Star Cluster Evolution, Dynamical Age Estimation and the Kinematical Signature of Star Formation

K3

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Abstract. We distribute 400 stars in $N_{\mathrm{bin}} = 200$ binary systems in clusters with initial half mass radii $0.077 \leq R_{0.5} \leq 2.53\,\mathrm{pc}$ and follow the subsequent evolution of the stellar systems by direct N-body integration. The stellar masses are initially paired at random from the KTG(1.3) initial stellar mass function. The initial period distribution is flat ranging from 10^3 to $10^{7.5}$ days, but we also perform simulations with a realistic distribution of periods which rises with increasing $P > 3$ days and which is consistent with pre-main sequence observational constraints. For comparison we simulate the evolution of single star clusters. Approximately after the first two initial relaxation times, all clusters evolve according to the same $n(t) \propto \exp(-t/\tau_e)$ curve, where $n(t)$ is the number density of stars in the central 2 pc sphere at time t and $\tau_e \approx 230$ Myrs. All clusters have the same lifetime τ. n(t) and τ are thus independent of (i) the inital proportion of binaries and (ii) the initial $R_{0.5}$. Mass segregation measures the dynamical age of the cluster: the mean stellar mass inside the central region increases approximately linearly with age. The proportion of binaries in the central cluster region is a sensitive indicator of the initial cluster concentration: it declines within approximately the first 10–20 initial relaxation times and rises only slowly with age, but for $R_{0.5} < 0.8$ pc, it is always significantly larger than the binary proportion outside the central region. If most stars form in binaries in embedded clusters that are dynamically equivalent to a cluster specified initially by $(N_{\mathrm{bin}}, R_{0.5}) = (200, 0.85\,\mathrm{pc})$, which is located at the edge of a $1.5 \times 10^5\,M_\odot$ molecular cloud with a diameter of 40 pc, then we estimate that at most about 10 per cent of all pre-main sequence stars achieve near escape velocities from the molecular cloud. The large ejection velocities resulting from close encounters between binary systems imply a distribution of young stars over large areas surrounding star forming sites. This 'halo' population of a molecular cloud complex is expected to have a significantly reduced binary proportion (about 15 per cent or less) and a significantly increased proportion of stars with depleted or completely removed circumstellar disks. In this scenario, the distributed population is expected to have a similar proportion of binaries as the Galactic field (about 50 per cent). If a distributed population shows orbital parameter distributions not affected by stimulated evolution (e.g. as in Taurus–Auriga) then it probably originated in a star-formation mode in which the binaries form in relative isolation rather than in embedded clusters. The Hyades Cluster luminosity function suggests an advanced dynamical age. The Pleiades luminosity function data suggest a distance modulus $m - M = 6$, rather than 5.5. The total proportion of binaries in the central region of the Hyades and Pleiades Clusters are probably 0.6–0.7. Any observational luminosity function of a Galactic cluster must be corrected for unresolved binaries when studying the stellar mass function. Applying our parametrisation for open cluster evolution we estimate the birth masses of both clusters. We find no evidence for different dynamical properties of stellar systems at birth in the Hyades, Pleiades and Galactic field stellar samples. Parametrising the depletion of low mass stars in the central cluster region by the ratio, $\zeta(t)$, of the stellar luminosity function at the 'H$_2$–convection peak' ($M_{\mathrm{V}} \approx 12$) and 'H$^-$ plateau' ($M_{\mathrm{V}} \approx 7$), we find good agreement with the Pleiades and Hyades $\zeta(t)$ values. The observed proportion of binary stars in the very young Trapezium Cluster is consistent with the early dynamical evolution of a cluster with a very high initial stellar number density.

Keywords: stars: low mass, formation, luminosity function – binary stars – Galactic clusters and associations: dynamical evolution, individual: Hyades, Pleiades, Trapezium

1 INTRODUCTION

We refer to the stellar mass function, the proportion of stellar systems (singles, binaries, triples, etc.) and the distribution of their orbital parameters as the *dynamical properties of stellar systems*. Studying Galactic clusters is important because they represent fossils of discrete star formation events. From them we can hope to obtain information on the dependence of the dynamical properties of stellar systems on the birth conditions.

The majority of stars in the Galactic disk may be born in embedded clusters rather than in isolation. This conclusion is drawn by Lada & Lada (1991) after studying the distribution of young stellar objects in the L1630 molecular cloud of the Orion complex. In it they find no significant distributed population of young stars but four embedded clusters containing at least about 627 objects. Strom, Strom & Merrill (1993), on the other hand, find a distributed population of about 1500 young stars in the L1641 molecular cloud in Orion, as well as seven clusters, in which 10–50 young stellar objects have been detected, and one partially embedded cluster, in which 150 young stellar objects have been detected. The ages (< 1 Myrs) of the stellar objects in the detected clusters appear to be younger than in the distributed population (5–7 Myrs). This is consistent with the latter having originated in aggregates which are now dissolved. Nevertheless, it is evident that the observational evidence for a predominant clustered star formation mode remains suggestive rather than conclusive. However, if it is assumed that star formation nearly always produces a binary star (as suggested by observations of pre-main sequence stars) then clustered star formation *must* be the dominant mode, rather than distributed star formation (Kroupa 1995a, hereafter K1).

This paper is the third in a series of three papers K1, K2 and K3. In K1 we study the evolution of a binary star population in stellar aggregates and use the results for inverse dynamical population synthesis on the observed dynamical properties of stellar systems in the Galactic field to deduce that a dominant clustered mode of star formation may exist. In K1 we suggest that most Galactic field stars may have been born in aggregates that are dynamically equivalent to the 'dominant mode cluster', which is defined by $(N_{\rm bin}, R_{0.5}) = (200, 0.85\,{\rm pc})$, where $N_{\rm bin}$ is the initial number of binaries and $R_{0.5}$ is the initial half mass radius. We also derive an initial distribution of periods. In K2 (Kroupa 1995b) we study in detail the dynamical properties of Galactic field stellar systems if they form in the dominant mode cluster. In this paper (K3) we make use of the N-body simulations of K1 and K2 to study the overall evolution of the stellar clusters.

The realistic KTG(1.3) stellar mass function (Section 2) is adopted, and all stars are initially in aggregates, or clusters, of binary systems. These have a period and mass ratio distribution consistent with pre-main sequence data. Our initial very large proportion of primordial binaries is also consistent with observational evidence that a large proportion of binaries may reside in the central region of at least one Galactic cluster (the Praesepe Cluster, Kroupa & Tout 1992). As a control experiment we simulate single-star clusters. We are interested to learn if and how the initial conditions of our simulations are observable in real Galactic clusters. By studying the rate at which the clusters dissolve and the evolution of mass and binary star segregation the evolutionary history of a Galactic cluster can probably be traced. The stellar luminosity function in the central regions of clusters evolves as a result of mass segregation, evaporation of stars and increase of the proportion of binary systems. We also investigate if the distribution of stellar systems in space and in velocity after dissolution of the aggregates can be used to obtain clues about the initial dynamical configuration.

We concentrate on readily observable properties of stellar clusters and do not mean to provide a comprehensive treatment of their evolution which has been extensively studied elsewhere: Mathieu (1985) describes the internal kinematics and the structure of Galactic clusters, Wielen (1985) discusses their dynamics, and Aarseth (1988a,b) reviews the dynamical evolution of open clusters and discusses core collapse, respectively, based on the extensive and detailed simulations of Terlevich (1987). The ejection of stars from open clusters containing binaries is studied in the context of OB runaway stars by Leonard & Duncan (1990). The hypothesis that blue stragglers may result from collisions and merging of finite sized stars in clusters is discussed by Leonard & Linnell (1992). Hut (1985) emphasises that binaries can be considered a dynamical energy source in a cluster similar to nuclear reactions in a star. A comprehensive review of the role of binaries in Globular cluster dynamics is provided by Hut et al. (1992). Heggie & Aarseth (1992) and McMillan & Hut (1994) consider the evolution of globular clusters consisting of equal-mass stars and containing an

initial proportion of binaries of up to 20 per cent. These authors, and the model of the binary star evolution in a globular cluster devised by Hut, McMillan & Romani (1992), provide many valuable insights into the processes that govern the interaction of the cluster with the population of binary systems.

In Section 2 we briefly summarise the assumptions, definitions and our method. The evolution of four binary star clusters and two single star clusters is discussed in Section 3 and compared with the evolution of the dominant mode cluster in Section 4, where we also elaborate on the concept of 'dynamical equivalence' introduced in K1. The kinematical signature of star formation is addressed in Section 5. In Section 6 we apply our simulations to three clusters and discuss the stellar luminosity function. We conclude with Section 7.

2. ASSUMPTIONS, DEFINITIONS AND METHOD

A detailed description of our assumptions and method can be found in section 3 of K1, which we briefly summarise here.

We distribute $N_{\rm bin} = 200$ binary systems according to a Plummer density law with half mass radii $R_{0.5} = 0.077, 0.25, 0.77, 2.53$ pc. $R_{0.5} = 0.08$ pc corresponds to an initially highly concentrated cluster similar to the Trapezium Cluster, whereas $R_{0.5} = 2.53$ pc corresponds to a loose cluster which approximates distributed star formation (e.g. Taurus–Ariga). We also distribute $N_{\rm sing} = 400$ single stars in clusters with $R_{0.5} = 0.077, 0.25$ pc. These are of academic interest only and serve as a comparison with the realistic binary star clusters. Virial equilibrium is assumed. For the N-body simulation of the dynamical evolution of each cluster we use the program *nbody5* written by Aarseth (1994). We model a standard Galactic tidal field (see K1).

Stellar masses in the range $0.1\,M_\odot \leq m \leq 1.1\,M_\odot$ are obtained from the initial mass function which is conveniently approximated by $\xi(m) \propto m^{-\alpha_i}$, $\alpha_1 = 1.3$ for $0.08\,M_\odot \leq m < 0.5\,M_\odot$, $\alpha_2 = 2.2$ for $0.5\,M_\odot \leq m < 1.0\,M_\odot$, $\alpha_3 = 2.7$ for $1.0\,M_\odot \leq m$ and $\xi(m)\,dm$ is the number of stars with masses in the range m to $m + dm$ (Kroupa et al. 1993). We refer to $\xi(m)\,dm$ as the KTG(α_1) mass function. Our adopted stellar mass range allows us to ignore post-main sequence stellar evolution which simplifies the computation and allows focusing on purely stellar dynamical evolution. If we have a population of stars with masses $0.1\,M_\odot \leq m < \mathcal{A}$, then for $\mathcal{A} = 10\,(50)\,M_\odot$, about 8 per cent of these are more massive than $1.1\,M_\odot$, and contribute 35 (39) per cent to the total mass. Stars more massive than about 5 (10) $M_\odot$ will affect the dynamical evolution of the clusters within the first 7×10^7 (2×10^7) yrs, but will be insignificant during most of the lifetime of the clusters studied here. The mean stellar mass in our models is $0.32\,M_\odot$ and the mass of each cluster is $128\,M_\odot$, which is near the peak of the mass function of Galactic clusters (Battinelli, Brandimarti & Capuzzo-Dolcetta 1994).

To build the initial binary stars we combine the stellar masses at random and distribute orbital periods, P (in days), from a flat distribution, $f_{\rm P}(\log_{10}P) = [\log_{10}(P_{\rm max}) - \log_{10}(P_{\rm min})]^{-1}$ (equation 3 in K1), with $\log_{10}P_{\rm min} = 3$, $\log_{10}P_{\rm max} = 7.5$ and $P_{\rm min} \leq P \leq P_{\rm max}$. The initial mass-ratio and period distribution are consistent with pre-main sequence binary star data (K1). The initial eccentricity distribution is assumed to be dynamically relaxed, $f_e(e) = 2\,e$ (equation 4 in K1), but is not critical here.

For each binary and single-star cluster we perform $N_{\rm run} = 5$ and 3 simulations, respectively. All results quoted here are averages of $N_{\rm run}$ simulations.

In addition to the simulations of the above six clusters we perform $N_{\rm run} = 20$ simulations of the dominant mode cluster studied by K2 which has $R_{0.5} = 0.85$ pc initially. The $N_{\rm bin} = 200$ binary systems per simulation have a mass-ratio distribution and a birth eccentricity distribution as above, but a birth period distribution given by equation 8 in K2. The birth orbital parameters of the short period ($\log_{10}P < 2 - 3$) binary systems are assumed to eigenevolve during the proto-stellar accretion phase on a timescale of approximately 10^5 yrs (see section 2 in K2). The resulting distribution of orbital elements is the initial ($t = 0$) distribution for the N-body simulation, and is consistent with the mass-ratio and eccentricity distributions observed for short-period G dwarfs, i.e. a bias towards equal mass components and a bell shaped eccentricity distribution, respectively. The minimum orbital period is about 3 days, and in our model about 3 per cent of all systems are merged binaries at the start of the N-body simulation. The initial period distribution can be approximated by $f_{\rm P,in}(\log_{10}P) = 3.50\log_{10}P\,/[100 + (\log_{10}P)^2]$ (equation 11 in K1, see also fig. 7 in K2). Stimulated evolution (i.e. the changes in orbital parameters due to interactions with other systems) does not significantly change the orbital parameters of binaries with $\log_{10}P < 3$.

Physical parameters for each cluster discussed here are listed in table 1 of K1, and in Table 1 we repeat some of these.

Table 1. Initial stellar clusters

$R_{0.5}$	N_{bin}	N_{sing}	f_{tot}	n	$\log_{10}(n_{\mathrm{c}})$	σ	T_{cr}	T_{relax}
pc			$t=0$	stars pc^{-3}	stars pc^{-3}	km sec^{-1}	Myrs	Myrs
0.08	200	0	1	12	5.6	1.7	0.094	0.30
0.25	200	0	1	12	4.1	0.9	0.54	1.8
0.77	200	0	1	11	2.7	0.5	3.0	9.5
2.53	200	0	1	5	1.1	0.3	17	56
0.08	0	400	0	12	5.6	1.7	0.094	0.30
0.25	0	400	0	12	4.1	0.9	0.54	1.8
0.85	200	0	1	10	2.5	0.5	3.5	11

n_{c} is the central number density; σ the average velocity dispersion; T_{cr} and T_{relax} are, respectively, the crossing and relaxation times (for further details see section 3 in K1).

We refer to a single star as a single star system and define the overall proportion of binary systems at time t to be $f_{\mathrm{tot}}(t) = N_{\mathrm{bin}}(t)/(N_{\mathrm{bin}}(t) + N_{\mathrm{sing}}(t))$ (equation 2 in K1). This definition extends to stellar systems in any sub-domain, e.g. if orbits only in a specific period range are accessible (cf. with equation 7 in K1) or if $f_{\mathrm{tot}}(t)$ is evaluated in a particular volume of space.

The simulations of the dynamical evolution of the above seven clusters allow an intercomparison of the evolution of overall properties of the stellar systems such as stellar number density, mean stellar mass and binary star segregation, and the distribution of centre-of-mass kinetic energies (in the local rest frame). To quantify the evolution of the clusters we measure the number density, $n(t)$, mean stellar mass, $\overline{m}(t)$, and the overall proportion of binaries, $f_{\mathrm{tot}}(t)$, within a standard spherical volume, V, with radius R centered on the number density maximum of the cluster. For example, if we take $R = 2$ pc then we refer to V as the *central 2 pc sphere*. $f_{\mathrm{tot}}(t)$ is defined above, and in this context $N_{\mathrm{sing}}(t)$ and $N_{\mathrm{bin}}(t)$ are the number of single and binary systems in V, respectively. Similarly, $\overline{m}(t) = M_{\mathrm{stars}}(t)/(N_{\mathrm{sing}}(t) + 2\,N_{\mathrm{bin}}(t))$, where $M_{\mathrm{stars}}(t)$ is the total stellar mass in V, and $n(t) = (N_{\mathrm{sing}}(t) + 2\,N_{\mathrm{bin}}(t))/V$. The three quantities $n(t)$, $\overline{m}(t)$ and $f_{\mathrm{tot}}(t)$ are readily accessible to an observer ($t =$cluster age). If an instrumental flux limit and/or a resolution limit limits the observations then we can in principle apply the same bias to our model values $n(t)$, $\overline{m}(t)$ and proportion of binaries. This will be necessary in a case-by-case treatment of individual Galactic clusters.

For conciseness we write $N_{\mathrm{bin}} = N_{\mathrm{bin}}(0)$, $N_{\mathrm{sing}} = N_{\mathrm{sing}}(0)$, and $R_{0.5} = R_{0.5}(0)$ in the knowledge that the half mass radius of a cluster evolves. We do not evaluate $R_{0.5}(t > 0)$ because this is non trivial requiring exact knowledge of escapers. In Section 3.1 we show that all clusters with 400 stars disintegrate completely after 800 Myrs, and from hereon we refer to the final distributions of, say binary star binding energies, as the distributions evaluated at $t = 1$ Gyr that result after cluster disintegration.

3. CLUSTER EVOLUTION

In this section we concentrate on the evolution of the number density, $n(t)$, mean stellar mass, $\overline{m}(t)$, and proportion of binaries, $f_{\mathrm{tot}}(t)$, in the four stellar clusters with $N_{\mathrm{bin}} = 200$ and $R_{0.5} = 0.077, 0.25, 0.77, 2.53\,\mathrm{pc}$, and in the two single-star clusters with $N_{\mathrm{sing}} = 400$ and $R_{0.5} = 0.077, 0.25$ pc.

3.1 Lifetime and birthrate

From Fig. 1 we infer that there is no significant difference in the evolution of $n(t)$ between the various clusters. After the first two relaxation times, $n(t)$ for the $R_{0.5} = 2.53$ pc cluster joins the other evolutionary curves and the clusters depopulate at the same rate, irrespective of the presence of primordial binaries (some of which are hard - see Fig. 5 below) and of the initial cluster concentration. A reasonable approximation of the depopulation of the central 2 pc sphere is

$$\log_{10} n(t) = 1.0 - 1.9 \times 10^{-3} \, t, \tag{1a}$$

where t is in Myrs and $t < 600$ Myrs. Equation 1a can be rewritten to

$$\frac{n(t)}{10} = \mathrm{e}^{-\frac{t}{\tau_e}}, \tag{1b}$$

where $\tau_e \approx 230$ Myrs is the exponential decay time.

We measure the disintegration time of each cluster to be the time taken until the number density has reached $0.1 \, \mathrm{stars \, pc^{-3}}$ (i.e. only three stars remain in the central 2 pc sphere), which is characteristic of the Galactic disk in the proximity of the Sun. The mean life time, $\tau_{0.1}$, is thus a strict upper limit and is listed in table 1 of K1. From an observational point of view we might expect that a stellar cluster with about 100 stars in the central 2 pc sphere (i.e. $n(t) \approx 3$ stars $\mathrm{pc^{-3}}$) might be closer to the detection limit of an open cluster. We therefore also consider the alternative definition of the lifetime of a cluster $n(\tau_{3.1}) = 3.1$ stars $\mathrm{pc^{-3}}$ (i.e. $1 \, M_\odot \, \mathrm{pc^{-3}}$, c.f. Lada & Lada 1991). From equation 1 and Fig. 1 we obtain $\tau_{3.1} \approx 250$ Myrs.

We conclude that clusters with $R_{0.5} \geq 0.077$ pc and initially with 400 stars have lifetimes $\tau_{0.1} \approx 700$ Myrs and $\tau_{3.1} \approx 250$ Myrs. McMillan & Hut (1994) also note that primordial binaries do not significantly affect the cluster evaporation timescale. The finding that $\tau_{0.1}$ and $\tau_{3.1}$ are independent of $N_{\mathrm{bin}}, N_{\mathrm{sing}}$ (as long as $N_{\mathrm{sing}} + 2 \, N_{\mathrm{bin}} = 400$) and $R_{0.5}$ disproves the theory of cluster lifetimes based on the assumption of a constant rate of evaporation from a cluster. Wielen (1988) shows that this theory implies a constant lifetime for $R_{0.5} > 0.2$ pc, and a steep decay of the lifetime for $R_{0.5} < 0.2$ pc.

Our $\tau_{0.1}$ is about 70 times as large as the lifetime of real clusters found by Battinelli & Capuzzo-Dolcetta (1991). In part this is due to our $\tau_{0.1}$ being a strict upper limit whereas the lifetime discussed by Battinelli & Capuzzo-Dolcetta (1991) refers to the time it takes for half of all clusters to be destroyed, but our result does confirm that other mechanisms than internal dynamical evolution must be responsible for cluster disintegration. For example, encounters with giant molecular clouds are destructive (Terlevich 1987) and can reduce cluster lifetimes on average to approximately 100 Myrs (Theuns 1992).

Little is known about the typical star-forming systems that eventually lead to the stellar population in the Galactic disk. Our inverse dynamical population synthesis in K1 indicates that the majority of Galactic field stars may result from clustered star formation. However, Wielen (1971) finds that only a few per cent of the Galactic disk stars are born in Galactic clusters by showing that there are too few Galactic clusters in total and that the lifetimes of the clusters are too long. Thus, if clustered star formation is the dominant mode then most birth clusters of young stellar objects are not bound gravitationally after dispersal of the remnant cloud material. Conversely, the Galactic clusters must remain gravitationally bound and probably result from rather rare incidences of high local star formation efficiency. These issues are discussed in greater detail by Lada, Margulis & Dearborn (1984), Mathieu (1986), Pinto (1987) and Verschueren & David (1989).

If the majority of low mass stars in the Galactic disc are born in initially unbound clusters of, say 200 binary systems, then we require a cluster birth rate of approximately $15 \, \mathrm{clusters \, kpc^{-2} \, Myr^{-1}}$, assuming a constant birth rate over 10^{10} yr, a vertical Galactic disc scale height of 300 pc and a local stellar number density of $0.1 \, \mathrm{pc^{-3}}$. This is approximately 30 times the birth rate for the Galactic clusters as obtained by Battinelli et al. (1994). If the majority of stars form in embedded clusters then their sample cannot be complete. The observational catalogues probably do not account for the total birth sample which includes initially embedded and later unbound aggregates of a few hundred low mass pre-main sequence stars.

3.2 Mass and binary star segregation

For a given Galactic cluster we are unable to observe the entire birth population, but we can obtain data from the central region quite readily. The distribution of stellar systems in the central region of a cluster is determined by its past dynamical evolution.

From Fig. 2 it is evident that the mean stellar mass within the central 2 pc sphere increases linearly until disintegration time when $\overline{m}(t) \approx 0.45 \, M_\odot$, whereas outside it is $0.32 \, M_\odot$. Mass segregation does not strongly depend on the initial cluster size (see also Fig. 4 below). Writing

$$\overline{m}(t) = 0.32 \, M_\odot + \mu \, t, \tag{2}$$

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with $t < 600$ Myrs in Myrs, we obtain $\mu = 0.28 \times 10^{-3} \, M_\odot \, \mathrm{Myr}^{-1}$ ($R_{0.5} = 2.53$ pc), $\mu = 0.23 \times 10^{-3} \, M_\odot \, \mathrm{Myr}^{-1}$ ($R_{0.5} = 0.77$ pc), $\mu = 0.22 \times 10^{-3} \, M_\odot \, \mathrm{Myr}^{-1}$ ($R_{0.5} = 0.25$ pc), $\mu = 0.20 \times 10^{-3} \, M_\odot \, \mathrm{Myr}^{-1}$ ($R_{0.5} = 0.08$ pc). However, mass segregation is significantly more pronounced in the single-star clusters where we obtain $\mu \approx 0.37 \times 10^{-3} \, M_\odot \, \mathrm{Myr}^{-1}$. This results because the mean mass per centre-of-mass particle is larger in the binary star clusters than in the single star clusters. In all panels of Fig. 2 the mean stellar mass outside the central 2 pc sphere decreases within the first 50 Myrs because low-mass stars are ejected preferentially. The 'escaped' low mass stars have a smaller mean stellar mass in the single-star clusters because in the binary star clusters the least massive stars are initially bound in binary systems. Some of these will form a halo population being unable to find an exit in the equipotential surface of the remnant cluster plus Galaxy (see e.g. Terlevich 1987).

In fig. 3 of K1 we show that the overall proportion of binary systems $f_{\mathrm{tot}}(t)$ (counting all cluster members and non-members) is depleted at a rate which is a sensitive function of the initial concentration. In Fig. 3 we see that the overall proportion of binaries within the central 2 pc sphere, $f_{\mathrm{in}}(t)$, is significantly larger than the proportion of binaries outside, $f_{\mathrm{out}}(t)$. The spatial segregation of binary systems results because their mean mass is larger than that of single stars. After the initially rapid ionisation of the wide binaries, f_{in} increases only by a small amount as the cluster ages. This result is also obtained by McMillan & Hut (1994). For the four clusters we define the ratio $r_1(R_{0.5}) = f_{\mathrm{out}}/f_{\mathrm{in}}$ and $r_2(R_{0.5}) = f_{\mathrm{tot}}^{\mathrm{obs}}/f_{\mathrm{in}}$ at approximately 600 Myrs. In the Galactic field $f_{\mathrm{tot}}^{\mathrm{obs}} = 0.47 \pm 0.05$ (K1, K2). The ratios r_1 and r_2 are tabulated in Table 2.

Table 2. Binary stars in the central 2 pc sphere at $t = 600$ Myrs

$R_{0.5}$ pc	r_1	r_2
0.08	0.53	0.93
0.25	0.63	0.74
0.77	0.75	0.58
2.53	1.0	0.57

Table 2 illustrates that the proportion of binaries outside the 2 pc sphere drops significantly with decreasing initial cluster size (r_1). The proportion of binaries in the central 2 pc sphere can, however, be similar in an initially highly concentrated cluster to that observed in the field (r_2) (compare with the Trapezium Cluster in Section 6.3). This population of binary stars is depleted significantly at $\log_{10} P \geq 5$ when compared to the Galactic field population (see fig. 5 in K1).

4 THE DOMINANT MODE CLUSTER AND DYNAMICAL EQUIVALENCE

In this section we study the evolution of number density, mass segregation and binary star proportion in the dominant mode cluster $[(N_{\mathrm{bin}}, R_{0.5}) = (200, 0.85 \, \mathrm{pc})]$, and we investigate which combination of $R_{0.5}$ and N_{bin} might define clusters that are dynamically equivalent to the dominant mode cluster.

4.1 Evolution of the Dominant Mode Cluster

In the top panel of Fig. 4 we show the average number density evolution, $n(t)$. It does not differ from the evolution of the six clusters discussed above, confirming our conclusion that after the first two initial relaxation times $n(t)$ does not depend on initial values of $R_{0.5}$, $\log_{10} P_{\mathrm{min}}$ or f_{tot}. Our dominant mode cluster has a lifetime $\tau_{0.1} = 740 \pm 150$ Myrs.

Mass segregation proceeds similarly to the six clusters discussed in Section 3 (Fig. 4). The initial mean stellar mass, $\overline{m}(0)$, is somewhat larger in the present simulations because a few per cent of all binaries have merged to single stars during pre-main sequence eigenevolution (K2). However, the slope of the approximately linear $\overline{m}(t)$ relation is about the same as for our $R_{0.5} = 0.77$ pc cluster (equation 2), and is smaller than for the single-star clusters.

The evolution of the overall proportion of binaries, $f_{\mathrm{tot}}(t)$, inside and outside the 2 pc sphere is shown in the bottom panel of Fig. 4. As found in Section 3, $f_{\mathrm{in}}(t)$ first decreases rapidly to a minimum ($f_{\mathrm{in}} \approx 0.6$)

after about 10 initial relaxation times and then rises slowly. Also, $r_1 = 0.73 \approx r_2 = 0.71$ as expected for the dominant mode cluster (compare with Table 2). However, $f_{\text{in}}(R_{0.5} = 0.85\,\text{pc}) < f_{\text{in}}(R_{0.5} = 0.77\,\text{pc})$ because our present initial binary star population extends to larger periods than in the $R_{0.5} = 0.77$ pc simulation discussed in Section 3. The large proportion of binaries in the central cluster region strengthens the finding by Kroupa & Tout (1992) that a large proportion of binary systems is consistent with the distribution of data in the colour–magnitude diagram of the Praesepe Cluster which has an age of about 8×10^8 yrs (Cayrel de Strobel 1990).

Fig. 4 demonstrates the following points: (i) $n(t)$ is invariant to the initial conditions; (ii) $\overline{m}(t)$ increases approximately linearly with time at a rate μ that depends primarily on whether the cluster is composed initially of binaries or single stars and only secondarily on $R_{0.5}$, and a zero point, $\overline{m}(0)$, that depends on the stellar mass function and on whether some of the birth binaries merge to more massive single stars; and (iii) $f_{\text{in}}(t)$ is approximately constant, but depends sensitively on the initial $R_{0.5}$ and on the initial distribution of periods.

4.2 Dynamically equivalent clusters?

We define a stellar aggregate or cluster, which is *dynamically equivalent* to the dominant mode cluster, to be an aggregate or cluster initially not necessarily in virial equilibrium with $R'_{0.5} \neq R_{0.5} = 0.85$ pc and $N'_{\text{bin}} \neq N_{\text{bin}} = 200$, in which the initial dynamical properties of stellar systems, as defined in Section 2, evolve to distributions after cluster disintegration, that are consistent with the observed distributions in the Galactic field.

Assuming the dynamical properties of stellar systems at birth are invariant to the initial conditions, we postulate that $n(t)$, $\overline{m}(t)$ and $f_{\text{in}}(t)$ uniquely specify the evolutionary path of a stellar cluster in virial equilibrium. That is, if we can measure these quantities for some Galactic cluster then we can probably uniquely specify the initial conditions ($N_{\text{bin}}, R_{0.5}$). In Section 3 we have shown that after two initial relaxation times, $n(t)$ is independent of the initial $R_{0.5}$. From Wielen (1988, fig. 2) and Terlevich (1987, fig. 3) we find that τ_l and $n(t)$ can be scaled to any initial $n'(0)$ (i.e. N'_{bin}) by a multiplicative factor:

$$\tau'_l = \frac{N'_{\text{bin}}}{N_{\text{bin}}}\, \tau_l \tag{3a}$$

and

$$n'(t) = \frac{N'_{\text{bin}}}{N_{\text{bin}}}\, n(t), \tag{3b}$$

where τ_l is $\tau_{0.1}$ or $\tau_{3.1}$. The exponential decay time τ_e is invariant to changes in initial conditions. The evolution of $\overline{m}(t)$ and $f_{\text{tot}}(t)$, however, depends on $R_{0.5}$.

We probably obtain the same overall stimulated evolution of orbital parameters if $n_c(0)\,\sigma(0) \approx 160$ stars pc^{-2} Myr^{-1}, where $n_c(0)$ and $\sigma(0)$ are the initial central number density and average velocity dispersion, respectively (Table 1). This implies $R_{0.5} \propto N_{\text{bin}}^{\frac{3}{7}}$, i.e.

$$R'_{0.5} \approx 0.088\, N_{\text{bin}}^{\frac{\prime 3}{7}}\ \text{pc}, \tag{4}$$

for aggregates that are dynamically equivalent to our dominant mode cluster. Alternatively, if we take the ratio of $\tau_{0.1}/T_{\text{relax}}$ to be an estimate of the number of relaxations the system undergoes in its lifetime $\tau_{0.1}$ then our dominant mode cluster is characterised by $\tau_{0.1}/T_{\text{relax}} \approx 70$. For constant $\tau_{0.1}/T_{\text{relax}}$ we obtain $R_{0.5} \propto N_{\text{bin}}^{\frac{1}{3}}\, [\log_{10}(0.8\, N_{\text{bin}})]^{\frac{2}{3}}$, i.e.

$$R'_{0.5} \approx 0.086\, N_{\text{bin}}^{\frac{\prime 1}{3}}\, [\log_{10}(0.8\, N'_{\text{bin}})]^{\frac{2}{3}}\ \text{pc}, \tag{5}$$

using the definition for T_{relax} (K1) and the scaling property of $\tau_{0.1}$ above.

Simulations of clusters with $R'_{0.5}$ and N'_{bin} scaled according to equations 4 and 5 are necessary to verify our postulate and assertions. Additional simulations including stars more massive than $1.1\,M_\odot$ and other initial configurations, such as subclustering, cold collapse and/or changing background potential owing to gas removal, which must be important in the first 10 Myrs of cluster evolution, will be necessary for a

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comparison with real open clusters. Furthermore, the evolution of real Galactic clusters can be affected by passing interstellar clouds (Terlevich 1987, Theuns 1992), which ought to be kept in mind when comparing Galactic clusters with models.

While we can apply the concept of dynamical equivalence to embedded clusters that disperse within about 10^7 yrs by our definition in the first paragraph of this section, we cannot apply equations 1–5 to these (see also section 6.4 in K1).

5 THE KINEMATICAL SIGNATURE OF STAR FORMATION

Past work (Heggie 1975, Hills 1975) has etablished that binary systems are ionised at a rate which is a function of the ratio of the internal binding energy, $-E_{\rm bin}$, of the binary and the centre of mass kinetic energy, $E_{\rm kin}$, or temperature of the surrounding field population. The hardening of relatively hard ($E_{\rm bin}/E_{\rm kin} > 1$) binaries heats the field thereby increasing the number of systems with relatively large $E_{\rm kin}$. In this section we study the observational consequences of these processes. Our aim is to understand the evolution of the binary star binding energy distribution, $f_{E_{\rm bin}}$, and of the centre of mass kinetic energy distribution, $f_{E_{\rm kin}}$, as the clusters evolve. We then apply the insights gained to star forming aggregates by invoking dynamical equivalence.

5.1 Energetics

For each of the four binary star clusters discussed in Section 3 we compute $f_{E_{\rm bin}}$ and $f_{E_{\rm kin}}$. For comparison we also compute $f_{E_{\rm kin}}$ for the single-star clusters discussed in Section 3. The final $f_{E_{\rm kin}}$ are tabulated in Table A-1. In Fig. 5 we show the initial distributions and the final distributions after cluster dissolution. Units of energy are $M_\odot$ km^2/sec^2 (i.e. in units of 1.99×10^{43}erg).

The initial $f_{E_{\rm bin}}$ are equal in all four binary star clusters, but the initial kinetic energies increase with decreasing cluster radius. The increasing overlap of the initial $f_{E_{\rm bin}}$ and $f_{E_{\rm kin}}$ with decreasing cluster size, and the increased number density, leads to an increasingly efficient destruction rate of binary systems. A substantial number of initial binaries are hard (i.e. have $E_{\rm bin}/E_{\rm kin}^{\rm max} > 1$), even in our most compact initial cluster ($R_{0.5} = 0.08$ pc, $E_{\rm kin}^{\rm max} \approx 4\,M_\odot$ km^2 sec^{-2}) and even though $P_{\rm min} > 2.7$ yrs. The final $f_{E_{\rm kin}}$ in the binary star clusters is significantly enhanced at $\log_{10}E_{\rm kin} > -1$ when compared to the single star clusters, which have initially the same $R_{0.5}$ (0.25 and 0.08 pc). Our most compact binary star cluster with $R_{0.5} = 0.08$ pc has a final high kinetic energy tail that is virtually identical to the high binding energy tail (Fig. 6). We also observe in Fig. 5 that increasingly harder binaries appear with decreasing $R_{0.5}$, being the result of enhanced stimulated evolution per unit time in a cluster with smaller initial crossing time.

The final high kinetic and binding energy tails come about because binaries with $E_{\rm bin} > E_{\rm kin}^{\rm max}$ on average acquire additional binding energy after interaction with a field particle and are not ionised (Heggie 1975). The perturbing star or system involved in the energy exchange gains kinetic energy. Energy exchange proceeds until the cluster is depopulated. While the average lifetime of the open cluster is not affected (Section 3.1), the final $f_{E_{\rm bin}}$ and $f_{E_{\rm kin}}$ contain information on the initial distribution of orbital periods, the initial velocity dispersion and the initial number density. Using the data in Table A-1 we compare in Table 3 for each cluster the final proportion of systems with $\log_{10}E_{\rm kin} \geq 1$.

Table 3. Fraction, $g_{E_{\rm kin}}$ in per cent, of center-of-mass systems with $E_{\rm kin} \geq 10\,M_\odot$ km^2 sec^{-2}

$R_{0.5}$ pc	$g_{E_{\rm kin}}$ $[f_{\rm tot}(t=0)=1]$	$[f_{\rm tot}(t=0)=0]$
0.08	8.0	2.2
0.25	4.6	1.1
0.77	2.0	–
2.53	0.2	–

In K2 we study in detail the distribution of dynamical properties of stellar systems that result if the majority of stars in the Galactic disk are born in the dominant mode cluster specified by $(N_{bin}, R_{0.5}) = (200, 0.85 \, pc)$. The distribution of binary star periods we adopt in K2 is more realistic than the flat distribution assumed for the four binary star clusters above, because it spans the entire observed range from $\log_{10} P_{min} \approx 0.5$ to $\log_{10} P \approx 9$. It is useful to compare this larger set of simulations of the dominant mode cluster with the results of the $(N_{bin}, R_{0.5}) = (200, 0.77 \, pc)$ cluster discussed above, and to extend our analysis to the observational plane, i.e. to study the distribution of velocities of centre-of-mass systems.

In Fig. 7 we compare the initial and final $f_{E_{kin}}$ and $f_{E_{bin}}$. The former are virtually identical while the latter differ significantly. The initial $f_{E_{bin}}$ extends to much higher values in the realistic case. These hard binaries, initially with $E_{bin}/E_{kin}^{max} > 25$ ($E_{kin}^{max} \approx 1 \, M_{\odot} \, km^2 \, sec^{-2}$), do not affect the dynamics of the cluster because their interaction cross sections are too small to be significant. The initially larger number of binaries in the $R_{0.5} = 0.77$ pc cluster, with $0.04 \leq E_{bin}/E_{kin}^{max} \leq 25$, do not significantly change the final distribution of E_{kin}, when compared with the $R_{0.5} = 0.85$ pc cluster (top panel of Fig. 7), because the clusters do not survive long enough to exhaust this binary star population. At cluster dissolution time a sufficient number of these binaries survives in both clusters. Nevertheless, it is these binaries which heat the cluster leading to the final high E_{kin} tail apparent in the top panel of Fig. 7. Finally, the binaries with $E_{bin}/E_{kin}^{max} < 0.04$ are ineffective energy sinks and are ionised without affecting the temperature. Thus, the different form of the initial $f_{E_{bin}}$ in the $R_{0.5} = 0.85$ pc cluster does not change the kinematical signature of star formation, which we now consider in greater detail.

In Fig. 8 we illustrate the changes in the E_{bin} and E_{kin} distributions that result after cluster dissolution. We observe a significant increase in the number of systems with $\log_{10} E_{kin} \leq -1.4$, which comes about because these systems had to overcome the cluster potential. Including gas removal during early cluster evolution would reduce this gain. There is also a significant increase in the number of systems with $E_{kin} > 0.32 \, M_{\odot} \, km^2 \, sec^{-2}$ which is due to binary star heating. The maximum final kinetic energy that results in our model is approximately $3.2 \times 10^3 \, M_{\odot} \, km^2 \, sec^{-2}$.

5.2 Kinematics

The interaction of multiple star systems, ionisation of binaries and the disintegration of a cluster changes the distribution of velocities of the stars. High velocity escapers, with velocities larger than $10 \, km \, sec^{-1}$ and up to $100 \, km \, sec^{-1}$ or larger (see Leonard & Duncan 1990), are the result of the internal dynamics of stellar systems. The distribution of velocities will be different for clusters consisting initially only of single stars than if these are composed of a large proportion of primordial binaries (Fig. 5). In this section we consider the $N_{run} = 20$ simulations of the dominant mode cluster $(N_{bin}, R_{0.5}) = (200, 0.85 \, pc)$.

In the top panel of Fig. 9 we show the initial distribution of centre of mass velocities. The dashed histogram shows the distribution in a Plummer sphere in virial equilibrium with velocity dipersion $\sigma \approx 0.5 \, km \, sec^{-1}$. The mean system mass at time $t = 0$, shown as open circles in the middle panel of Fig. 9, is $0.65 \, M_{\odot}$. It is independent of velocity and has the value expected for random pairing from our mass function (equation 1 in K1). After cluster disintegration we are left with essentially the same low-velocity tail of the centre of mass velocity distribution (solid histogram in the top panel of Fig. 9; note the distributions are normalised to unit area so that the increased number of systems with small velocity owing to cluster dissolution (see Figs. 7 and 8) is thus not apparent here). However, there is now an appreciable high velocity tail. About 1.5 per cent of all systems have a velocity greater than $10 \, km \, sec^{-1}$. The mean system mass decreases systematically with increasing velocity up to about 1–2 $km \, sec^{-1}$ (filled circles in the middle panel of Fig. 9) reflecting the establishment of equipartition of energy in a quasi-equilibrium system. For larger velocities there is no correlation because of the stochastic process of near encounters. The mean mass of stars in the second highest velocity bin is $0.82 \, M_{\odot}$, whereas the fastest star ejected from the cluster has a velocity of about $70 \, km \, sec^{-1}$ and a mass of $0.18 \, M_{\odot}$. Thus the change in character of the final velocity data at about 1–2 $km \, sec^{-1}$, which approximately corresponds to the maximum kinetic energy available in the initial cluster (Figs. 7 and 8 below), is a result of the process of equipartition of kinetic energy being replaced by the stochastic shedding of energy from relatively hard binaries to the field population.

The bottom panel of Fig. 9 shows the number of single stars and binaries initially and after cluster dissolution as a function of centre of mass velocity. After cluster disintegration most of the high velocity systems are single stars, with the two highest velocity bins containing no binaries. The small number of

single stars at $t = 0$ results from merging during pre-main sequence eigenevolution (see K2) and wide pairs being ionised immediately in the crowded central region of the cluster, where we have about 320 stars pc^{-3} (Table 1).

5.3 Discussion

Kinematical data of all stars of equal age in the vicinity of a star forming region should reveal similar distributions if the present scenario is correct, although we must keep in mind the bias introduced here by neglecting massive stars, stellar evolution effects and a changing background potential.

Including massive stars will probably raise the maximum velocity of ejected stars because more binding energy is available in massive binaries. Leonard & Duncan (1990) obtain runaway stars with velocities up to about 200 km sec^{-1} for $5\,M_\odot$ stars and up to about $50\,\mathrm{km}\ \mathrm{sec}^{-1}$ for $20\,M_\odot$ stars corresponding to $\log_{10} E_{\mathrm{kin}} = 5$ and $\log_{10} E_{\mathrm{kin}} = 4.4$, respectively. Stellar evolution effects tend to unbind the cluster reducing the work to be done when leaving the cluster potential, and would appear to be effective in Galactic clusters rather than embedded clusters, which have ages less than about 1–10 Myrs. Also, the initial dynamical evolution of a cluster of proto-stars may be dominated by a cold collapse rather than our assumed virial equilibrium. Aarseth & Hills (1972) demonstrate that higher ejection velocities are achieved in this case, but that the cluster relaxes within about two free-fall times to a configuration it would have had if it had been in virial equilibrium initially. In this case we would expect a larger number of high velocity escapers, but not a significantly different distribution of final kinetic energies. The overall distribution of E_{kin} should not change significantly if stars more massive than $1.1\,M_\odot$ are included, because of the steeply decreasing initial mass function with increasing stellar mass.

A much more significant effect is the expulsion of a significant amount of binding mass during the first 10 Myrs (Mathieu 1986, see also section 6 in K1). This would imply that the young cluster expands without having to overcome its own binding energy thereby more or less freezing $f_{E_{\mathrm{kin}}}$, as discussed in greater detail by Verschueren & David (1989, see also Pinto 1987).

Given our results in K1, we now postulate that most stars form in aggregates that are dynamically equivalent to the dominant mode cluster, and we contemplate the implications this has for the kinematics and distribution of young stars. The mass in stars is $M_{\mathrm{stars}} = \epsilon\,M_{\mathrm{tot}}(0)$, where $M_{\mathrm{tot}}(0)$ is the total initial mass of the star-forming core. Observations indicate that the star formation efficiency $\epsilon \approx 0.1 - 0.3$ (see e.g. Mathieu 1986) so that virial velocities of stellar systems in a very young aggregate will be dictated by the mass remaining in the gas, $M_{\mathrm{gas}}(t) = M_{\mathrm{tot}}(t) - M_{\mathrm{stars}}$. The initial velocity dispersion for such a system is $\sigma = 0.042 (M_{\mathrm{tot}}(0)/R_{0.5})^{\frac{1}{2}}$ km sec^{-1} (assuming virial equilibrium and a Plummer density profile), where $M_{\mathrm{tot}}(0)$ is in $M_\odot$ and $R_{0.5}$ is in pc. Thus, if $\epsilon = 0.1$, $R_{0.5} = 0.5$ pc and $M_{\mathrm{stars}} = 128\,M_\odot$, then $\sigma = 2.1$ km sec^{-1}. Assuming a mean system mass of $0.64\,M_\odot$ then about 95 per cent of all systems will have $E_{\mathrm{kin}} < 6\,M_\odot$ km^2 sec^{-2}. After 10 Myrs most stars will retain $E_{\mathrm{kin}} < 6\,M_\odot$ km^2 sec^{-2} irrespective of how rapidly $M_{\mathrm{gas}}(t)$ tends to zero. Dynamical equivalence implies that the number of systems with $E_{\mathrm{kin}} > 10\,M_\odot$ km^2 sec^{-1} will probably not be significantly different to the value given in Table 3. Bearing in mind possible cold collapse and massive stars, we roughly estimate $g_{E_{\mathrm{kin}}}(E_{\mathrm{kin}} > 10\,M_\odot$ km^2 sec$^{-2}) \approx 5$ per cent.

A kinetic energy $\log_{10} E_{\mathrm{kin}} = 1$ corresponds to the potential energy a young stellar system of mass $0.32\,M_\odot$ has at the edge of a spherical giant molecular cloud that has a typical mass of $1.5 \times 10^5\,M_\odot$ and diameter of 40 pc (Blitz 1993). At a distance of 8 kpc from the Galactic center, and assuming a mass of $10^{11}\,M_\odot$ within this distance, the tidal radius of the molecular cloud would be roughly 60 pc. This crude estimate suggests that the great majority of young stars may remain in the vicinity of a molecular cloud for its entire life time (a few 10^7 yrs) even though the high kinetic energy tail is enhanced by binary star heating. The molecular cloud will probably have a halo of young stars which have been ejected with velocities near to the escape velocity at formation site. They might be on orbits which are either bound to the molecular cloud, or they might have escape energy with escape delayed significantly until the stars "find an exit" in the openings of the equipotential surface formed by the cloud and the Galaxy (see e.g. Terlevich 1987). The presumed halo population will have had close encounters with relatively hard binary systems and will probably have lost much of the circumstellar material. The halo of young stars is thus expected to be enriched with wTTS (weak-lined T-Tauri stars). Given our simulations we expect of order of 5 per cent of

all stars formed in the molecular cloud to be deposited in this halo, but the details depend on the mass, shape and extend of the molecular cloud, and on the dynamics of the dominant mode embedded cluster (see also section 6 in K1). Of particular importance in this context is the detection of wTTS over a wide area surrounding the Orion star-forming region by Sterzik et al. (1995).

A kinetic energy of $E_{\mathrm{kin}} = 10$ km^2 sec^{-2} corresponds to a velocity $v = 8$ km sec^{-1} for a star of mass $0.32\,M_\odot$. Systems with $v < 5$ km sec^{-1} are less likely to escape the molecular cloud complex and systems with $v > 5$ km sec^{-1} have gained kinetic energy because of binary star heating. From our data (Table A-2) we find that about 95 per cent of all systems have a velocity $v < 5$ km sec^{-1} with respect to the star formation site. Bearing in mind a possible cold collapse and massive stars we roughly estimate that 90 per cent of all young systems are trapped in the molecular cloud complex. These systems, of which 50 per cent are binaries, may diffuse over a region 25 pc in radius over a time span of 5 Myrs. A young star may acquire additional kinetic energy if it falls towards the potential minimum of the molecular cloud and may cover a larger distance in 5 Myrs, and the formation site may have an additional velocity of a few km sec^{-1} with respect to the centre of mass of the molecular cloud. The remaining 5–10 per cent of all systems with velocities larger than 5 km sec^{-1} have a binary proportion of about 15 per cent and will spread throughout the molecular cloud, with only roughly 3 per cent of all systems having large enough velocities ($v > 8$ km sec^{-1}) to leave the cloud altogether.

This entire population may resemble the 5–7 Myrs old distributed population found by Strom et al. (1993) in the L1641 molecular cloud, and may not necessarily imply distributed star formation (see also Section 1). Given our reasoning here, such a distributed population is consistent with star formation proceeding in that cloud for about 7 Myrs with stars currently being born in the much younger aggregates. The first embedded dominant mode clusters may have dissolved by now. The lack of an apparent distributed population in the L1630 molecular cloud (Lada & Lada 1991) may simply be due to star formation beginning recently in that cloud with not enough time being available for a distributed population to be established.

Whether these considerations are correct can be determined observationally. By measuring the proportion of binaries in the distributed population of L1641, and their distribution of periods, we can determine whether these have passed through the dynamics of an aggregate dynamically equivlant to the dominant mode cluster. That is, the distributed population ought to have a proportion of about 50 per cent binaries, and a period distribution similar to the main sequence period distribution. The distributed population observed in Taurus–Auriga, on the other hand, must have been born in an isolated star formation mode, because the observed distribution of periods for these young systems appears to be unevolved (see sections 1 and 2 in K1). Precise radial velocity and proper motion measurements of a large sample of young stars in the vicinity of a molecular cloud will be very important to quantify the kinematics.

6 EXAMPLES: HYADES, PLEIADES AND TRAPEZIUM CLUSTERS

As suggested in Section 4.2 (see also Section 2) and assuming the dynamical properties of stellar systems are invariant to the star forming conditions in the Galactic disk, the set of observables $n(t)$, $\overline{m}(t)$ and $f_{\mathrm{tot}}(t)$ probably uniquely define the past dynamical evolution of the cluster.

We consider the Hyades, Pleiades and Trapezium Clusters because these have been observed extensively. Unfortunately we do not have a complete census of all cluster members, nor do we have a complete census of all binary systems in these clusters. A very detailed and insightful investigation of the dynamics of the Pleiades Cluster is provided by Limber (1962a, 1962b). Given the lack of observational data, Limber neglects binary systems and has to extrapolate to stars fainter than $M_{\mathrm{V}} \approx 10$. Limber (1962b) argues that the cluster has relaxed sufficiently so that the massive stars have settled near the cluster center. Their present spatial distribution thus needs not reflect the birth configuration. The binary proportion in the Pleiades Cluster is 13 per cent for spectroscopic binaries ($P < 10^3$ days, Mermilliod et al. 1992), and about 46 per cent for photometric binaries (Steele & Jameson 1995). For the Hyades Cluster, Griffin et al. (1988) estimate that 30 per cent of all cluster members are radial velocity binaries. For systems brighter than $M_{\mathrm{V}} \approx 13$, Eggen (1993) finds a photometric binary proportion $f_{\mathrm{tot}}^{\mathrm{phot}} \approx 0.4$. These binaries are concentrated towards the cluster center and their proportion is higher than in the field (compare with Figs. 1 and 4). f_{tot} is thus likely to be significantly larger in both clusters (Kroupa & Tout 1992). The best determined observational quantity, however, is the low-mass stellar luminosity function for the Hyades and Pleiades Clusters, which contains information on all three observables above.

In this section we consider the $N_{\rm run} = 20$ simulations of the dominant mode cluster $(N_{\rm bin}, R_{0.5}) = (200, 0.85\,{\rm pc})$. We convert stellar masses to absolute magnitudes using the mass–luminosity relation derived and tabulated in Kroupa et al. (1993), and obtain I- and K-band absolute magnitudes as in Kroupa (1995c). The single star luminosity function, $\Psi_{\rm sing}$, is obtained by binning all individual stars into magnitude bins, and the system luminosity function, $\Psi_{\rm sys}$, is obtained by binning all single star systems and all binary systems (which are assumed to be unresolved) into magnitude bins.

6.1 The theoretical luminosity function

In this section we discuss the various features of the stellar luminosity function.

In the upper panel of Fig. 10 we plot the K-band luminosity functions for all individual stars and systems. It is evident from this figure, that after cluster dissolution, the surviving unresolved binary stars, which have a period distribution as shown in fig. 7 in K2 and a mass ratio distribution plotted in fig. 12 in K2, lead to a decay of the field star luminosity function at $M_{\rm K} > 7$. This is exactly the effect one observes in the Galactic field (Kroupa et al. 1993, Kroupa 1995c). The flattening of the luminosity function at $M_{\rm K} \approx 4.4$ ($M_{\rm V} \approx 7$) is the 'H$^-$ plateau', and the maximum at $M_{\rm K} \approx 7$ ($M_{\rm V} \approx 12$) is the 'H$_2$–convection peak' (Kroupa, Tout & Gilmore 1990). The model Galactic field star luminosity functions are tabulated in appendix 1 in Kroupa (1995c).

In the lower panel of Fig. 10 we compare the model field star luminosity functions, shown in the upper panel, with the single star and the system luminosity functions inside the central 2 pc sphere of the dynamically highly evolved dominant mode cluster at age $t = 480\,{\rm Myr}$ (i.e. after 44 initial relaxation times).

The luminosity function of all individual stars in the central cluster region is highly depleted in low mass stars owing to advanced mass segregation and evaporation of stars. The system luminosity function in the central cluster region is highly biased towards bright systems when compared to the field system luminosity function.

Consider the ratio which is independent of the initial $N_{\rm bin}$ (i.e. the initial number of stars in the cluster):

$$\zeta = \frac{\Psi_{\rm H_2}}{\Psi_{\rm H^-}}, \tag{6}$$

where $\Psi_{\rm H_2}$ and $\Psi_{\rm H^-}$ are the luminosity functions at the H$_2$–convection peak and the H$^-$ plateau, respectively. In principle, Ψ could be evaluated at different magnitudes, and we choose the H$_2$–convection peak and the H$^-$ plateau because these features are universal, being determined by stellar structure. Thus $\zeta_{\rm K} \approx \Psi(M_{\rm K,1})/\Psi(M_{\rm K,2})$, where $\Psi(M_{,\rm K})\,dM_{\rm K}$ is the number of systems in the magnitude interval $M_{\rm K}$ to $M_{\rm K}+dM_{\rm K}$, $M_{\rm K,1} \approx 7$ and $M_{\rm K,2} \approx 4.4$.

From the bottom panel of Fig. 10 we note that $\zeta_{\rm K} = 8.5/2 = 4.3$ for the model system luminosity function of the Galactic field, whereas $\zeta_{\rm K} = 3.5/1.7 = 2.1$ for the model system luminosity function in the central 2 pc sphere of our dynamically highly evolved cluster. Thus, ζ is a measure of the state of dynamical evolution of any cluster provided the stellar mass function and initial proportion of binaries is universal.

6.2 Comparison with the Hyades and Pleiades Clusters

The stellar populations in open clusters lack the considerable disadvantage of cosmic scatter inherrent to studies of the luminosity function in the Galactic disk. Stars in Galactic clusters all have approximately the same age, metallicity and distance. The effects of unresolved binary stars are, however, also severe (Kroupa & Tout 1992), and the bottom panel of Fig. 10 shows that cluster luminosity functions suffer from mass segregation and evaporation of stars. Thus, when contemplating the universality of the stellar mass function we need to keep the shortcomings of each sample in mind.

We consider recent determinations of the luminosity function in two open clusters. Hambly, Hawkins & Jameson (1991) and Reid (1993) measure the luminosity functions in the Pleiades and Hyades clusters, respectively. Leggett, Harris & Dahn (1994) obtain a luminosity function for the Hyades for $M_{\rm V} \geq 11$ which is consistent with the data of Reid (1993). The luminosity function of the Pleiades cluster is shown in apparent I-band magnitudes as solid circles in the top panel of Fig. 11. The Hyades luminosity function,

converted to absolute V-band magnitudes by Reid (1993), is shown as solid circles in the bottom panel of Fig. 11.

6.2.1 Mass segregation and distance estimation

In both panels of Fig. 11 we plot our model for the Galactic field system luminsity function $k\,\Psi_{\mathrm{mod,sys}}(t = 1\,\mathrm{Gyr})$, our initial system luminosity function $k\,\Psi_{\mathrm{mod,sys}}(t = 0)$, and also the fully resolved, i.e. single star, luminosity function, $k\,\Psi_{\mathrm{mod,sing}}$. The constant k we determine by scaling $\Psi_{\mathrm{mod,sys}}(t = 1\,\mathrm{Gyr})$ to the data at $m_{\mathrm{I}} \approx 13$ in the top panel, and at $M_{\mathrm{V}} \approx 10$ in the bottom panel. In the top panel we assume a distance modulus $m - M = 5.5$.

The observed luminosity function for the Pleiades cluster has the shape we expect, apart from a small overabundance of relatively bright systems at $m_{\mathrm{I}} \approx 12.5$. However, its peak lies at $m_{\mathrm{I}} = 15$, whereas in our model it lies at $m_{\mathrm{I}} = 14.4$. If we were to model pre-main sequence brightening then the discrepancy between our model and the observed luminosity function would be larger still. Comparing their luminosity function with the Galactic field photometric luminosity function, Hambly et al. (1991) conclude that the stellar mass function in the Pleiades cluster must be similar to the mass function of stars in the field. Assuming the same stellar mass function for the Pleiades as for the Galactic field, four reasons might be responsible for the discrepancy in the location of the peak: (i) Our mass–M_{V} relation is wrong. However, comparison of the model luminosity functions with the Galactic field luminosity function in fig. 1 of Kroupa (1995c) suggests that the peak in our model cannot be wrong by more that $\Delta M_{\mathrm{V}} \approx 0.3\,\mathrm{mag}$, i.e. $\Delta m_{\mathrm{I}} \approx 0.2\,\mathrm{mag}$. (ii) The $M_{\mathrm{V}}, V - I$ relation derived by Stobie et al. (1989) using trigonometric parallax data might be systematically wrong. However, that their relation is a reasonable approximation is easily demonstrated by plotting it together with the data published by Monet et al. (1992). To check whether we have made a mistake when transforming mass to M_{V} to M_{I} to m_{I} we also explicitly transform the luminosity function of Stobie et al. (1989) to m_{I} and show it in the top panel of Fig. 11 as crosses, after scaling as above. The same discrepancy is evident. Leggett et al. (1994) point out that, at $V - I \approx 3$, the $M_{\mathrm{V}}, V - I$ relation appears to steepen. This would alleviate some of the discrepancy found here. We also note that Kroupa et al. (1993) show that the $M_{\mathrm{V}}, V - I$ relation derived by Stobie et al. (1989) has to be corrected for systematic bias owing to cosmic scatter and unresolved binary systems. Even the most extreme corrected relation (table 6 in Kroupa et al. 1993), however, does not change M_{I} at $M_{\mathrm{V}} = 12$ by more than 0.16 mag. (iii) The photometric calibration of the photographic data might be inaccurate. The photographic data were obtained in the R-band, and in their fig. 8, Hambly et al. (1991) compare their luminosity function with the Galactic field photographic luminosity function observed in the R-band by Hawkins & Bessell (1988). Both data sets agree, suggesting both make the same systematic error, or point (ii) above. (iv) Following the suggestion by Kroupa & Tout (1992) that the 'H$_2$–convection peak' in the stellar luminosity function may be used as a distance indicator we consider the possibility that the distance modulus of the Pleiades might be closer to $m - M = 6$ than to 5.5. Although the most often quoted value is 5.5 there is a recent measurement suggesting $m - M = 5.9 \pm 0.26$ (Gatewood et al. 1990, see also Giannuzzi 1995). In the top panel of Fig. 11 we plot, as the short-long dashed curve, the model Galactic field system luminosity function, $k\,\Psi_{\mathrm{mod,sys}}(t = 1\,\mathrm{Gyr})$, assuming a distance modulus of $m - M = 6$, and obtain a much improved representation of the data.

The observed luminosity function for the Hyades cluster has a peak at the correct location. The overabundance of bright stars is very apparent, and is interpreted by Reid (1993) to be due to mass segregation and stellar evaporation. In his elegant paper, Eggen (1993) also shows that the luminosity function for Hyades stars is depleted at the faint end when compared to the observed field star luminosity function.

In what follows we adopt a distance modulus of $m - M = 6$ for the Pleiades.

6.2.2 Dynamical age estimation via luminosity function fitting

We now focus our attention on the system luminosity function within a central sphere with radius 5 pc in our dominant mode cluster. This volume corresponds approximately to the survey volumes of both the Hambly et al. (1991) and Reid (1993) samples. We plot in Fig. 12 the model system luminosity functions at times $t = 87, 260$ and $476\,\mathrm{Myr}$ (they are tabulated in Table A-3).

As our cluster evolves we observe in Fig. 12 a drop in the number of stars (compare to top panel of Fig. 4), and an increasing deficiency in faint stars. From Fig. 12 we see that the Hyades cluster is best represented by a dynamically advanced model. For the Pleiades we obtain a dynamical age between 90 and 260 Myrs (i.e. between 8 and 24 initial crossing times), and for the Hyades about 500 Myrs (i.e. about 44 initial crossing times).

The fainter location of the peak in the observational Hyades data is accounted for by the higher metallicity of the Hyades stars. From fig. 10 of VandenBerg & Poll (1989) we obtain for the Hyades stars, which have [Fe/H]=0.15 (VanDenBerg & Poll 1989), approximately 0.3 mag fainter luminosities in the V-band for a mass of $0.33\,M_\odot$, at which the peak in the luminosity function lies (see also fig. 5 in Kroupa et al. 1993). The Pleiades cluster has [Fe/H]=0.03 (Cayrel de Strobel 1990), which cannot account for the residual ≈ 0.4 mag fainter location of the peak in the luminosity function in Fig. 12.

For each of the models shown in Fig. 12 we parametrise the depletion of low-mass stars by evaluating ζ (equation 6: $\zeta_V = \frac{\Psi_{max}}{\Psi(M_V=7)}$ and $\zeta_I = \frac{\Psi_{max}}{\Psi(M_I=5.5)}$, where Ψ_{max} is the maximum of the luminosity function). The results are plotted in Fig. 13. For the Hyades and Pleiades data (Fig. 11) we evaluate

$$\zeta_V^{Hy} = \frac{\Psi(M_V = 11.75) \pm \delta\Psi}{\Psi(M_V = 6.75) \pm \delta\Psi} = 2.24 \pm 0.20,$$

and

$$\zeta_I^{Pl} = \frac{\Psi(m_I = 15.0) \pm \delta\Psi}{\Psi(m_I = 11.45) \pm \delta\Psi} = 4.42 \pm 0.35,$$

where $\delta\Psi$ is the Poisson uncertainty at the respective magnitude. The nuclear ages of the Hyades and Pleiades Clusters based on isochrone fitting are, respectively, 655 ± 55 Myrs and 100 ± 30 Myrs (Cayrel de Strobel 1990). The data are compared with our $\zeta(t)$ model in Fig. 13.

6.2.3 Discussion

Concerning the comparison of our models with the observed Hyades and Pleiades luminosity functions we must, apart from not modelling the higher metallicity of the Hyades and high mass stars, keep in mind the following caveats: (i) The dynamical evolution of real clusters is affected by perturbations from passing molecular clouds. (ii) The published Hyades (and to a lesser degree Pleiades) luminosity functions may be incomplete or contaminated by Galactic field stars (see discussion in Reid 1993). (iii) The Hyades cluster has a distance of about 46 pc (VanDenBerg & Poll 1989), so that some binary systems are probably resolved (a binary system with a mass of $0.64\,M_\odot$ and $\log_{10}P > 5.2$ has a semi major axis $a > 48$ AU, i.e. larger than 1 arc sec). (iv) The real clusters are likely to have had an initial $R_{0.5}$ different to that assumed here (0.85 pc). This has little affect on $n(t)$ and $\overline{m}(t)$, but determines $f_{tot}(t)$, and thus influences the shape of the luminosity function.

We thus treat the dynamical ages suggested in Fig. 12 with reservation, although our dynamical dating provides encouraging age estimation (Fig. 13). Extension of the model $\zeta(t)$ to $t > 500$ Myrs will require simulations with $N_{bin} > 200$, but Fig. 13 suggests that $\zeta(t)$ may continue evolving linearly with the same slope. Using an alternative approach, Buchholz & Schmidt-Kaler (1980) suggest that the radial mass distribution as a function of time can be a reliable age estimator for open clusters.

The excellent agreement of our model with the observed Hyades luminosity function (Fig. 12) suggests that the proportion of Hyades systems in the central 2 pc sphere that are binary stars may be 65 per cent (Fig. 4) which is larger than in the Galactic field (47 ± 5 per cent). This would be consistent with the conclusion by Kroupa & Tout (1992) that a large proportion of binaries may reside in the Praesepe Cluster, which is somewhat older than the Hyades Cluster. Similarly, we expect that the total proportion of binaries in the central 2 pc sphere of the Pleiades Cluster is probably close to 60 per cent (Fig. 4) and may still be decreasing.

Our models do not suggest that the initial dynamical properties of the stellar systems born in the Hyades and Pleiades Clusters were significantly different from the Galactic field birth population. It would thus appear that we can, in principle, estimate the initial number of stars that formed in the two clusters (see Section 4). In doing this, we have to keep the above caveats in mind.

To verify our procedure we first of all consider our models for which we have complete data. Our $t = 476$ Myrs system luminosity function contains 54 systems with $M_V > 5$ within the central 5 pc sphere (Table A-3, or Fig. 12). From Fig. 4 we estimate $f_{tot} \approx 0.65$, so that the number of stars in the central 5 pc sphere is $N_{5pc} = 89$. Comparison with Fig. 4 allows us to estimate the spatial configuration correction factor to be $s = N_{2pc}(476\,\text{Myrs})/N_{5pc}(476\,\text{Myrs}) \approx 40/89 = 0.45$. We can now symbolically map our 'observed' 89 stars in the central 5 pc sphere to the $\log_{10} n(t)$ curve in Fig. 4, and apply equation 1 to solve for the number of stars in the cluster at birth. The result is $0.45 \times 89 \times \exp[476/230] = 317$ stars. This compares favourably with our initial 400 stars.

In the Hyades Cluster Reid (1993) counts about 210 systems with $M_V \geq 5$ in the central 5 pc sphere. Following the above method, with $s = 0.45$, $f_{tot} = 0.7$ and adopting the nuclear age, we estimate $0.45 \times 357 \times \exp[655/230] = 2771$ stars at birth. Since the mass of our cluster is $128\,M_\odot$, we estimate the mass of the Hyades Cluster to have been roughly $900\,M_\odot$ at birth. Stars more massive than $1.1\,M_\odot$ contribute about 30–40 per cent to the total mass, so that the birth mass of the stellar component of the Hyades Cluster may have been about $1300\,M_\odot$, in good agreement with Reid's (1993) estimate. Studying the distribution of white dwarfs and estimating their loss from the cluster, Weidemann et al. (1992) estimate the number of stars to have been about 3000–4000 at birth, again in good agreement with our estimate.

Similarly, Hambly et al. (1991) count about 600 systems with $M_I \geq 4.5$ in the central 5 pc sphere of the Pleiades Cluster. Assuming $f_{tot} = 0.7$ (Fig. 4), we estimate that the Pleiades may have contained about $0.45 \times 1020 \times \exp[100/230] = 709$ stars at birth. The total stellar birth mass may thus have been about $300\,M_\odot$. Limber (1962a) estimates a total present stellar mass of about $800\,M_\odot$, and van Leeuwen (1980) derives a present value of about $2000\,M_\odot$. Both estimates assume virial equilibrium and imply significantly larger birth masses than our estimate. Further numerical simulations will be needed to clarify this issue.

6.3 The Trapezium Cluster

The Trapezium Cluster has been extensively observed and dated (Zinnecker, McCaughrean & Wilking 1993, Prosser et al. 1994). Prosser et al. find for the central sphere with a radius of 0.25 pc that the proportion of binaries with projected separations in the range of approximately 44–440 AU is $f \approx 0.11$ which is similar to the Galactic field. Assuming a system mass of $1.3\,M_\odot$ this separation range corresponds to $5 < \log_{10} P < 6.5$. Their result would appear to be a lower limit (i.e. $f_{tot}(t) > 0.11$, with $t \approx 1$ Myr) because they include all apparently single stars down to the flux limit. These may have fainter undetected companions.

The present central number density is $\log_{10} n_c \approx 4.5$ so that the roughly 1 Myrs old Trapezium Cluster may be compared with our $R_{0.5} = 0.08$ and 0.25 pc models (Table 1). For these clusters we see from fig. 3 in K1 and Fig. 3 here that most destruction of orbits occurs within the first few Myrs ($R_{0.5} = 0.08$ pc) and within the first few tens of Myrs ($R_{0.5} = 0.25$ pc), and from fig. 5 in K1 we deduce that the distribution of periods must be similar to the G dwarf distribution for $\log_{10} P < 6$ after a few Myrs and about 20 Myrs, respectively. We remember that fig. 5 needs to be modified at $\log_{10} P < 3$ as suggested in sections 5 in K1 and 2.2 in K2. From fig. 5c in K1 we expect that the Trapezium Cluster contains no binaries with approximately $\log_{10} P > 7$ and from fig. 4b in K1 we expect the mass ratio distribution to be depleted significantly at small values. More detailed comparisons and predictions will be possible when more realistic initial conditions such as the presence of massive stars and a changing background potential are included.

The binary population of the Trapezium Cluster is undergoing rapid stimulated evolution (fig. 3 in K1), given that it is between 2 and 10 crossing times old (Table 1), and will prove an interesting laboratory for the study of the interplay of stimulated evolution and eigenevolution (cf. to discussion of the $e - \log_{10} P$ diagram in section 3.1 in K2).

Comparison of our model luminosity function with the observed Trapezium Cluster luminosity function by Prosser et al. (1994) is not attempted here owing to the very difficult and uncertain treatment of pre-main sequence luminosity evolution (Kroupa, Gilmore & Tout 1992). We refer the reader to Zinnecker et al. (1993), who show that the stellar luminosity function for a sample of stars younger than 2 Myrs can be significantly distorted because deuterium burning delays contraction.

7 CONCLUSIONS

Obtaining observational data on clusters of stars is very important because the same age, metallicity and distance of the stars ease analysis. A cluster of stars is a fossil of one star-formation event. We need to study as many of these as is possible in order to find out, if there is variation of the spectrum of masses formed, or of the proportion of binaries, and of their distribution of periods. The stellar population in one cluster samples the birth distribution of dynamical properties of stellar systems $\mathcal{D}$ (section 4 in K1). In this paper we identify generic features of the evolution of the stellar number density, mean stellar mass, stellar luminosity function and binary proportion in Galactic clusters with the special aim of addressing observable properties.

We assume that all stars form in binary systems with component masses paired at random from the KTG(1.3) mass function. The initial period distribution is flat in the range $3 \leq \log_{10}P \leq 7.5$, P in days. For the purposes of the present study, we assume that 200 binaries initially populate stellar clusters with half mass radii 0.08 pc $\leq R_{0.5} \leq 2.5$ pc that are initially in virial equillibrium. We consider stellar masses in the range $0.1\,M_\odot \leq m \leq 1.1\,M_\odot$ to avoid complications concerning stellar evolution. We also distribute 400 single stars for a comparison with the binary star clusters. Using direct N-body integration we follow the evolution of these stellar systems until they disperse, repeating the experiment five times for the binary star clusters and three times for the single star clusters. We also perform 20 simulations of the dominant mode $(N_{\rm bin}, R_{0.5}) = (200, 0.85\,{\rm pc})$ cluster, in which the binary stars have a rising period distribution with increasing $\log_{10}P \geq 0.48$. The parameters for these clusters are listed in Table 1.

We find that the number density evolution in the central 2 pc sphere of both the binary and single star clusters is indistinguishable (Fig. 1) and has an exponential decay time $\tau_e \approx 230$ Myrs. Both decay to less than 0.1 stars pc^{-3} within about 700 Myr. Cluster lifetime and evolution of number density within the central 2 pc sphere are invariant to changes of initial $R_{0.5}$ if $0.08 \leq R_{0.5} \leq 2.53$ pc (Section 3). After the initial rapid ionisation of the less bound binary systems the proportion of binary systems rises slowly in the central region of the clusters (Figs. 3 and 4). However, even for our most compact cluster ($R_{0.5} = 0.08$ pc) the binary proportion in the central region does not decrease below about 30 per cent. The central proportion of binary systems in a cluster is a sensitive function of the initial concentration of the cluster, whereas segregation of mean stellar mass is a sensitive function of the dynamical age of the cluster (Figs. 2 and 4). Evolution of the number density and mean stellar mass, and scaling to other initial cluster parameters, are explored in Section 4.

The *kinematical signature of star formation* reflects the initial configuration (Fig. 5): the clusters consisting initially of 100 per cent primordial binaries lead to a different distribution of centre of mass kinetic energies than the clusters that initially have no primordial binaries. The former have a distinct high kinetic energy tail which is a function of the initial cluster configuration. The high velocity tail ought to be apparent in the distribution of young stars in the vicinity of star forming regions.

If the majority of stars form in aggregates that are dynamically equivalent to the dominant mode cluster then only a few per cent of all stars are ejected from the cluster with a large enough velocity to escape the molecular cloud. Roughly 90 per cent of all stars have a velocity smaller than about 5 km sec^{-1} and remain trapped in the vicinity of their parent giant molecular cloud, until it disperses after a few 10^7 years. This population of stars may appear as a distributed population of pre-main sequence stars either while star formation continues in the cloud, or after it has ceased. We expect about 50 per cent of young stellar systems in the apparently distributed population to be binaries. In this light, it seems possible that the L1630 molecular cloud in the Orion complex is void of a distributed population of young stars because star formation has only just begun in the four locations, where Lada & Lada (1991) report embedded clusters which are similar to our dominant mode cluster (K1). The L1641 molecular cloud in the southern region of Orion A may have been forming stars over a time span of about 7 Myrs, which may explain the significant distributed population of young stars with an age of 5-7 Myrs which Strom et al. (1993) detect. The stellar dynamical properties of this population will prove very useful in discriminating its birth dynamical structures.

A halo of young stars around a molecular cloud may be expected because even stars with escape velocities (implying a close encounter with a binary system and probable loss of circumstellar material thus becoming a weak-lined T-Tauri star, i.e. wTTS) may remain trapped by the equipotential surface of the molecular cloud

and Galaxy. Only about 15 per cent of these systems are binaries with the proportion of binaries decreasing with increasing ejection velocity (Fig. 9). The discovery of wTTS distributed over the entire region of the Orion molecular cloud complex by Sterzik et al. (1995) is thus of particular interest.

If most stars are born in embedded clusters similar to our dominant mode cluster then we require a birth rate of roughly 15 clusters $\mathrm{kpc}^{-2}\,\mathrm{Myr}^{-1}$ (Section 3.1).

The stellar luminosity function in the central region of the dominant mode cluster flattens with time (i.e. an overabundance of bright systems develops) although the 'H$_2$–convection peak' at $M_V \approx 12, M_K \approx 6.5$ remains. Assuming a universal initial mass function we thus expect the luminosity function in Galactic clusters to differ from that in the Galactic field (Figs. 10 and 11). Recently published data on the Hyades luminosity function (Reid 1993) are consistent with a dynamically evolved cluster (Fig. 12). The location of the peak in the Pleiades luminosity function data (Hambly et al. 1991) suggests a distance modulus of $m - M = 6$ rather than 5.5 (top panel of Fig. 12).

Parametrising stellar mass dependent evaporation from the central region of the dominant mode cluster by the ratio ζ (equation 6) of the luminosity function at the 'H$_2$–convection peak' and at the 'H$^-$ plateau', we find good agreement with $\zeta(t)$ obtained from the Hyades and Pleiades luminosity functions (Fig. 13). This result, together with our results on the evolution of the stellar number density, mean stellar mass and binary star proportion, suggests: (i) that our initial assumptions about stellar mass function and binary stars are consistent with the data, and (ii) for each cluster a unique evolutionary history probably exists which may be found by simple scaling to pre-computed histories (Section 4). Thus, using Fig. 4 we may expect that about 60–70 per cent of all systems in the central 2 pc sphere are binaries in the Pleiades and Hyades Clusters. A rough estimate of the initial number of stars in the Hyades and Pleiades Clusters suggests birth masses of roughly 1200 $M_\odot$ and 300 $M_\odot$, respectively. The former value is consistent with other estimates (Weidemann et al. 1992, Reid 1993), but the latter value is significantly smaller than estimates by Limber (1962a) and van Leeuwen (1980).

The simulation of a cluster with high initial stellar number density ($R_{0.5} < 0.25$ pc) implies that the period distribution in the range $\log_{10}P < 6$ is similar to the main sequence period distribution after a few initial relaxation times, which is consistent with observations of the Trapezium Cluster (Section 6.3). The binary population in this cluster must be undergoing significant stimulated evolution.

More detailed comparisons with observational data (e.g. colur–magnitude diagrams, spatial distribution of bright and faint stars, spatial distribution of velocities), and modeling of individual Galactic clusters, as well as embedded clusters, and including massive stars, a varying background potential during the first 10 Myrs and non-virial equilibrium initial conditions, will mature our present ideas and results. By adopting the nuclear ages of clusters and comparing with realistic N-body models, we can hope to identify the physics responsible for shaping the cluster.

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Table A-1: The Distribution of Centre of Mass Kinetic Energies After Cluster Dissolution
(Section 5.1)

$R_{0.5}$

$\log_{10}E_{kin}$ $M_\odot$ km^2 sec^{-2}	2.53 pc	0.77 pc	0.25 pc	0.08 pc	0.25 pc	0.08 pc	$\log_{10}E_{kin}$ $M_\odot$ km^2 sec^{-2}	0.85 pc
$N_{bin} =$	200	200	200	200	0	0		200
$N_{sing} =$	0	0	0	0	400	400		0
−4.68	0.0	0.0	0.0	0.0	0.0	0.0	−4.80	0.0
−4.32	0.0	0.0	0.0	0.0	0.0	0.0	−4.46	0.0
−3.96	0.0	0.0	0.2	0.6	0.3	0.0	−4.13	0.0
−3.61	0.0	0.6	0.0	0.4	0.6	2.0	−3.79	0.1
−3.25	1.0	1.2	2.6	2.0	4.0	3.3	−3.45	0.5
−2.89	3.4	2.8	5.4	3.4	8.3	8.0	−3.11	1.9
−2.54	9.0	11.4	14.0	13.0	32.0	23.0	−2.77	5.7
−2.18	20.0	26.0	34.8	26.6	63.0	56.0	−2.43	15.0
−1.82	36.0	44.4	50.4	52.0	87.6	88.3	−2.10	31.1
−1.46	50.2	51.0	58.0	54.6	91.3	79.3	−1.76	49.2
−1.11	46.2	50.8	41.6	50.8	56.0	56.0	−1.42	56.3
−0.75	32.0	25.4	24.8	37.0	23.0	31.3	−1.08	48.7
−0.39	10.0	12.4	12.4	15.2	8.0	14.6	−0.74	24.7
−0.04	4.2	8.0	9.4	14.4	5.0	9.6	−0.40	10.8
+0.32	2.2	4.0	9.6	10.4	5.0	6.3	−0.07	4.9
+0.68	1.8	4.8	9.0	9.0	3.0	4.6	+0.27	4.8
+1.04	0.2	2.6	5.2	8.0	2.6	2.6	+0.61	3.2
+1.39	0.2	1.2	3.4	7.8	1.6	2.3	+0.95	2.8
+1.75	0.0	0.6	2.0	4.2	0.0	2.3	+1.29	2.1
+2.11	0.0	0.6	1.8	3.2	0.3	1.3	+1.63	1.0
+2.46	0.0	0.0	0.6	1.4	0.0	0.0	+1.96	0.5
+2.82	0.0	0.0	0.0	0.6	0.0	0.0	+2.30	0.4
							+2.64	0.2
							+2.98	0.1
							+3.32	0.0

The $R_{0.5} = 0.85$ pc data are binned somewhat differently for historical reasons

Table A-2: Velocity Distribution After Disintegration of the Dominant Mode Cluster (Section 5.2)

$\log_{10} v$ km sec^{-1}	f_v	δf	$< m >_v$ $M_\odot$	$N_{\mathrm{sing},v}$	$N_{\mathrm{bin},v}$
-1.775	0.0000	0.0000	0.000	0	0
-1.625	0.0004	0.0003	0.799	0	2
-1.475	0.0004	0.0003	0.278	1	1
-1.325	0.0015	0.0005	0.555	0	8
-1.175	0.0055	0.0010	0.688	6	23
-1.025	0.0138	0.0017	0.645	18	55
-0.875	0.0299	0.0024	0.617	28	130
-0.725	0.0658	0.0036	0.608	95	253
-0.575	0.1225	0.0049	0.534	225	423
-0.425	0.1641	0.0057	0.500	353	515
-0.275	0.1934	0.0062	0.469	505	518
-0.125	0.1664	0.0058	0.451	534	346
$+0.025$	0.1043	0.0046	0.401	408	144
$+0.175$	0.0486	0.0031	0.439	211	46
$+0.325$	0.0170	0.0018	0.601	69	21
$+0.475$	0.0198	0.0020	0.664	84	21
$+0.625$	0.0136	0.0016	0.684	50	22
$+0.775$	0.0108	0.0015	0.496	47	10
$+0.925$	0.0076	0.0012	0.565	34	6
$+1.075$	0.0059	0.0011	0.504	26	5
$+1.225$	0.0036	0.0008	0.736	14	5
$+1.375$	0.0026	0.0007	0.293	14	0
$+1.525$	0.0015	0.0005	0.740	7	1
$+1.675$	0.0009	0.0004	0.817	5	0
$+1.825$	0.0002	0.0002	0.184	1	0
$+1.975$	0.0000	0.0000	0.000	0	0

The data are plotted in Fig. 9 and are a mean of 20 simulations. Column 1 contains the centre of each velocity bin; Column 2 the proportion of systems in each velocity bin; Column 3 the Poisson uncertainty; Column 4 the mean mass per bin; Columns 5 and 6, respectively, list the number of single stars and binary systems per bin.

$< m >_v = \frac{M_v}{N_{\mathrm{bin},v} + N_{\mathrm{sing},v}}$, where M_v is the total mass in velocity bin v.

Table A-3: The System Luminosity Function in the Central 5 pc Sphere of the Dominant Mode Cluster (Section 6.2.2)

M_V	$\Psi_{\mathrm{mod,sys}}$ $t = 87\,\mathrm{Myr}$	$\delta\Psi$	$\Psi_{\mathrm{mod,sys}}$ $t = 260\,\mathrm{Myr}$	$\delta\Psi$	$\Psi_{\mathrm{mod,sys}}$ $t = 476\,\mathrm{Myr}$	$\delta\Psi$
0.0	0.00	0.00	0.00	0.00	0.00	0.00
0.5	0.00	0.00	0.00	0.00	0.00	0.00
1.0	0.05	0.05	0.00	0.00	0.00	0.00
1.5	0.20	0.10	0.10	0.07	0.05	0.05
2.0	0.25	0.11	0.10	0.07	0.05	0.05
2.5	0.30	0.12	0.10	0.07	0.10	0.07
3.0	0.25	0.11	0.05	0.05	0.05	0.05
3.5	0.65	0.18	0.40	0.14	0.20	0.10
4.0	0.90	0.21	0.55	0.17	0.35	0.13
4.5	2.55	0.36	2.40	0.35	1.10	0.24
5.0	4.05	0.46	2.85	0.38	1.80	0.30
5.5	2.30	0.34	1.40	0.27	0.85	0.21
6.0	4.40	0.48	3.45	0.42	1.90	0.31
6.5	5.20	0.52	3.40	0.42	2.45	0.35
7.0	5.25	0.52	4.25	0.47	2.50	0.36
7.5	3.25	0.41	2.05	0.32	1.30	0.26
8.0	2.10	0.33	1.60	0.29	0.85	0.21
8.5	7.35	0.62	4.85	0.50	2.85	0.38
9.0	6.30	0.57	4.70	0.49	2.45	0.35
9.5	8.20	0.65	5.80	0.55	2.70	0.37
10.0	9.85	0.72	7.00	0.60	3.80	0.44
10.5	17.00	0.94	11.25	0.76	4.90	0.50
11.0	18.65	0.99	13.25	0.83	6.20	0.57
11.5	22.60	1.09	12.40	0.80	4.75	0.50
12.0	22.35	1.08	11.60	0.78	4.75	0.50
12.5	13.50	0.84	8.25	0.65	2.85	0.38
13.0	10.80	0.75	5.70	0.54	1.45	0.27
13.5	9.10	0.69	4.50	0.48	1.30	0.26
14.0	6.70	0.59	4.10	0.46	1.75	0.30
14.5	5.45	0.53	2.85	0.38	0.80	0.20
15.0	5.75	0.55	2.25	0.34	0.80	0.20
15.5	5.20	0.52	2.70	0.37	0.75	0.19

M_{I}	$\Psi_{\mathrm{mod,sys}}$	$\delta\Psi$	$\Psi_{\mathrm{mod,sys}}$	$\delta\Psi$	$\Psi_{\mathrm{mod,sys}}$	$\delta\Psi$
	$t = 87\,\mathrm{Myr}$		$t = 260\,\mathrm{Myr}$		$t = 476\,\mathrm{Myr}$	
0.5	0.00	0.00	0.00	0.00	0.00	0.00
1.0	0.15	0.08	0.05	0.05	0.00	0.00
1.5	0.20	0.10	0.15	0.08	0.10	0.07
2.0	0.35	0.13	0.10	0.07	0.05	0.05
2.5	0.35	0.13	0.05	0.05	0.10	0.07
3.0	0.65	0.18	0.45	0.15	0.20	0.10
3.5	1.65	0.29	1.25	0.25	0.65	0.18
4.0	3.60	0.43	3.15	0.40	1.75	0.30
4.5	4.15	0.46	2.35	0.35	1.50	0.28
5.0	5.75	0.55	4.40	0.48	2.45	0.35
5.5	7.30	0.61	5.35	0.53	3.40	0.42
6.0	5.25	0.52	3.65	0.43	2.20	0.34
6.5	5.55	0.54	3.75	0.44	2.55	0.36
7.0	10.55	0.74	7.65	0.63	3.95	0.45
7.5	12.25	0.80	8.95	0.68	4.35	0.47
8.0	19.30	1.00	12.65	0.81	5.90	0.55
8.5	31.25	1.28	20.05	1.02	9.15	0.69
9.0	29.65	1.24	15.80	0.91	6.30	0.57
9.5	20.85	1.04	12.20	0.80	4.00	0.45
10.0	15.60	0.90	8.40	0.66	2.60	0.36
10.5	8.45	0.66	4.80	0.50	1.60	0.29
11.0	9.00	0.68	4.45	0.48	1.60	0.29
11.5	6.95	0.60	3.10	0.40	1.05	0.23
12.0	4.15	0.46	2.65	0.37	0.55	0.17

Pre-main sequence stellar evolution is not modelled. Column 1 lists the absolute magnitudes. Columns 2 and 3 list the system luminosity function and the standard deviation of the mean (see appendix 1 in Kroupa 1995c), respectively, at time $t = 87\,\mathrm{Myr}$. The following two pairs of columns contain the system luminosity function at 260 Myr and 476 Myr. The luminosity functions are averages of 20 simulations.

Figure captions

Figure 1. Evolution of the number density of stars within the central 2 pc sphere of the binary star clusters (shown by the different lines) and the two single star clusters (shown by the open and solid circles) (Section 3.1). The horizontal dotted line marks the density below which the cluster is considered completely disintegrated.

Figure 2. Evolution of mass segregation in the four binary star clusters (**top four panels**) and the two single star clusters (**bottom two panels**), respectively (Section 3.2). The mean stellar mass within the central 2 pc sphere is plotted as open circles and the mean stellar mass outside this sphere is plotted as solid circles. The apparent drop in the mean mass within the central 2 pc sphere after approximately 600 Myrs results from our avaraging technique in which the mean mass is computed from the $N_{\rm run}$ simulations (see table 1 in K1) at a time when some of the simulations have lead to completely dissolved clusters.

Figure 3. Evolution of the overall proportion of binaries within (open circles, $f_{\rm in}(t)$) and outside (filled circles, $f_{\rm out}(t)$) the central 2 pc sphere for the four binary star clusters (Section 3.2).

Figure 4. The average evolution of 20 simulations of the dominant mode cluster initially with $(N_{\rm bin}, R_{0.5}) = (200, 0.85\,{\rm pc})$ (Section 4). **Top panel:** The number density evolution within the central 2 pc sphere (solid curve) is compared to the number density evolution of the clusters discussed in Section 3 (dotted curves, Fig. 1). **Middle panel:** The evolution of $\overline{m}(t)$ within the central 2 pc sphere (open circles) and outside this sphere (solid circles). The evolution of $\overline{m}(t)$ inside the central 2 pc sphere for the four binary star clusters (dotted curve: $R_{0.5} = 2.53$ pc; short dashed curve: $R_{0.5} = 0.77$ pc; long dashed curve: $R_{0.5} = 0.25$ pc; dot dashed curve: $R_{0.5} = 0.08$ pc) and the two single star clusters (dot long dashed curves) shown in Fig. 2 is also plotted here. **Bottom panel:** The evolution of the overall proportion of binaries within the central 2 pc sphere (open circles), and outside this sphere (solid circles). The evolution of $f_{\rm in}(t)$ for the four binary star clusters shown in Fig. 3 is plotted here using the same symbols as in the middle panel.

Figure 5. The energy distributions (Section 5.1). Units of energy are $M_\odot$ km^2 sec^{-2}. **Top panel:** The initial $(t = 0)$ distribution of centre of mass kinetic energies is shown by the solid curves for the two binary star clusters with initial $R_{0.5} = 2.53$ pc (left distribution) and 0.08 pc (right distribution). The distributions after cluster dissolution are represented by the dotted curve ($R_{0.5} = 2.53$ pc), the short dashed curve ($R_{0.5} = 0.77$ pc), the long dashed curve ($R_{0.5} = 0.25$ pc) and the dot dashed curve ($R_{0.5} = 0.08$ pc). The final distribution of kinetic energies for the single star clusters is shown by the solid triangles: long dashed curve ($R_{0.5} = 0.25$ pc) and dot dashed curve ($R_{0.5} = 0.08$ pc). These have been scaled to the binary star curves at $\log_{10}E_{\rm kin} \approx -1.6$. The binary star clusters with $R_{0.5} = 0.08$ and 0.25 pc have a significantly larger relative population with $\log_{10}E_{\rm kin} > 0$ than the single star clusters. **Bottom panel:** The distribution of initial (solid curve) and final binding energy distributions of binaries for the four binary star clusters. The initial $R_{0.5}$ are represented by the same symbols as in the top panel. The increased erosion of the binary star population is apparent as the initial kinetic energy distribution (solid curves in top panel) shift to higher energies.

Figure 6. The final centre of mass kinetic (thin histogram) and binary system binding (thick histogram) energy distributions for the two binary star clusters with initial $R_{0.5} = 0.25$ pc (**top panel**) and $R_{0.5} = 0.08$ pc (**bottom panel**). Units of energy are $M_\odot$ km^2 sec^{-2}.

Figure 7. Comparison of the $(N_{\rm bin}, R_{0.5}) = (200, 0.77\,{\rm pc})$ cluster shown in Fig. 5 with the $(200, 0.85\,{\rm pc})$ cluster (Section 5.1). Although the initial distribution of binary binding energies (**bottom panel**) differs in both cases the final distribution of centre of mass kinetic energies (**top panel**) is indistinguishable. The initial kinetic energy and binding energy distributions are shown by thin solid curves for the $R_{0.5} = 0.77$ pc cluster, and as thick solid curves with small solid dots for the $R_{0.5} = 0.85$ pc cluster. The final kinetic energy and binding energy distributions are shown as the thin dashed curves for the $R_{0.5} = 0.77$ pc cluster, and as thick dashed curves with large solid dots for the $R_{0.5} = 0.85$ pc cluster. Systems have to overcome

the potential of the cluster so that after cluster dissolution we have an increased number of systems with $\log_{10} E_{\mathrm{kin}} < -1.4$.

Figure 8. The distributions of centre of mass kinetic energies are plotted as thin lines and the distribution of binding energies of the binaries are plotted as thick lines (Section 5.1). The mean of 20 simulations is shown for the $R_{0.5} = 0.85$ pc cluster. **Top panel**: The initial distributions. **Middle panel**: The distributions after cluster dissolution. **Bottom panel**: The difference of the kinetic energy distributions is shown as the thin line, and the difference of the binding energy distributions is shown as the thick line. Negative numbers correspond to a gain. Units of energy are $M_\odot$ km^2/sec^2.

Figure 9. The mean distribution of velocities of 20 simulations of the $R_{0.5} = 0.85$ pc cluster (Section 5.2). **Top panel**: The initial proportion of systems as a function of their centre of mass velocity is shown as the dashed histogram. After cluster disintegration the distribution of centre of mass velocities is shown as the solid histogram (note that both distributions are normalised to unit area). **Middle panel**: the mean stellar mass as a function of centre of mass velocity is plotted as open cicles for the initial distribution, and as solid dots after cluster dissolution. **Bottom panel**: The initial distribution of centre of mass velocities of single stars and binaries is shown as the thin and thick dashed histogram respectively. The sum of these two gives the initial distribution shown in the top panel. The distribution after cluster disintegration of the single stars and binaries are shown as the thin and thick solid histogram, respectively. The sum of these two gives the final distribution shown in the top panel. The data presented here are available in Table A-2.

Figure 10. The K-band luminosity function averaged from 20 simulations (Section 6.1). **Top panel**: Counting all stars separately we obtain the luminosity function shown as the long-dashed histogram. Pairing all stars at random to binary systems we obtain the initial system luminosity function shown as the dotted histogram. This luminosity function (neglecting pre-main sequence stellar evolution) evolves in the environment of our $R_{0.5} = 0.85$ pc dominant mode cluster to the system luminosity function shown as the thick solid line histogram. This luminosity function represents a mixture of single stars (52 per cent) and unresolved binary systems (48 per cent, assuming all bound binaries remain unresolved), which have the period distribution shown as the solid histogram in fig. 7 in K2 and the mass ratio distributions shown in figs. 8 and 12 in K2. It is the luminosity function obtained from deep photographic surveys if our model is a true representation of the Galactic field star population. **Bottom panel**: The luminosity function within the central 2 pc sphere of the dominant mode cluster after 44 initial relaxation times (see also Fig. 4). Counting all stars individually we obtain the long-dashed histogram, but counting all systems we obtain the solid histogram. To show that significant mass and system segregation and loss has occurred (see Fig. 4) we scale the luminosity functions from the top panel to the present luminosity functions at $M_{\mathrm{K}} \approx 4$ and plot them as curves using the same symbols as in the top panel. An observer looking at the central region of a dynamically highly evolved cluster would observe a luminosity function similar to the thick solid line histogram instead of the luminosity function shown by the solid line. The preferential loss of low-mass stars is evident.

Figure 11. Comparison of our model Galactic field luminosity functions (tabulated in appendix 1 in Kroupa 1995c) with the observed Pleiades (top panel) and Hyades (bottom panel) luminosity functions (Section 6.2.1). In both panels $\Psi_{\mathrm{mod,sing}}$ is shown as the long-dashed curve, $\Psi_{\mathrm{mod,sys}}(t = 0)$ is the dotted curve and $\Psi_{\mathrm{mod,sys}}(t = 1\,\mathrm{Gyr})$ is shown as the solid curve. **Top panel**: The solid dots are the observed luminosity function for the Pleiades Cluster (Hambly et al. 1991). The model luminosity functions are plotted assuming a distance modulus $m - M = 5.5$, except for the short-long-dashed model, which is identical to the solid curve apart from assuming $m - M = 6$. The large crosses are the photometric luminosity function for the Galactic field (Stobie et al. 1989) transformed to m_{I}. **Bottom panel**: The solid dots are the observed luminosity function for the Hyades open cluster (Reid 1993).

Figure 12. The time evolution of the model system luminosity function (assuming all binaries are unresolved) in the central 5 pc sphere of the dominant mode cluster (Section 6.2.2). **Top panel**: Assuming a

distance modulus of $m - M = 6$ we plot the model luminosity functions in the I-band, and compare with observational data of the Pleiades Cluster shown as open and solid symbols (Hambly et al. 1991) normalised to the models at $m_I \approx 15$. **Bottom panel**: The observed luminosity function for the Hyades Cluster (Reid 1993) shown as open and solid symbols is normalised to the models at $M_V \approx 12$. Taking account of the higher metallicity of the Hyades Cluster would shift the model luminosity functions to fainter luminosities by 0.3 mag.

Figure 13. Evaporation of low mass stars from the central 5 pc volume in the $R_{0.5} = 0.85$ pc cluster (Section 6.2.2). $\zeta(t)$ is defined by equation 6. ζ_V and ζ_I represent our model of the dynamical evolution of the dominant mode cluster. The Hyades datum is the lower right cross, and the Pleiades datum is the upper left cross.