

GALAXY CLUSTERS AND MICROWAVE BACKGROUND ANISOTROPY

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Abstract

Previous estimates of the microwave background anisotropies produced by freely falling spherical clusters are discussed. These estimates are based on the Swiss-Cheese and Tolman-Bondi models. It is proved that these models give only upper limits to the anisotropies produced by the observed galaxy clusters. By using spherically symmetric codes including pressureless matter and a hot baryonic gas, new upper limits are obtained. The contributions of the hot gas and the pressureless component to the total anisotropy are compared. The effects produced by the pressure are proved to be negligible; hence, estimations of the cluster anisotropies based on N-body simulations are hereafter justified. After the phenomenon of violent relaxation, any realistic rich cluster can only produce small anisotropies with amplitudes of order 10^{-7} . During the rapid process of violent relaxation, the anisotropies produced by nonlinear clusters are expected to range in the interval $(10^{-6}, 10^{-5})$. The angular scales of these anisotropies are discussed.

Key words: cosmic microwave background (12.03.1) – methods: numerical (03.13.4)

1 Introduction

Cosmological overdensities on supercluster scales are pancake-like; nevertheless, galaxy clusters cannot be considered as planar structures; clusters are similar to ellipsoids, which become quasispherical in some cases (Coma cluster). This shape suggests the use of spherical symmetry as an approximating condition. This symmetry strongly reduces the computational cost with respect to the general tridimensional (3D) case, but the resulting evolution is too fast. This problem with the evolution of spherical clusters is a result of the radial structure of the velocity field. Even for vanishing initial velocities, gravitational forces generate a too rapid infalling radial motion. As it is proved in Section 4, the fast evolution of the spherical model leads to an overestimate of the nonlinear anisotropies of the Cosmic Microwave Background (CMB) produced by galaxy clusters. Even for very accurate spherical models (including hot gas), the nonlinear anisotropy is always overestimated.

In the last decade, there has been a lot of work on the CMB anisotropies produced by nonlinear cosmological structures. Nearby voids, clusters and Great Attractor-like objects have been considered. These structures are often modeled by using spherically symmetric pressureless solutions of the Einstein equations. Two popular models are based on this kind of solutions: (i) The Swiss-Cheese (SC) model proposed by Rees & Sciama (1968) is based on a matching of three exact solutions. This model involves rather particular initial conditions, and (ii) the Tolman-Bondi (TB) model is based on the solution obtained by Tolman (1934) and Bondi (1947); in this second case, initial conditions are general. The initial profiles of the energy density and the velocity can be arbitrarily chosen (Arnau et al. 1993). Chodorowski (1991) used a Newtonian version of the TB model. This version

applies when relativistic effects are negligible and, consequently, it applies in the case of nonlinear galaxy clusters. Nottale (1984) used the SC model to predict relative temperature variations $\Delta T/T \sim 10^{-4}$ for very dense and fast collapsing objects, but the masses of these objects are much greater than that of the richest cluster. In the case of feasible rich clusters, recent estimates based on the TB model and its Newtonian version are presented in Table 1, where D is the distance from the observer to the cluster and M is the mass inside a sphere of radius R_M . The resulting predictions depend on the amplitudes (normalization) and the shapes of the initial profiles. From Table 1, it follows that the predicted $\Delta T/T$ values corresponding to realistic rich clusters are a few times 10^{-6} . These values are below presently observable levels. The main limitations of the SC and the TB models are the spherical symmetry condition and the absence of pressure.

In the case of very large structures, such as voids and Great Attractor-like objects, the spherical pressureless model seems to be acceptable, but in the case of clusters, the combined effect of high density contrasts and radial infalling velocities lead to an unavoidable collapse (see below); furthermore, the hot gas content in clusters is around 10% and may even reach up to 30% of the estimated virial mass (Böhringer & Wiedenmann 1991); hence, the hot gas component is not clearly negligible. The pressure of this component could produce very large gradients –even shocks– which could be comparable to the gradients of the dominant dark component. Since the nonlinear anisotropy produced by a cluster essentially depends on time variations of the spatial gradients of the total gravitational potential, the contribution of the hot gas component should not be neglected without any justification. This component should be gravitationally coupled to the pressureless matter in order to estimate its importance in the calculation of anisotropies. A spherically symmetric coupling

is studied in this paper. Some conclusions can be extended to the general nonsymmetric case.

In a previous paper (Quilis, Ibáñez & Sáez 1994), it was shown that some shocks develop in the hot gas component of simplified planar 1D structures, the same could occur in the spherically symmetric case. The possible formation of shocks strongly motivates the use of modern *high-resolution shock-capturing* techniques; nevertheless, the use of these techniques is preferable in any case, even in the absence of shocks. Some advantages of these codes are described by Ryu et al. (1993).

Other estimates of the CMB anisotropy produced by clusters are based on N-body simulations. Van Kampen & Martínez-González (1991) simulated a rich cluster producing an effect on the order of a few times 10^{-7} . Anninos et al. (1991) considered a distribution of nonlinear objects but they did not study the effect produced by a single rich cluster. The main problems with N-body simulations are: the absence of a hot gas component and the uncertainties in both the spectrum and the statistics of the energy density distribution. A lot of information about clusters is concerned with the temperature, the density contrast and the energy radiated by the hot gas component, but these quantities are not considered in N-body simulations. This information should be compared with theoretical predictions given by a suitable model based on the 3D coupling between the hot gas (described by modern *high-resolution shock-capturing* techniques) and the pressureless (described by *N-body simulations*) component. This model is complicated. The comparison between its predictions and the observations (including those related to the hot gas component) should be very important in order to simultaneously test the spectrum, the statistics, and all the usual hypothesis about the composition and properties of the hot gas component.

In this paper, the possible production of shocks and the importance of the hot gas component are studied in the framework of the spherically symmetric model. Some conclusions can be extrapolated to the general 3D case.

The anisotropy produced by a distribution of nonlinear density perturbations is being currently studied; recently, Martínez-González, Sanz & Silk (1994) have reported that these nonlinear anisotropies range from 10^{-6} to 10^{-5} on degree angular scales. The main problems with this kind of calculations arise as a result of the current uncertainties about the time evolution of both the spectrum and the statistics of the nonlinear density distribution. Given a null geodesic, any nonlinear object located near this line influences the corresponding microwave photon; hence, taking into account the existence of nonlinear structures from the observer position to distances of order $\sim 10^3 Mpc$ –the beginning of nonlinearity–, estimates of the total nonlinear anisotropy would require: (1) N-body simulations in very large boxes including the observer and all the structures influencing the CMB and (2) information about the time evolution of a nonlinear spectrum involving all the nonlinear scales. Since such a spectrum and simulations are not yet available (even if the hot gas component is neglected), other approaches giving useful *indications* about the anisotropy produced by a realistic distribution of nonlinear structures are very useful. One of these approaches is the estimate of the anisotropy produced by a single nonlinear cluster.

Here, the spherical freely falling model for cluster evolution is improved by introducing a hot gas component. The limitations of this model are pointed out and some applications to the estimation of CMB anisotropies are presented and discussed; furthermore, another spherical model (Section 4.2) is used in order to find significant upper limits to the anisotropies produced by nearby virialized clusters.

The main features of our improved spherical model are: (i) The clusters are assumed to be nonlinear structures formed by a pressureless component –cold dark matter plus point-like galaxies– and a hot rarified gas emitting in the X–band, (ii) both components evolve in the total gravitational field created by themselves, and (iii) modern *high-resolution shock-capturing* techniques are used in order to solve the partial differential equations governing the evolution of the system.

Hereafter, t stands for the cosmological time, t_0 is the age of the Universe, $a(t)$ is the scale factor. \dot{X} stands for the derivative of the function X with respect to the cosmological time. Function \dot{a}/a is denoted by H . Hubble constant is the present value of H ; its value in units of $100 \text{ Km s}^{-1} \text{ Mpc}^{-1}$ is h . The background is flat. Velocities are given in units of the speed of light.

The plan of this paper is as follows: In Section 2, our numerical code is described. In Section 3, it is proved that any spherically symmetric model produces a too fast cluster evolution. Relevant upper limits to the nonlinear anisotropies produced by galaxy clusters are shown in Section 4. A discussion about the evolution of the hot gas component is presented in Section 5, and the main conclusions are summarized and discussed in Section 6.

2 Basic equations and numerical code

Our model consists of a hot baryonic fluid and a pressureless one, both fluids are gravitationally coupled. The hot component is described as a fluid with pressure, while the dark matter and the point-like galaxies are two components of an unique pressureless fluid.

The evolution of each of these fluids is described by their corresponding system of hydrodynamical equations. Hereafter, ρ_T is the total energy density. ρ_b , ρ_{PL} and ρ_B stand for the energy densities of the hot gas, the pressureless matter and the background (critical density), respectively; the same subscripts are used for the velocities, the density contrasts and any other quantity defined for each fluid. The subscript i stands for initial conditions. All the density contrasts are defined with respect to the background energy density, for example, the baryonic density contrast of the hot gas is $\delta_b = (\rho_b - \rho_B)/\rho_B$. The pressure is denoted p , ϵ is the specific internal energy, T_7 is the temperature in units of 10^7 K, and L_1 is the luminosity produced by an sphere of radius h^{-1} Mpc centred at the point where the cluster is.

2.1 Basic equations

For small enough spatial scales (see below), each of the gravitationally coupled fluids obeys the following Newtonian equations:

$$\frac{\partial \delta}{\partial t} + \frac{1}{a} \nabla \cdot (1 + \delta) \vec{v} = 0 \quad (1)$$

$$\frac{\partial \vec{v}}{\partial t} + \frac{1}{a} (\vec{v} \cdot \nabla) \vec{v} + H \vec{v} = -\frac{1}{\rho a} \nabla p - \frac{1}{a} \nabla \phi \quad (2)$$

$$\frac{\partial E}{\partial t} + \frac{1}{a} \nabla \cdot [(E + p) \vec{v}] = -3H(E + p) - H \rho \vec{v}^2 - \frac{\rho \vec{v}}{a} \nabla \phi - \Lambda \quad (3)$$

$\vec{v} = a(t) \frac{d\vec{r}}{dt}$, \vec{r} , $\Lambda(\rho_b, T)$, and δ being the peculiar velocity, the Eulerian dimensionless coordinates, the cooling rate, and the density contrast of the fluid, respectively. $E = \rho\epsilon + \frac{1}{2}\rho\mathbf{v}^2$ is the addition of the internal and the kinetical energy densities. Quantities without subscripts correspond to a generic fluid. The equations governing each particular

fluid have the form (1)-(3), but the subscripts mentioned in Section 2 must be included in each case. The total energy density contrast δ_T is the source of the peculiar gravitational potential $\phi(t, \mathbf{x})$, which satisfies the following equation

$$\nabla^2 \phi = \frac{3}{2} H^2 a^2 \delta_T, \quad (4)$$

this potential is involved in the differential equations of each fluid (gravitational coupling). Only pressure gradients and gravitational forces act on the system.

The Newtonian description given by Eqs. (1)-(4) applies if the following conditions are satisfied (Peebles 1980): a) The inhomogeneity size is much smaller than the causal horizon size; thus, background curvature is negligible, and velocities are much smaller than c , and b) no strong local gravitational fields are present. These conditions make unnecessary a relativistic approach. In all our applications we have verified that the above conditions are satisfied.

In the spherically symmetric case, Eqs. (1)-(3) can be easily written as follows:

$$\frac{\partial \vec{u}}{\partial t} + \frac{\partial \vec{f}(\vec{u})}{\partial r} = \vec{s}(\vec{u}) \quad (5)$$

the vector of unknowns \vec{u} being

$$\vec{u} = [\delta, m, E] \quad (6)$$

where $m = (\delta + 1)v$. The vector-valued function $\vec{f}(\vec{u})$ (the fluxes) is

$$\vec{f}(\vec{u}) = \left[\frac{m}{a}, \frac{m^2}{(\delta + 1)a} + \frac{p}{a\rho_B}, \frac{(E + p)m}{a(\delta + 1)} \right] \quad (7)$$

and the sources $\vec{s}(\vec{u})$ are

$$\vec{s}(\vec{u}) = \left[-\frac{2m}{ar}, -\frac{(\delta + 1)}{a} \frac{\partial}{\partial x} \phi - Hm - \frac{2m^2}{ar(\delta + 1)}, -3H(E + p) - \frac{\rho_B H m^2}{(\delta + 1)} - \frac{m\rho_B}{a} \frac{\partial \phi}{\partial r} - \frac{2(E + p)m}{ar(\delta + 1)} - \Lambda \right] \quad (8)$$

Written in this way, we have displayed the conservative character of the system, in the sense of Lax (1973). Its hyperbolic property was already pointed out in Quilis, Ibáñez & Sáez (1993, 1994). Thus, the above system is a one-dimensional (1D) *hyperbolic system of conservation laws* with sources. In order to solve numerically these kind of systems, powerful tools have been developed, the so-called modern *high-resolution shock-capturing* techniques (see below).

The hot gas component located inside the clusters is a fluid obeying Eqs. (5)-(8). An equation of state of the form $p_b = (\gamma - 1)\rho_b\epsilon$ is assumed. The pressureless component is another fluid obeying the same equations. The equation of state is $p_{PL} = 0$. The peculiar gravitational potential involved in the equations governing the evolution of both fluids is the same; it is the total peculiar gravitational field produced by the total density contrast $\delta_T = (\rho_b + \rho_{PL} - \rho_B)/\rho_B$ and, consequently, this potential satisfies the following 1D spherical version of Poisson's equation:

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left[r^2 \frac{\partial \phi}{\partial r} \right] = \frac{3}{2} H^2 a^2 \delta_T . \quad (9)$$

At each instant, the term $\nabla\phi$ is computed as $\nabla\phi = G \frac{M}{R^2}$; where $R = ar$ and $M = \int 4\pi\delta_T\rho_B R^2 dR$ is the total peculiar mass located inside a sphere of radius R at time t .

2.2 Some details about our code

We have built up a hydro-code based on a modern *high-resolution shock-capturing* method. Our code involves the "minmod" cell reconstruction (it is a version of the MUSCL algorithm derived by van Leer, 1979), Roe's prescription for evaluating the numerical fluxes (Roe 1981), and a second order Runge-Kutta algorithm for advancing in time. It can be proved

that these elements set out a global second order accurate algorithm. Reader interested in details can be addressed to Quilis, Ibañez & Sáez (1994).

Some comments about the time steps and the spatial grids used in this paper are now presented. The time step used in our numerical integrations is chosen to be the smallest of the following times:

1. The Courant time Δt_c , the minimum of the steps Δt_{cj} defined by

$$\Delta t_{cj} = CFL_1 \frac{\Delta r}{|\lambda_1(r_{j+\frac{1}{2}}) - \lambda_3(r_{j-\frac{1}{2}})|} \quad (10)$$

where j labels the cell and $j + \frac{1}{2}$ is the interface between the cells j and $j + 1$. λ_1 and λ_3 are the minimum and maximum characteristic speeds (see Quilis, Ibañez & Sáez 1994). CFL_1 is a correction factor whose value is experimentally fixed in order to obtain the best results. Correction factors range in the interval $(0, 1)$. In our computation, the factor CFL_1 has varied from 0.6 to 0.9.

2. The dynamical time

$$\Delta t_d = CFL_2 \sqrt{\frac{3\pi^2}{4\rho_T}} \quad (11)$$

where ρ_T is the maximum total energy density appearing in the previous time iteration. Typical values of the factor CFL_2 are of the order 10^{-4} .

The first epoch of the evolution is governed by the time step Δt_c . During the ulterior very nonlinear epoch, that is, when δ_{pL} reaches large values, the most restrictive time step is Δt_d .

In all the calculations displayed in this paper, a geometric spatial grid with 400 cells is used. It has been verified that a grid with 800 cells does not lead to physically significant differences with respect to the 400 cells grid. Hence, this number of cells warranties the

convergence to the solution.

Our hydro-code passed successfully the standard shock tube tests, both in the Newtonian and relativistic 1D cases (Martí, Ibáñez & Miralles 1990, 1991; Marquina et al. 1993). Pure cosmological tests were presented in Quilis, Ibáñez & Sáez (1993, 1994).

3 Cluster model

In this section, the initial conditions for cluster evolution are fixed in such a way that the resulting cluster appears to be similar to the observed ones; however, the evolution of these structures is not realistic. A very rich Abell cluster is simulated from a particular choice of the initial conditions.

Radio observations at 21 *cm* show that the level of neutral hydrogen located inside clusters is very low (Gunn-Peterson test). In the framework of a cold dark matter model (Cen & Ostriker 1992), the high degree of ionization of the hot baryonic gas cannot be produced by shocks, bremsstrahlung or free-bound radiation; hence, this ionization must be assumed to be an initial condition. The high amount of iron in the hot gas requires that a large fraction of this comes from galaxies in bursts and winds (see Böhringer & Wiedenmann 1991 and references cited therein). On account of these considerations, the hot gas is assumed to be mainly formed by high energetic particles produced by stellar evolution –winds, supernovae– and by processes in active galactic nuclei. This part of the gas and the residual primordial gas (which is not confined inside galaxies during galaxy formation) are strongly ionized by ultraviolet radiation emitted by stars, the resulting gas remains ionized until present time. It is also assumed that the formation of the ionized gas

and its localization in the intergalactic space occurs during a cosmologically short period; thus, this gas is assumed to be instantaneously formed at a given initial redshift z_i (the feasibility of this assumption is discussed in Section. 5).

It is assumed that galaxies trace mass; thus, the initial density of galaxies is proportional to that of dark matter. Since galaxies were formed in the primordial baryonic gas and they afterwards produced the main part of the ionized component observed in clusters, the initial density of this component is assumed to be proportional to that of galaxies; hence, all the initial densities are assumed to be proportional to the total energy density. Density contrasts are accordingly related among them.

Initial velocities are assumed to be identical for the three components. The dark matter and the point-like galaxies are pressureless components obeying the same equations; hence, their velocities are identical at any time. As a result of the absence of pressure, these velocities can be obtained by using the spherical freely falling solution of the Newtonian hydrodynamics equations (Peebles 1980). This solution admits an arbitrary density contrast. The chosen velocities correspond to vanishing decaying modes. Finally, since the hot gas is mainly formed inside galaxies (and galaxies are formed in the initial primordial gas), all the hot gas component participates –at the beginning– of the same motions as the point-like galaxies; during evolution, the hot gas and the pressureless component evolve in a different way and, consequently, their velocity fields become different.

The initial profile of the total density contrast is chosen to be:

$$\delta_T(R, t_i) = \frac{\delta_{T_i}}{1 + \left(\frac{R}{R_V}\right)^{1.8}} , \quad (12)$$

where δ_{T_i} is the amplitude of $\delta_T(R, t_i)$, and R_V is the radial distance at which the initial

density contrast reduces to one-half of its amplitude δ_{Ti} . This profile gives suitable present profiles for galaxies.

There is a set of free parameters in the initial conditions. The values of these parameters must be chosen in such a way that the simulated and the observed clusters be comparable.

The free parameters are: the initial redshift z_i , δ_{Ti} , R_V , the initial specific internal energy ϵ_i , and the ratio between the amplitudes of the hot gas and the total energy densities.

As the observations in the X-band and the observations of the Sunyaev -Zel'dovich (1980) effect suggest, the hot gas component is very rarified. This gas is mainly formed by protons, electrons, and helium nuclei. It is highly ionized. The abundance of protons and He nuclei are assumed to be of 9 to 1 in number. The resulting gas is treated as a monoatomic gas (adiabatic coefficient $\gamma = 5/3$) with an averaged atomic weight calculated from the above standard abundances (Sherman 1982).

The temperature evolution of the rarified hot gas is only sensitive to the Compton cooling and the thermal Bremsstrahlung. As a result of the small density of neutral elements, other interactions like recombinations and ionizations are not important; hence, the term Λ – involved in some equations of Section 2.2– can be written as follows: $\Lambda = \Lambda^C + \Lambda^{Br}$, where Λ^C and Λ^{Br} are the contributions of the Compton cooling and the thermal Bremsstrahlung, respectively. According to Umemura & Ikeuchi (1984) Λ^C and Λ^{Br} are:

$$\Lambda^C = 5.4 \times 10^{-36} (1+z)^4 n_e T \quad (\text{erg cm}^{-3} \text{s}^{-1}) \quad (13)$$

and

$$\Lambda^{Br} = 1.8 \times 10^{-27} n_e T^{\frac{1}{2}} (n_{HII} + 4n_{HeIII}) \quad (\text{erg cm}^{-3} \text{s}^{-1}) \quad (14)$$

where n_e , n_{HII} and n_{HeIII} are the number density of electrons, protons, and helium nuclei,

respectively.

The main observational features of galaxy clusters are the following: The density of electrons –in units of cm^{-3} – ranges in the interval $(10^{-3}, 10^{-2})$ (Sunyaev & Zel’dovich 1980) and the total mass within a radius of $1.5h^{-1} Mpc$ is $\sim 10^{15} M_{\odot}$. These data are only compatible with δ_b values ranging in the interval $(10^2, 10^3)$. The radius of the core is $R_c \sim 0.2h^{-1} Mpc$, R_c being the distance at which the present energy density reduces to one-half of its maximum value (Peebles 1994). The temperature of the hot gas component, T_7 , and the luminosity, L_1 , in units of erg/s range in the intervals $(2, 10)$ and $(10^{43}, 3 \times 10^{45})$, respectively (Böhringer 1991). The present density contrast of the galaxy distribution has the form (12).

For the following initial conditions: $z_i = 7$, $\epsilon_i = 5 \times 10^{-6}$, $\delta_{T_i} = 0.26$, $R_v = 0.6h^{-1} Mpc$, and $\rho_{bi} = 0.2\rho_{T_i}$, Table 2 shows the features of the resulting structure at the times $0.94t_0$, $0.96t_0$, and $0.98t_0$.

At time $0.96t_0$, the resulting structure looks like a very rich Abell cluster (perhaps a too rich cluster, but see below for a justification of this choice); hence, our spherically symmetric model can reproduce the main features of realistic clusters; nevertheless, the evolution of the chosen cluster is not admissible. At time $0.98t_0$, the densities, the luminosity, and the temperature are too great, while the size is too small; hence, an unavoidable collapse is developing. At times smaller than $0.94t_0$, the densities and the temperature become too small and the size becomes too large. The existence of rather stable clusters located between $z \sim 1$ to $z \sim 0$ is not compatible with the spherical symmetry condition. This occurs because the pressureless component has a radial infalling motion accelerated by gravity and, consequently, this component fast collapses, forcing the collapse of the subdominant

hot gas component, and making the clusters very unstable structures. As it is proved in the next section, any quickly evolving spherical model (including the model of this paper plus the SC and TB models) leads to overestimates of the anisotropies produced by clusters. In spite of these facts, the spherical model can be appropriately used in order to get significant upper limits for the nonlinear anisotropy.

Figure 1 shows the profiles of the hot gas and the pressureless density contrasts at time $0.96t_0$. No shocks have appeared in the hot gas component. Although this component is gravitationally dragged by the rapid freely falling pressureless component, pressure has not produced spatial gradients larger than those of the pressureless fluid; hence, the presence of pressure does not seem to be important in order to compute cluster anisotropies (see Section 4.1 for a quantitative verification of this statement). As discussed in the introduction, the study of the hot gas component should be important in order to study the evolution and normalization of 3D realistic clusters, but it does not seem to be directly relevant in the calculation of nonlinear anisotropies. Since the largest gradients and the shocks are expected to be favored by the rapid infalling induced by the spherical symmetry condition, the above conclusion about the importance of the hot gas component in the calculation of nonlinear anisotropies can be extended to the case of realistic clusters. This important fact justifies the computations of nonlinear anisotropies based on N-body simulations (excluding the hot baryonic gas). The evolved profile of the pressureless density contrast has the form (12), hence, the predicted profile of point-like galaxies is compatible with observations.

Since shock formation is expected to be more feasible in the case of the richest clusters, we have preferred the study of a very rich one (perhaps too rich); thus the absence of shocks in the chosen case ensures the absence of these phenomena in the case of any realistic rich

cluster.

Objects similar to Abell clusters can be simulated at any time by using our freely falling model. Admissible times would range from $z \sim 1$ to $z \sim 0$. The evolution appears to be wrong in any case.

4 CMB anisotropies

Any overdensity located between the observer and his last scattering surface produces: (1) a Sachs-Wolfe effect, (2) A Doppler effect, which appears as a result of the peculiar velocity produced by the overdensity on the last scattering, (3) a second Doppler effect due to the peculiar velocity induced on the observer, (4) a Sunyaev-Zel'dovich effect produced by the interaction between the CMB photons and the free electrons of the overdensity, and (5) a gravitational nonlinear effect.

In the case of a nonlinear cluster, the effects (1) and (2) are negligible because these structures are very far from the last scattering surface. The effect (3) is only relevant when the cluster is located very near to the observer (Virgo cluster), it has an exactly dipolar form. The effect (4) is the dominant one in the case of a single object, its estimate is easy and its detection has been claimed by Uson & Wilkinson (1988) and Birkinshaw (1990). The total Sunyaev-Zel'dovich effect produced by a realistic distribution of clusters is a matter of current study. The study of the effect (5) is the main subject of this section.

When the nonlinear effect (5) dominates, the anisotropy is (Martínez-González, Sanz & Silk 1990):

$$\frac{\Delta T}{T} \sim -2 \int_e^o \vec{\nabla} \phi(\vec{x}, t) d\vec{x} \sim 2 \int_e^o \frac{\partial \phi(\vec{x}, t)}{\partial t} dt, \quad (15)$$

where ϕ is the potential involved in the line element

$$ds^2 = -(1 + 2\phi)dt^2 + (1 - 2\phi)a^2 \delta_{ij} dx^i dx^j; \quad (16)$$

if this line element and the energy-momentum tensor of a perfect fluid are introduced in the Einstein equations and the powers and products of the potential and its gradients are neglected, the Newtonian equations (1)-(4) are obtained; hence, the function ϕ involved in Eqs. (15) and (16) is the Newtonian gravitational potential created by the cluster.

The integral involving $\frac{\partial \phi(x, t)}{\partial t}$ points out the importance of the time evolution of the gravitational potential. Any overestimate of $\frac{\partial \phi(x, t)}{\partial t}$ leads to an overestimate of $\Delta T/T$.

The integral involving the spatial gradients of the potential ϕ is the most appropriated in order to be evaluated by using our numerical methods. These gradients are directly involved in the differential equations describing the cluster evolution; hence, when these equations are numerically solved, the spatial gradients are directly calculated at the nodes of the spatial grid and at each time step; afterwards, the gradients can be calculated at arbitrary positions and times by using suitable interpolations.

The integrals involved in Eq. (15) must be carried out along each null geodesic from the emitter (e) to the observer (o). The emitter is located on the last scattering surface. The equations of these geodesics can be derived in the background (at zero order).

In the spherically symmetric case, a direction of observation (a null geodesic) is defined by the angle ψ formed by the line of sight and the line pointing towards the cluster centre.

The origin of coordinates is located in the cluster centre and the position of the observer is fully determined by its radial distance to the origin.

4.1 Previous upper limits

If the spherical cluster simulated in Section 3 is located at $70h^{-1} Mpc$ from the observer, the resulting amplitude of the nonlinear anisotropy is $|\Delta T/T(\psi = 0)| = 1.2 \times 10^{-5}$ (see Fig. 2). The CMB photons passed near this cluster at time $0.96t_0$, when the structure had the features of a very rich Abell cluster (see Table 2). The masses inside three spheres of radius $1.5h^{-1} Mpc$, $2h^{-1} Mpc$ and $4h^{-1} Mpc$ are $0.95 \times 10^{15}h^{-1} M_\odot$, $2.0 \times 10^{15}h^{-1} M_\odot$ and $6.2 \times 10^{15}h^{-1} M_\odot$, respectively. These data facilitate comparisons with the predictions of Table 1. Let us discuss in detail some of these comparisons. We begin with Panek's predictions (1992). In the model described in the second row of Table 1, there is a total mass $M = 5.7 \times 10^{15}h^{-1} M_\odot$ inside a sphere of radius $4h^{-1} Mpc$, this mass is very similar to that of the cluster considered in this section; however, the amplitude of the density contrast corresponding to our cluster (see Table 2) is much greater than that of Panek's model (~ 674); as a result of this discrepancy, the amplitude of the anisotropy obtained by Panek (1992) is one-half of our amplitude. Let now consider Chodorowski's estimates (1991). In the cases presented in the rows 3 and 4 of Table 1, the masses located inside spheres of radius $\sim 2h^{-1} Mpc$ are smaller than the mass corresponding to our model ($2.0 \times 10^{15}h^{-1} M_\odot$) and, consequently, our results cannot be directly compared with those of Table 1; nevertheless, an indirect comparison can be established taking into account that the amplitude of the anisotropy computed by Chodorowski (1991) scales with the mass like $\sim M^{3/2}$ (see Nottale 1984), this fact can be easily verified from the data exhibited in the rows 3 and 4 of Table

1. The mentioned scaling leads to the conclusion that, in Chodorowski's models (1991), the amplitude of the anisotropy corresponding to $M = 2.0 \times 10^{15} h^{-1} M_{\odot}$ is $\sim 1.14 \times 10^{-5}$; this value is in very good agreement with our estimate. After these comparisons, we can conclude that our prediction seems to be compatible with those of Chodorowski (1991) and Panek (1992). The differences between our amplitude and those of the above authors appear as a result of the differences in the normalization and slope of the density and velocity profiles. Our normalization takes into account the features of the hot gas component (see Section 3). Previous amplitudes and the amplitude computed in this section have been overestimated (as the ϕ evolution). They are upper limits.

The spatial gradients of the peculiar gravitational potentials produced by the hot gas and the dark matter components have been separately calculated. These gradients have been used in order to compute the anisotropy produced by each of these components according to Eq (15). In Fig. 2, the total temperature contrast $\Delta T/T$ and the part of this contrast produced by the pressureless matter are shown. The difference ($\sim 18.5\%$ of the total effect) is produced by the hot gas component. Since the 20% of the total mass is contained in the hot gas and this gas produces the $\sim 18.5\%$ of the total anisotropy, the contribution of the hot baryonic component to $\Delta T/T$ is very similar to the expected contribution ($\sim 20\%$) corresponding to the same proportion of pressureless matter; hence, the effect of the pressure is not important. This conclusion is in good agreement with the absence of shocks and large gradients in the density profile corresponding to the hot gas (see Fig. 1)

4.2 New upper limits

In order to obtain more stringent upper limits to the nonlinear anisotropy produced by clusters, it must be taken into account that: (a) Any overestimate of the time evolution of the gravitational potential ϕ leads to an upper limit for $\Delta T/T$, and (b) the smaller the differences between the true evolution and the overestimated one, the smaller and more stringent the resulting upper limit. After virialization, the upper limits of Section 4.1 are not stringent enough because they are based on too great overestimates of $\frac{\partial\phi}{\partial t}$; stringent limits corresponding to this period are obtained in this section.

In realistic clusters, the particles (point-like galaxies and small gravitationally bounded amounts of cold dark matter) do not move radially; after the process of violent relaxation (Lynden-Bell 1967), these particles reach quasistable orbits in the total gravitational field of the structure and the inertial forces almost compensate the gravitational ones; thus the fall towards the centre becomes very slow and the clusters appear to be highly stable structures undergoing modest changes. The shape of the system can be quasispherical.

After violent relaxation, the total density contrast, δ_T , of the resulting cluster has been estimated to be $\sim 2 \times 10^2$ (Böhringer and Wiedenmann 1991); hence, according to our definition of the contrasts (see Section 2), in a model containing 20% of baryonic hot gas, the corresponding δ_b value is $\delta_b = (\rho_b/\rho_B) - 1 = (0.2\rho_T/\rho_B) - 1 = 0.2\delta_T - 0.8 \sim 40$. These estimates are based on a very simple spherical homogeneous model. In more realistic models based on nonuniform density profiles (see Fig. 1 and Eq. 12), the distribution of mass is not homogeneous and the expected amplitudes of δ_T and δ_b should reach values greater than those predicted by the homogeneous model. On account of these facts, it is

hereafter assumed that, after virialization, the amplitude of δ_b is ~ 100 . This conservative assumption is also compatible with the limits on δ_b discussed in Section 3, which are based on observations.

As discussed in Böhringer and Wiedenmann (1991), all the clusters do not virialize at the same redshift; according to these authors, the virialization finishes at a certain time depending on the initial conditions defining the protocluster in the linear regime. A conservative hypothesis compatible with observations and theoretical considerations is that any cluster undergoes virialization at a redshift $z_{vir} \leq 1$.

During the violent relaxation, the gravitational field undergoes rapid variations and, consequently, the upper limits based on the spherically symmetric model of Section 3 are much better than in the subsequent slowly evolving period. After violent relaxation, the situation is very different; the evolution is not known, but two possibilities can be imagined: (a) the cluster tends to a stationary state and, (b) the cluster tends to collapse but at a rate which is much slower than during the relaxation. In case (a), the evolution of virialized clusters at $z \ll z_{vir}$ would be very slow. In case (b), these clusters would evolve faster than in case (a) as a result of the progressive instability of the system. In order to find upper limits for the anisotropy produced by virialized clusters at $z \ll z_{vir}$, a certain evolution of kind (b) is appropriately simulated.

According to the observations, all the galaxy clusters –from $z \sim 1$ to $z \sim 0$ – have a hot baryonic component with density contrast δ_b ranging from 10^2 to 10^3 ; hence, the fastest admissible evolution of kind (b) would produce a present cluster with $\delta_b \sim 10^3$ starting from the value $\delta_b \sim 10^2$ corresponding to z_{vir} (see above). Any quantity being bounded by the observations –as the total mass inside a sphere of $1.5h^{-1} Mpc$ – can be used to define

overestimated evolutions in the same way as δ_b . Any overestimated evolution is fictitious. It does not describe the true evolution of the clusters. It is only used with the essential aim of finding upper limits to the nonlinear anisotropy. This kind of evolution can be produced by fictitious suitable forces. The use of fictitious forces is not a newness in Cosmology, they are introduced in the *Adhesion model* (Gurvato, Saichev & Shandarin 1989) with the essential aim of simulating structure formation; nevertheless, in our case, the role of the fictitious forces is more modest than in the adhesion case.

An evolution of kind (b) can be simulated by means of a fictitious force producing a partial cancelation of the gravitational one. A perfect cancelation leads to stationarity. The strongest the cancelation, the slowest the evolution. If the fictitious force \vec{F}_f is assumed to be proportional to the gravitational one ($\vec{F}_f = (1-\zeta)\vec{\nabla}\phi$, $0 < \zeta < 1$), the inner regions becomes more stable than the outer ones, where gravitational forces are greater; thus, some kind of accretion on the core is simulated. This core is not fully stable because the gravitational force increases as a result of the accretion, but the instability does not produce a too rapid collapse as in the absence of any compensating action. The residual force leads to a certain degree of evolution. The value of the parameter ζ can be numerically determined –in each case– by assuming a final δ_b value of order 10^3 ; thus a too fast evolution is simulated (at least for $z \ll z_{vir}$). $\zeta = 0$ corresponds to stationarity.

In practice, the computations are carried out as follows: (1) the model described in Section 3 is used in order to obtain suitable density contrasts ($\delta_b \sim 10^2$) at $z = z_{vir}$. These contrasts plus vanishing velocities are the initial conditions for the subsequent evolution, in which, the pressure is neglected and a fictitious antigravity force is introduced. These conditions are easily included in the codes described in Sections 2 and 3. The parameter ζ

is fitted in order to get suitable overestimates of the evolution; namely, suitable final values $\delta_b \sim 10^3$.

The case described in Fig. 3 corresponds to a cluster with contrasts $\delta_b \sim 100$ and $\delta_b \sim 1200$ at redshifts $z = z_{vir} = 1$ and $z = 0$, respectively. At the small redshifts corresponding to the assumed locations of the cluster ($z < 0.1$), δ_b is greater than 10^3 . A parameter $\zeta \sim 5.5 \times 10^{-3}$ has been necessary in order to simulate this evolution. This quantity gives a qualitative idea about the high degree of stability being necessary in order to explain the observations about clusters. Since the gravitational forces are very strong, the main part of them must be cancelled in order to keep acceptable levels of accretion (variations of ϕ). The simulated object has a total mass $\sim 0.6 \times 10^{15} h^{-1} M_\odot$ inside a radius of $1.5 h^{-1} Mpc$. It is a rich cluster. Fig. 3 shows the nonlinear anisotropy for three positions of the system. In these positions, the distances from the cluster centre to the observer are $25 h^{-1} Mpc$, $100 h^{-1} Mpc$, and $300 h^{-1} Mpc$. The amplitude of the nonlinear anisotropy is $\sim 4 \times 10^{-7}$ in all the cases, but the angular scales consistently change.

The results displayed in Fig. 3 have been obtained in the case $z_{vir} = 1$. This high value of z_{vir} has been arbitrarily chosen. Values of z_{vir} smaller than 1 have been also considered, but the results corresponding to these values are not displayed in Figures by the sake of brevity. These results lead to the following conclusion: If a rich cluster is normalized in the same way as in the case $z_{vir} = 1$ and located at $z \ll z_{vir} < 1$, the resulting anisotropy is $\sim 4 \times 10^{-7}$ for any admissible choice of $z_{vir} < 1$. This anisotropy is very similar to that predicted by van Kampen and Martínez-González by using N-body simulations. Since δ_b increases from 10^2 to $\sim 10^3$ in all the cases, the fictitious evolution corresponding to $z_{vir} = 1$ is slower than that of the cases $z_{vir} < 1$ and, consequently, the resulting anisotropy should

be an increasing function of z_{vir} ; since this dependence on z_{vir} is not found, the fictitious evolution –although faster than the true evolution– should be negligible in all the cases. In order to verify this possibility, the gravitational force has been completely canceled ($\zeta = 0$) and the anisotropy has been calculated again; as expected, the results are indistinguishable from those presented in Fig. 3. Two main conclusion can be obtained: (1) At $z \ll z_{vir}$, any admissible evolution is too slow to produce a significant contribution to $\Delta T/T$ and (2) the resulting $\Delta T/T$ values obtained in our computations are produced by a non-evolving spherical cluster located at various positions. The time delay of the CMB photons crossing a nonlinear stationary structure produces the estimated anisotropy. This effect was first described by Rees and Sciama (1968).

From the above results, the following conservative conclusion can be obtained: Any rich cluster produces an effect of a few times 10^{-7} at $z \ll z_{vir}$. The angular scale of this anisotropy depends on the position of the cluster (see Fig. 3 for the angular scales corresponding to three locations). Any realistic distribution of virialized clusters cannot produce relevant effects ranging in the interval $(10^{-6}, 10^{-5})$.

Nearby clusters located at distances smaller than $300h^{-1} \text{ Mpc}$ ($z < 0.1$) can only produce relevant anisotropies if they are undergoing violent relaxation. Taking into account that the distances between clusters are of various tens of Megaparsecs, this kind of nearby objects can only appear in some isolated directions. The effect of these clusters could be separately considered after detection in observational surveys; hence, we are hereafter concerned with non virialized clusters located at $z > 0.1$, which would produce anisotropies on scales smaller than $\sim 1^\circ$. Are these anisotropies cosmologically relevant?. Some comments about this question are worthwhile.

During the epoch of violent relaxation, cluster evolution is faster than in the subsequent epoch; hence, the most important anisotropies must be produced during relaxation. In this epoch, the upper limits given by the freely infalling model of Section 3 are much more stringent than at $z < z_{vir}$. These limits can be obtained in the same way as in Section 4.1, but the epoch of infalling and the location of the cluster must be appropriately fixed near z_{vir} . The resulting upper limits are of the same order as in Section 4.1; therefore, the anisotropies arising during relaxation could reach values near 10^{-5} for the richest clusters. If a cluster is located at $z \sim 1$, the angular scale of the corresponding anisotropy is about $10'$, while the angular scale corresponding to $z = 0.1$ is $\sim 1^\circ$; hence, the total anisotropy produced by clusters undergoing violent relaxation at different redshifts $0.1 < z_{vir} < 1$ is a superposition of angular scales ranging from $10'$ to 1° . According to this discussion, we claim that the main part of the nonlinear effect (5) should be produced during a brief period ($0.1 < z < 1$) corresponding to violent relaxation of clusters; in other words, only the clusters located in a narrow interval of redshifts would produce significant anisotropies and each of these cluster would only produce anisotropy during the short period of violent relaxation. These facts could guide and simplify future estimates based on N-body simulations.

The anisotropy produced by a distribution of clusters cannot be calculated from that of a single cluster, but amplitudes at the level of 10^{-5} or 10^{-6} (see Martínez-González, Sanz & Silk 1994), on angular scales between $10'$ and 1° are suggested by the above arguments. More work is necessary in order to see if the currently observable level ($\sim 10^{-5}$) is reached.

5 Some comments on the hot gas component

If the pressure of the hot gas component is not neglected from z_{vir} to $z = 0$ –in order to obtain temperatures and luminosities–, and the compensating antigravity force is introduced –in order to stabilize the system–, the above conclusions about anisotropies do not change; however, it has been verified that the Compton cooling and the thermal Bremsstrahlung strongly cool the system. Starting from initial admissible values of the temperatures and luminosities, the evolved values become too low. The clusters are radiating energy during a long period –from z_{vir} to the redshift defining the cluster location– and, consequently, a too strong cooling is produced. This effect decreases as z_{vir} decreases. The same effect is expected in realistic nonsymmetric cases (although the stabilization is not produced by fictitious forces). This problem is a result of the hypothesis that all the hot gas component was generated and distributed at a certain initial redshift (see Section 3). The hot gas component must be continuously ejected from galaxies and a substantial part of this component must arrive to the intracluster medium after z_{vir} . This is a necessary ingredient of future nonsymmetric simulations.

6 Conclusions

Cluster evolution has been proved to be slower than the evolution predicted by spherical models; on account of this fact, these models can only be used in order to obtain upper limits to the anisotropy produced by galaxy clusters.

The CS and TB models lead to very fast evolutions, which only give stringent enough upper limits to $\Delta T/T$ during the brief period of violent relaxation. Nottale (1984), Chodor-

owski (1991), Panek (1992) and Sáez, Arnau & Fullana (1993) presented some estimates based on these models (see Table 1).

Appropriate spherical models evolving faster than the observed clusters are used in order to obtain upper limits to the microwave background anisotropies produced by some clusters. In the case of a virialized nearby clusters ($0 \leq z \leq 0.1$), the angular scale of the anisotropy is larger than 1° and its amplitude is a few times 10^{-7} . The anisotropy produced by a realistic distribution of these structures is too small to be detected. Nonlinear clusters undergoing virialization in the period $0 \leq z \leq 0.1$ cannot be very abundant, these objects can be only observed in isolated directions (see Section 4.2), they lead to small local spots with amplitudes of a few times $\sim 10^{-6}$ (perhaps near 10^{-5} for very rich clusters). Only clusters undergoing violent relaxation at redshifts $0.1 < z < 1$ can produce significant anisotropies on large regions. The total effect of all these clusters is a superposition of anisotropies on angular scales ranging from $\sim 1^\circ$ to $\sim 10'$. Upper limits based on the spherical freely infalling model prove that this anisotropy is smaller than 10^{-5} (likely a few times 10^{-6}). Realistic cluster models are necessary in order to improve these conclusions.

In order to simulate realistic clusters, it would be desirable a suitable coupling between a 3D code based on modern *high-resolution shock-capturing* techniques and a N-body code. The first code would describe the evolution of the hot gas component and the second one would describe the pressureless matter. Both components must evolve in their total gravitational field. Here, it has been verified that the hot gas component must be gradually generated. In our opinion, the most interesting feature of these realistic models is the study of the temperature and the luminosity of the hot baryonic component, whose evolution –for given cosmological spectra and (or) statistics– can be compatible (or not) with observational

data. Furthermore, these future models could be basic in order to estimate the Sunyaev-Zel'dovich effect produced by the hot gas located inside clusters. Fortunately, according to our estimates, neglecting the hot gas component (N-body simulations) must lead to an accurate enough computation of the nonlinear gravitational anisotropy produced by clusters; of course, a model including this component would give more accurate estimates of these anisotropies.

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Figure captions

Figure 1. Density contrasts –at time $0.96t_0$ – of the hot baryonic gas (continuous line) and the pressureless component (dashed line) as functions of the radial distance R in units of $h^{-1} Mpc$.

Figure 2. Dotted line is a plot of $-(\Delta T/T) \times 10^5$ as a function of the angle ψ (in degrees) for the rich cluster described in Table 2 located at $70h^{-1} Mpc$ from the observer. Dashed line displays the contribution of the pressureless component to the total effect (dotted line).

Figure 3. Plot of $-(\Delta T/T) \times 10^7$ as a function of the angle ψ (in degrees) for the quasisustainable rich cluster of Section 4.2 located at the distances from the observer displayed inside the panel.

Table 1. Previous estimates of $\Delta T/T$.

Authors	D	R_M	M	$\Delta T/T$
	$(h^{-1} Mpc)$	$(h^{-1} Mpc)$	$(\times 10^{15} h^{-1} M_\odot)$	$(\times 10^6)$
Panek	100	4	1.8	-1.0
Panek	100	4	5.7	-6.0
Chodorowski	~ 70	2	0.77	-2.6
Chodorowski	~ 70	2	1.54	-7.7

Table 2. The spherically symmetric model.

time	δ_b	δ_{PL}	L_1	T_7	R_c
			$(\times 10^{44} \text{ erg/s})$		$(\times h^{-1} Mpc)$
$0.94t_0$	110	510	0.37	1.2	0.75
$0.96t_0$	520	3220	4.6	3.3	0.23
$0.98t_0$	4000	1.9×10^5	42.0	120.0	0.02