a discrete symmetry on the multi-Higgs models under which no Higgs doublet couples to both u-type and d-type quark sectors. This restriction is not inevitable, however, we can give up the discrete symmetry and impose flavour symmetries on fermions to suppress FCNC instead. Without Yukawa interaction terms, the SM lagrangian manifests a set of global U(1) flavour symmetries. When we switch on Yukawa couplings, the flavour symmetries are weakly broken and we have approximate symmetries because of smallness of Yukawa couplings. Recently it is proposed that the violations of the flavour symmetries are parametrized by a set of small parameters $\{\epsilon\}$ [1–3]. It turns out that proper choice of the small parameters can give suppressions of FCNC.

Hall and Weinberg [2] provide the analysis with particular formulars for the small parameters on quark sectors from observed masses and mixing angles. They impose the suppression factors ϵ_{Qi} for the left-handed quark doublets, ϵ_{ui} and ϵ_{di} for the up-type and down-type right-handed quarks respectively. Then the Yukawa coupling constant λ^U_{ijn} is doubly suppressed by both ϵ_i and ϵ_j such that the Yukawa couplings are of order $\lambda^U_{ijn} \approx \epsilon_{Q_i}\epsilon_{u_j}$ and $\lambda^D_{ijn} \approx \epsilon_{Q_i}\epsilon_{d_j}$ where n is the index related to Higgs bosons. They also show that CP must also be a good approximate symmetry in their framework. The problem of CP violation from multi-Higgs potential with the approximate flavour symmetries is also pointed out in the recent work by Wu and Wolfenstein [4].

On the lepton sector, it is hard to specify the leptonic suppression factors explicitly due to ignorance of lepton mixing angles and neutrino masses. Assuming neutrino masses with the famous see-saw mechanism [5], some statements can be made. Rašin and Silva [3] studied the approximate flavour symmetries in the lepton sector under the see-saw mechanism and gave some predictions of neutrino masses and mixings with additional Ansätze.

In this letter we study the approximate flavour symmetries on lepton sectors with the see-saw mchanism in detail and apply them to several neutrino phenomenologies. We show that the possible windows for the neutrino masses and mixings obtained by the MSW solution for the solar neutrino problem can give some constraints on the neutrino mass under the assumption of the approximate flavour symmetries. We also discuss the possible existence of the neutrinoless double beta decays, $(\beta\beta)_{0\nu}$ and possibilities to find the neutrino oscillation in CHORUS.

II. SEE-SAW MECHANISM WITH APPROXIMATE FLAVOUR SYMMETRIES

We consider the neutrino mass terms with see-saw mechanism. After the spontaneous symmetry breaking the mass terms of neutrinos are given by

$$\mathcal{L}_\nu = -\frac{1}{2}\bar{\nu}^c_{Li} M^L_{ij} \nu_{Lj} - \frac{1}{2}\bar{\nu}_{Ri} M^R_{ij} \nu^c_{Rj} - \bar{\nu}_{Ri} M^D_{ij} \nu_{Lj} \tag{1}$$

where the mass matrix M^L_{ij} is of the left-handed Majorana neutrinos, M^R_{ij} of the right-handed Majorana neutrinos and M^d_{ij} of the Dirac neutrnos. Let us assume that $M^L \sim 0$, $M^D \sim M_{fermions}$ and $M^R \gg M_{fermions}$. We write it as the compact form

$$\mathcal{L}_\nu = -\frac{1}{2}\left(\bar{\nu}^c_L, \bar{\nu}_R\right) M \begin{pmatrix} \nu_L \\ \nu^c_R \end{pmatrix} \tag{2}$$

where

$$M = \begin{pmatrix} 0 & M^D \\ M^{D\dagger} & M^R \end{pmatrix} \equiv \begin{pmatrix} 0 & m \\ m^\dagger & M^R \end{pmatrix} \tag{3}$$

We impose the suppression factors ϵ_{Li} for the left-handed lepton doublets and ϵ_{li} for the right-handed charged leptons with respect to the ref. [2]. The orderings $\epsilon_i \leq \epsilon_j$ for $i \leq j$ are assumed. Let us impose the factor ϵ_{ν_i} for right-handed neutrinos. Since the suppression factors for the left-handed neutrinos ν_L are common with the charged lepton partners, we can assume that $\epsilon^c_{Li} = \epsilon_{Li}$. We also let $\epsilon^c_{Ri} = \epsilon_{Ri}$ with no loss of generality. Then the mass matrices elements are written by

$$M^D_{ij} \approx \epsilon_{Li}\epsilon_{\nu j}\langle\varphi\rangle_1, \qquad M^R_{ij} \approx \epsilon_{\nu i}\epsilon_{\nu j}\langle\varphi\rangle_2.$$

Now we diagonalize the 6 × 6 mass matrix (3). After block-diagonalization, we obtain the matrix $M^{(bd)}$

$$M^{(bd)} \sim \begin{pmatrix} -mM^{R^{-1}}m & 0 \\ 0 & M^R \end{pmatrix} + \mathcal{O}\left(m\left(\frac{m}{M^R}\right)^2\right) \tag{4}$$

where $\mathcal{O}(m(m/M)^2)$ describes matrices of which elements have the values of order of magnitude of the square ratio of the typical Dirac mass scale to the typical right-handed Majorana mass scale. Thus the block matrix elements are written as

$$\begin{aligned} (mM^{R^{-1}}m)_{ij} &\sim \epsilon_{Li}\epsilon_{Lj}\frac{\langle\varphi\rangle_1^2}{\langle\varphi\rangle_2} \\ M^R_{ij} &\sim \epsilon_{\nu i}\epsilon_{\nu j}\langle\varphi\rangle_2. \end{aligned} \tag{5}$$

Now we diagonalize the 3 × 3 matrices, $(mM^{R^{-1}}m)_{ij}$ and M^R_{ij} which are left- and right-handed Majorana mass matrices. In fact, we are interested in the left-handed Majorana mass matrix and we obtain the relations as

$$\begin{aligned} m_i &\approx \epsilon^2_{Li}\langle\varphi\rangle \\ V^{lep}_{ij} &\approx \epsilon_{Li}/\epsilon_{Lj} \qquad i,j = 1,2,3. \end{aligned} \tag{6}$$

Therefore we get the simple relations between masses and mixings for the neutrinos.

$$V^{lep}_{ij} \approx \frac{\epsilon_{Li}}{\epsilon_{Lj}} \approx \sqrt{\frac{m_i}{m_j}} \tag{7}$$

Note that this is not a precise numerical calculations but only order of magnitude estimates.

III. MSW SOLUTIONS AND APPROXIMATE FLAVOR SYMMETRIES

The detection of neutrinos from the sun is a possible probe to neutrino mixing hypothesis. These experiments are sensitive to values of neutrino masses and mixing angles so small that could not be reached in laboratory experiments. Ever since Davis [6], several results are reported from Kamiokande II [7], SAGE [8] and GALLEX [9] collaborations. Mikheyev and Smirnov [10] and Wolfenstein [11] have proposed a solution to the solar neutrino problem from the point of view of resonance transitions of neutrinos in matter. All existing solar neutrino experimental data could be described by the MSW mechanism if we assume the standard solar model is valid. The recent analysis found the following three windows for the allowed values of the parameters Δm^2 and $\sin^2 2\theta$ [12].

$$\text{I.} \qquad 3.2\times10^{-6}\text{eV}^2 \leq \Delta m^2 \leq 1.2\times10^{-5}\text{eV}^2$$

$$5.0\times10^{-3} \leq \sin^2 2\theta \leq 1.6\times10^{-2} \tag{8}$$

$$\text{II.} \qquad 5.4\times10^{-6}\text{eV}^2 \leq \Delta m^2 \leq 1.1\times10^{-4}\text{eV}^2$$

$$0.18 \leq \sin^2 2\theta \leq 0.86 \tag{9}$$

$$\text{III.} \qquad 1.0\times10^{-7}\text{eV}^2 \leq \Delta m^2 \leq 1.8\times10^{-6}\text{eV}^2$$

$$0.74 \leq \sin^2 2\theta \leq 0.93 \tag{10}$$

We look into our result in above windows. Following relation is derived from eq. (7)

$$\sin^2 2\theta = \frac{4}{\sqrt{1+\frac{\Delta m^2}{m_1^2}}}\left(1-\frac{1}{\sqrt{1+\frac{\Delta m^2}{m_1^2}}}\right) \tag{11}$$

m_1 is the mass of the electron neutrino and plays the role of an input parameter. Keep in mind that the eq. (7) is not the exact one but only the order of magnitude estimate and the relation (11) is also the case. Thus we cannot expect to extract the exact value of m_1 with the relation (11). But we may estimate the order of magnitude of the mass of the electron neutrino and it is sufficient at present since we do not know even whether ν_e has non-zero mass or not. The results are shown in Fig. 1 for the different values of m_1. We find that: *i)*

Window I favors the value $m_1 \sim \mathcal{O}(0.01)$ eV. *ii*) Window II is matched with the values of $m_1 < 0.1$ eV. *iii*)Window III favors small values of m_1, $m_1 < 0.01$.

IV. POSSIBILITIES OF NEUTRINOLESS DOUBLE-BETA DECAYS

Searches of the neutrinoless double-beta decays ($(\beta\beta)_{0\nu}$) are known to be sensitive to the existence of massive Majorana neutrinos coupled to the electron in the weak charged lepton currents. Since neutrino masses are expected to be small, $m_\nu << 30$ MeV, the decay amplitude is proportional to the (1,1) element of the Majorana mass matrix of neutrinos, m_{ee}. We have the upper bound for m_{ee}, $m_{ee} < (1-2)$ eV from the results of $(\beta\beta)_{0\nu}$ decay searches. Future experiments will cover the range of values of $|m_{ee}|$ as small as $|m_{ee}| \simeq (0.1-0.3)$ eV.

Petkov and Smirnov [13] studied the possibilities to observe the $(\beta\beta)_{0\nu}$ decay in the present and future experiments with the windows obtained by the solar neutrino experiments. If one assume that the Majorana neutrinos obey the mass hierarchy relation, $m_1 << m_2 << m_3$, the value of $|m_{ee}|$ can only due to a sufficiently large admixture of the ν_3 state with a mass $m_3 \geq 0.1$ eV in the ν_e state: $|m_{ee}| \sim m_3|U_{e3}|$. In this case, one have the mass hierarchy $m_1/m_3 \leq 10^{-3}$ and the large mixing between the first and the third families, $|V_{13}| \leq 1$. This result is not compatible with the eq. (7) from approximate flavour symmetries.

The case of highly degenerate neutrino mass spectrum is also considered, $m_1 \simeq m_2 \simeq m_3 \geq 0.1$ eV. The solar neutrino deficit is then explained by ν_e conversion to ν_μ if $|m_2-m_1| \leq (0.5-5) \times 10^{-5}$ eV. And the atmospheric neutrino deficit by the oscillation $\nu_\mu \leftrightarrow \nu_\tau$ can also be explained with $|m_3 - m_2| \simeq (0.5-5) \times 10^(- 2)$ eV. With these conditions eq. (7) requires large mixings $|V_{ij}| \sim 1$ and the solar neutrino windows II and III are permitted. In the fig. 1, we find that an appreciable region may be allowed around the window II which is compatible with the approximate flavour symmetries with $m_1 \sim (0.1-0.01)$ eV even though the value of $|m_{ee}|$ is somewhat small for observation of $(\beta\beta)_{0\nu}$ decay. We conclude that under the assumption of the approximate flavour symmetries on the lepton sector observations of the $(\beta\beta)_{0\nu}$ decay imply the degenerate mass spectrum of neutrinos with the mass $m_\nu < 0.1$ eV.

V. SEARCHES OF $\nu_\mu \leftrightarrow \nu_\tau$ OSCILLATIONS

Direct searches of neutrino oscillations at accelerator proceed. CHORUS [14] shall perform the experiment to explore the domain of small mixing angles down to $\sin^2 2\theta_{\mu\tau} \sim 3 \times 10^{-4}$ for mass parameters $\Delta m^2 > 1$ eV^2.

Like eq. (11), we have the following relation for $\nu_\mu \leftrightarrow \nu_\tau$ mixing.

$$\sin^2 2\theta = 4\sqrt{1 - \frac{\Delta m^2}{m_3^2}} \left(1 - \sqrt{1 - \frac{\Delta m^2}{m_3^2}}\right) \tag{12}$$

where m_3 is the tau neutrino mass. We show that this relation for different values of m_3 with the parameter space which will be covered by CHORUS experiment. We find that we can estimate the tau neutrino mass m_3 under the assumption of the approximate flavour symmetries if CHORUS observe the oscillation events in their experiment.

VI. SUMMARY

We studied the see-saw mechanism under the assumption of approximate flavor symmetry. This approximate symmetry provide guidelines for various neutrino phenomenologies. We derived a simple relation between neutrino masses and mixing angles. Using the relation, we gave several predictions for the neutrino masses and mixings without additional Ansätze. We showed that the relation is consistent with MSW solution of the solar neutrino problem and gave some constraints on the electron neutrino mass. The neutrinoless double-beta decay seems not to be compatible with the flavour symmetry well but it would give strong conditions on neutrino masses and mixings with the approximate flavour symmetry if $(\beta\beta)_{0\nu}$ decay is observed. We also have another condition for the tau neutrino mass if the neutrino oscillation is observed in CHORUS.

Though our calculations given here are not exact, we can provide many useful estimates for the neutrino masses and mixings which are tested in the near future.

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FIGURES

FIG. 1. Plot of the eq.(11) on the parameter space of Δm^2 and $\sin^2 2\theta$. It is calculated with the parameter $m_1 = 1$ eV , 0.1 eV , 0.01 eV , 0.001 eV and 0.0001 eV. The dashed line boxes denote the windows permitted by the solar neutrino experiments coupled to the MSW solution.

FIG. 2. The predictions of approximate flavour symmetry for $\nu_\mu \leftrightarrow \nu_\tau$ oscillation and the region covered by CHORUS experiment on the parameter space of Δm^2 and $\sin^2 2\theta$. It is calculated with the parameter $m_3 = 1$ eV, 10 eV, 100 eV and 1000 eV.