Perturbative Approach for Nonrenormalizable Theories.

J. Gegelia, G. Japaridze, N. Kiknadze and K. Turashvili

High Energy Physics Institute, University street 9, Tbilisi 380086, Georgia.

Abstract

Renormalization procedure is generalized to be applicable for nonrenormalizable theories. It is shown that introduction of an extra expansion parameter allowes to get rid of divergences and to express physical quantities as series of finite number of interdependent expansion parameters. Suggested method is applied to quantum (Einstein's) gravity.

1 Introduction

Existence of divergences is one of the basic problems of quantum field theories (QFT). The renormalization procedure handles these divergences only for some class — renormalizable theories. Although it is not *a priory* clear that nonrenormalizable theories lack physical significance. Moreover, in spite of the fact that most of the fundamental interactions are described by renormalizable QFT-s, the problem of the quantum gravity is still open — while Einstein's classical theory of gravity has substantial success, the corresponding quantum theory is nonrenormalizable.

We share the opinion that the renormalizability is just a technical requirement and it has nothing to do with the physical content of the QFT [1]. A lot of people believe that in meaningful theories divergences arise due to the perturbative expansion. It was noted in various papers and various contexts [1], [2]. Of course not all of the nonrenormalizable theories are meaningful. But the same is true for the renormalizable ones. E.g. the scalar ϕ^3 theory is renormalizable for space-time dimensions up to six [3], but has spectrum unbound from below. On the other hand there exist nonrenormalizable theories which can be handled in some other approach (e.g. the four-fermion interaction in $(2+1)$ dimensions is nonrenormalizable if the conventional renormalization procedure is applied but can be renormalized after performing $1/N$ expansion with N being the number of flavours [4]).

Below we are going to present a method of extracting physical information out of the perturbative series of nonrenormalizable theories. For renormalizable ones it just coincides with the usual renormalization procedure and only in that case can be interpreted in terms of counterterms.

The basic assumption in the further discussion would be that the referred nonrenormalizable theory is finite and so the regularized series can be summed up to some function which remains finite when the regularization is removed. This is just an assumption of mathematical consistency of the theory — if divergences are present even in exact solutions, then such theory can not be regarded to be fundamental [1]. Though the method will work for any nonrenormalizable theory, we choose to illustrate how the method must be applied on the example of Quantum Gravity based on celebrated Einstein's classical Lagrangian. This choice is motivated by our belief that this theory has much more chance to be consistent then any other nonrenormilizable theory known to us.

In Sec. 2 we briefly review conventional renormalization procedure in a way that suits best for our purposes. Sec. 3 is devoted to general description of the suggested method. In Sec. 4 application of the method to the abelian gauge field coupled to gravity is described and in Sec. 5 we give some final remarks and conclusions.

2 Renormalization procedure

Consider self-interacting scalar field ϕ with some Lagrangian $\mathcal{L}(\phi, m_0, g_0)$ (m_0 and g_0 are the bare mass and coupling constant, respectively). The perturbation theory produces diverging expressions for the Green's functions. So some regularization is required. For definiteness let us work with dimensional regularization [5]. After regularization, the S-matrix elements σ_i can be calculated (with the help of LSZ reduction technique [6]):

$$\sigma_i(g_0, m_0, p_k, \epsilon) = \sum_l \sigma_i^l(m_0, p_k, \epsilon) g_0^l \tag{1}$$

where p_k are momenta and ϵ is the regularization parameter. In the limit $\epsilon \to 0$ coefficients σ_i diverge. Let us introduce some functions:

$$m = \sum_l M_l(m_0, \mu, \epsilon) g_0^l \tag{2}$$

$$g = \sum_l G_l(m_0, \mu, \epsilon) g_0^l \tag{3}$$

Here μ is normalization point. We can solve g_0 and m_0 from (2) and (3):

$$m_0 = \sum_l M_l^*(m, \mu, \epsilon) g^l \tag{4}$$

$$g_0 = \sum_l G_l^*(m, \mu, \epsilon) g^l \tag{5}$$

Now, substitute (4)-(5) into (1):

$$\sigma_i(g, m, p_k, \mu, \epsilon) = \sum_l \sigma_i^{*l}(m, p_k, \mu, \epsilon) g^l \tag{6}$$

If it is possible to choose the functions (2) and (3) in such a way that all divergences in (6) cancel, theory is renormalizable. Of course if there exists one pair of functions m, g that satisfies this condition, the infinite number of such pairs can be found and they are some finite functions of initial g and m expandable in positive power series in g — this is manifestation of the freedom in choosing renormalization scheme. The choice that seems natural is to take some physical quantities (e.g. the pole mass in (2)) as m and g. Of course quite often for technical reasons other schemes are more convenient.

To reproduce the counter term technique of renormalization, let us recall that LSZ technique prescribes to divide N-point Green's functions by a factor $Z^{1/2}$ with Z being the residue of the propagator at the pole. Alternatively we could define renormalized field as $\phi_R = \phi Z_1^{-1/2}$ where Z_1 can differ from Z by a finite multiplier.

Now if we rewrite Lagrangian in terms of ϕ_R and substitute instead of bare parameters their expansions (4) and (5) we will recover Lagrangian with counter terms. Although the described formulation of renormalization procedure is fully equivalent to the counter term technique, for our purposes it is more convenient.

The feature of renormalization procedure that we want to underline can be formulated as follows: *After regularizing renormalizable theory it is enough just to express all physical quantities in terms of few observables (their number equals to the number of bare parameters) and divergences will disappear.* In the next section we are going to demonstrate that the same is true for nonrenormalizable theories too with the exception that the number of expansion parameters is more (but finite) than that of bare ones.

3 Renormalization of nonrenormalizable

First of all let us formulate an assumption which will help us to argument our method. We will assume that nonrenormalizable theory under consideration is consistent — i.e. nonperturbatively finite and hence all divergences appearing in perturbative series are due to the nonanalytic dependence of the expanded quantities on the bare parameters. Of course we do not expect that all of the nonrenormalizable theories are of that kind, but we want to emphasize that the method removes divergences from *any* theory. In fact the method is not explicitely based on this assumption — as long as we do have no information about nonperturbative finiteness it can be applied blindly just hoping that the resulting finite series will be meaningful. Anyway we believe that it will be case only if the nonrenormalizable theory under consideration is selfconsistent.

The question we want to answer is whether it is possible to extract any reliable information about the relations between different physical quantities σ_i even if they are given by the series with divergent coefficients. In order to see that sometimes the answer is 'yes', let us consider a simple mathematical example. Suppose we have two functions f_1 and f_2 given by series with divergent coefficients (we are interested in taking $\epsilon \to 0$ limit):

$$\begin{aligned} f_1 &= -\frac{g^3}{\epsilon} + \frac{g^5}{\epsilon} + \frac{1}{2}\frac{g^5}{\epsilon^2} + \cdots \\ f_2 &= 1 + g + \frac{g^2}{\epsilon} - \frac{g^4}{\epsilon} + \cdots \end{aligned} \tag{7}$$

Note that k-th inverse power of ϵ goes together with at least k-th power of g^2. Denoting $x \equiv g^2$ we can rewrite (7) as (in each term containing ϵ^{-k}, g^{2k} is substituted by x^k):

$$\begin{aligned} f_1 &= -g\frac{x}{\epsilon} + g^3\frac{x}{\epsilon} + \frac{g}{2}\frac{x^2}{\epsilon^2} + \cdots \\ f_2 &= 1 + g + \frac{x}{\epsilon} - g^2\frac{x}{\epsilon} + \cdots \end{aligned} \tag{8}$$

Now let us for a moment consider x as an independent parameter and express iteratively x from the second line in (8) as power series in g and $\alpha \equiv f_2 - 1 - g$ (note that the definition of α is automatically implied from (8) and substitute it into the expression of f_1. It is easy to see that divergences disappear. We get:

$$\begin{aligned} x &= \epsilon\left(\alpha + \alpha g^2 + \cdots\right) \\ f_1 &= -(g\alpha - \frac{g}{2}\alpha^2 + \cdots) \end{aligned} \tag{9}$$

The righthandside of (9) is the expansion of

$$f_1 = -gln(1+\alpha) = -gln(f_2 - g) \tag{10}$$

Indeed, we have obtained (7) by 'regularizing' and expanding the following functions:

$$\begin{aligned} f_1(g) = gln g^2 &\rightarrow gln\frac{g^4/\epsilon + 1}{g^2/\epsilon + 1} \\ f_2(g) = g + \frac{1}{g^2} &\rightarrow g + \frac{g^2/\epsilon + 1}{g^4/\epsilon + 1} \end{aligned} \tag{11}$$

So we have recovered correct relation between f_1 and f_2 — (10) starting from series with divergent coefficients. We considered this simple example to illustrate that the diverging series may as well contain some information about relations between functions and this relations may be extracted. It is worth mentioning that the method deals well with different singular functions and reproduces correct series for different 'regularizations'.

We would like to note, that although initially in (11) we had dependence over one parameter g, the expansion with finite coefficients became possible only after introduction of one extra expansion parameter α. In fact parameters g and α are not independent.

The series in quantum field theory have a nice feature — analogue to the one that turned out useful in above example — inverse powers of ϵ always come together with at least some nonzero power of coupling constant (bare or renormalizable). In other words any renormalized Green's function or amplitude can be written as:

$$G_R = \sum_{i,k} f_{ik} g^i \left(\frac{g_R^\beta}{\epsilon} \right)^k \tag{12}$$

Here coefficients f_{ik} are finite in $\epsilon \to 0$ limit and β is determined by simple power counting.

Now it is clear how we can proceed in nonrenormalizable theory. Consider some consistent nonrenormalizable theory with single coupling constant. Acting along the lines of conventional renormalization in the spirit described in previous section, write expansion of e. g. pole mass m in bare coupling and solve from this expansion bare mass m_0. Also we can express bare coupling g_0 from some physical amplitude or Green's function (in the latter case renormalization of wave function is also required) at some kinematics — usually in renormalizable theories it is an effective vertex g_R.

If the theory were renormalizable, then performing wave function renormalization and inserting expressions of bare parameters in m and g_R would make finite any Green's function. In nonrenormalizable theory we are left with series for Green's functions that still contain divergences.

Next introduce in (12) x instead of g_R^β and express it from any convenient Green's function or amplitude as series in g_R and α (where definition of α would be automatically implied just like in the example above). Evidently, inserting this series into any other Green's function will lead to series free of divergences. The price we have to pay for it is introduction of an extra expansion parameter. Of course g_R and *alpha* are not independent. We do not know whether the relation between them can be established perturbatively.

Of course the status of final series will depend on the theory under consideration — hopefully, for consistent theories they will not be worse than asymptotic.

The method can be generalized for the case of several bare couplings avoiding introduction of more then one extra expansion parameter.

So the suggested method coincides with the ordinary renormalization procedure for renormalizable theories and implies introduction of an extra effective parameter for nonrenormalizable ones. It works not only for nonrenormalizable theories which are finite outside perturbation theory but, unfortunately, it will produce finite coefficients for the theories where infinities are present even in exact solutions. So special care is needed if one tryes to apply it for infrared divergences — one must be sure of the origin of them.

The described method is easily applied within the framework of any regularization where divergences appear only as powers of some regulator. For other regularizations more (but finite number) of extra expansion parameters will be required.

4 Application to Quantum Gravity

Let us illustrate the general ideas presented in previous section on the example of Quantum Gravity. The nonrenormalizability of Einstein's gravity coupled to scalar field and to fermion or photon fields was demonstrated in [8] and [9], respectively. We will consider the latter case — photon field. First let us derive the Feynman rules (we follow ref. [7]). The Lagrangian density has the form:

$$\mathcal{L} = \mathcal{L}_G + \mathcal{L}_A$$

where $\mathcal{L}_G$ is the familiar Einstein Lagarngian:

$$\mathcal{L}_G = \frac{2}{k^2}\sqrt{-g}g^{\mu\nu}R^{\sigma}_{\mu\sigma\nu}$$

with $g^{\mu\nu}$ being the metric tensor and R the curvature tensor. $\mathcal{L}_A$ denotes the generally covariant photon Lagrangian, defined by minimal substitution

$$\mathcal{L}_A = -\frac{1}{4}\sqrt{-g}g^{\mu\nu}g^{\alpha\beta}F_{\mu\nu}F_{\alpha\beta}$$

with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. (We work in Euclidian space.) Defining $\mathcal{G}^{\mu\nu} \equiv \sqrt{-g}g^{\mu\nu}$ we can rewrite $\mathcal{L}_G$ and $\mathcal{L}_A$ in arbitrary n dimensions as

$$\mathcal{L}_G = \frac{1}{2k^2}\left(\mathcal{G}^{\rho\sigma}\mathcal{G}_{\lambda\mu}\mathcal{G}_{\kappa\nu} - \frac{1}{n-2}\mathcal{G}^{\rho\sigma}\mathcal{G}_{\mu\kappa}\mathcal{G}_{\lambda\nu} - 2\delta^{\sigma}_{\kappa}\delta^{\rho}_{\lambda}\mathcal{G}_{\mu\nu}\right)\partial_\rho\mathcal{G}^{\mu\kappa}\mathcal{G}^{\lambda\nu}_{\sigma}$$

$$\mathcal{L}_A = -\frac{1}{4}\left(-det\mathcal{G}\right)^{-\frac{1}{n-2}}\mathcal{G}^{\mu\nu}\mathcal{G}^{\alpha\beta}F_{\mu\alpha}F_{\nu\beta}$$

where δ-s denote n-dimensional Kronecker symbols. We can write the generating functional as follows

$$\begin{aligned} Z[\mathcal{J}_{\mu\nu}, J_\mu] &= \int \mathcal{D}(\mathcal{G}^{\mu\nu})\mathcal{D}(A^\mu)\Delta[\mathcal{G}^{\mu\nu}, A_\alpha] \\ &\quad \delta(\partial_\mu\mathcal{G}^{\mu\nu})\delta(\partial_\alpha A^\alpha)e^{i\int d^n x(\mathcal{L}_G+\mathcal{L}_A)+J_\mu A^\mu+\mathcal{J}_{\mu\nu}\mathcal{G}^{\mu\nu})} \end{aligned}$$

$$\Delta[\mathcal{G}^{\mu\nu}, A_\alpha]\int \mathcal{D}\Omega\delta(\partial_\mu\mathcal{G}^{\mu\nu}_{\Omega})\delta(\partial_\alpha A^{\alpha\Omega}) = 1$$

Defining graviton field $\phi^{\mu\nu}$ by $k\phi^{\mu\nu} = \mathcal{G}^{\mu\nu} - \delta^{\mu\nu}$ we can expand $\mathcal{G}_{\mu\nu}$ as

$$\mathcal{G}_{\mu\nu} = \delta_{\mu\nu} - k\phi_{\mu\nu} + k^2\phi_{\mu\lambda}\phi_{\lambda\nu} + k^3\phi_{\mu\alpha}\phi_{\alpha\beta}\phi_{\beta\nu} + O(k^4)$$

(We work in Euclidian space so there is no need to distinguish between upper and lower indices of ϕ). In terms of graviton field Lagrangian takes the form:

$$\mathcal{L} = \sum_{i=2}^{\infty} k^{i-2}\mathcal{L}_{(i)} = \sum_{i=2}^{\infty} k^{i-2}\left(\mathcal{L}_{G(i)} + \mathcal{L}_{A(i)}\right)$$

To define graviton and photon propagators we need quadratic parts:

$$\mathcal{L}_{G(2)} = \frac{1}{2}\partial_\mu\phi_{\nu\lambda}\partial_\mu\phi_{\nu\lambda} - \frac{1}{2(2-n)}\partial_\mu\phi_{\nu\nu}\partial_\mu\phi_{\lambda\lambda} - \partial_\mu\phi_{\mu\nu}\partial_\rho\phi_{\rho\nu}$$

$$\mathcal{L}_{A(2)} = -\frac{1}{4} F^{\mu\nu} F^{\mu\nu}$$

The ghost propagator is defined from the expression (note that using particular type of gauge $\partial_\alpha A_\alpha = 0$ no ghost corresponding to the photon field is required):

$$\begin{aligned} \Delta[\mathcal{G}^{\mu\nu}, A_\alpha] &= \int \mathcal{D}(\zeta_\lambda)\mathcal{D}(\eta_\nu) exp\Big\{ i \int d^n x \eta_\nu \Big[\delta_{\mu\nu}\partial^2 - k\Big(\partial_\lambda \partial_\mu \phi_{\mu\nu} - \\ &- \phi_{\mu\rho}\delta_{\nu\lambda}\partial_\rho\partial_\mu - \partial_\mu\phi_{\mu\rho}\delta_{\nu\lambda}\partial_\rho + \partial_\mu\phi_{\mu\nu}\partial_\lambda \Big) \Big] \zeta_\lambda \Big\} \end{aligned} \tag{13}$$

Accordingly propagators have the following form:

photon propagator

$$D_{\mu\nu}(q^2) = \frac{1}{q^2}\left(-\delta_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right)$$

ghost propagator

$$\Delta_{\mu\nu} = \frac{\delta_{\mu\nu}}{q^2}$$

graviton propagator

$$\begin{aligned} \mathcal{D}_{\alpha\beta,\lambda\nu}(p) &= \frac{1}{2p^2}(\delta_{\lambda\alpha}\delta_{\beta\mu} + \delta_{\alpha\mu}\delta_{\beta\lambda} - 2\delta_{\alpha\beta}\delta_{\lambda\mu}) - \\ &- \frac{1}{2p^4}(p_\lambda p_\alpha \delta_{\mu\beta} + p_\mu p_\alpha \delta_{\lambda\beta} + p_\lambda p_\beta \delta_{\mu\alpha} + p_\mu p_\beta \delta_{\lambda\alpha}) + \\ &+ \frac{1}{p^4}(p_\lambda p_\mu \delta_{\alpha\beta} + p_\alpha p_\beta \delta_{\lambda\mu}) \end{aligned}$$

Vertices are defined from the $\mathcal{L}_{(i>2)}$ and also from $\eta\phi\zeta$ terms in (13). It is easy to see that any N-particle vertex has the order k^{N-2} — this fact is important for our further analysis.

Consider a Feynman diagram containing N_i i-particle (graviton or photon) vertices, with E external legs and I internal ones. It is straightforward to relate these quantities to the number of loop integrations l:

$$l = \frac{1}{2}(\sum_i N_i(i-2) - E + 2)$$

If we use dimensional regularization, then l loop integrations may produce at most $(\frac{1}{\epsilon})^l$ divergence. As we have mentioned above any i-particle vertex has the order

$k^{(i-2)}$, so comparing powers of k and $\frac{1}{\epsilon}$ we see that any N-point Green's function can be written as

$$G_N = k^{N-2} \sum_{m,n} \left(\frac{k^2}{\epsilon} \right)^m k^n C_{m,n}$$

where coefficients $C_{m,n}$ are free of divergences.

Let us consider amputated Green's function at symmetric point q^2:

$$\Gamma_{\alpha\beta\sigma\lambda} \equiv < \mathcal{G}_{\alpha\beta} A_\sigma A_\lambda >$$

After performing wave function renormalization it takes the form:

$$\begin{aligned} \Gamma_{\alpha\beta\sigma\lambda} &\sim \delta_{\alpha\beta}\delta_{\lambda\sigma} C_1(q^2) + (\delta_{\alpha\lambda}\delta_{\beta\sigma} + \delta_{\alpha\sigma}\delta_{\beta\lambda}) C_2(q^2) + \delta_{\alpha\beta} p'_\sigma p_\lambda C_3(q^2) + \\ &+ \delta_{\lambda\sigma}(p_\alpha p'_\beta + p'_\alpha p_\beta) C_4(q^2) + \\ &+ (\delta_{\alpha\sigma} p_\lambda p\beta' + \delta_{\beta\sigma} p_\lambda p'_\alpha + \delta_{\alpha\lambda} p_\beta p'_\sigma + \delta_{\beta\lambda} p_\alpha p'_\sigma) C_5(q^2) \end{aligned} \tag{14}$$

We suppose that there exist some finite exact formfactors C_i that stand in (14) as coefficients of independent tensor structures. The perturbative expansion gives:

$$\begin{aligned} 2(2-n)C_1 &= kq^2 + k^3 q^4 \left(\frac{a_1}{\epsilon} + a_2 + a_3(\epsilon) \right) + \\ &+ k^5 q^6 \left(\frac{a_4}{\epsilon^2} + \frac{a_5}{\epsilon} + a_6 + a_7(\epsilon) \right) + \cdots \end{aligned} \tag{15}$$

$$C_2 = \frac{kq^2}{2} + k^3 q^4 \left(\frac{b_1}{\epsilon} + b_2 + b_3(\epsilon) \right) + k^5 q^6 \left(\frac{b_4}{\epsilon^2} + \frac{b_5}{\epsilon} + b_6 + b_7(\epsilon) \right) + \cdots \tag{16}$$

Let us introduce renormalized coupling k_R as:

$$k_R = \frac{2(2-n)C_1(\Lambda^2)}{\Lambda^2} \tag{17}$$

here Λ is normalization point. Solving (15) iteratively for k we obtain:

$$\begin{aligned} k &= k_R - k_R^3 \Lambda \left(\frac{a_1}{\epsilon} + a_2 + a_3(\epsilon) \right) + \\ &+ k_R^5 \Lambda^4 \left(\frac{3a_1^2 - a_4}{\epsilon^2} + \frac{6a_1 a_2 - a_5}{\epsilon} + \tilde{a}(\epsilon) \right) + \cdots \end{aligned} \tag{18}$$

Inserting this expansion for bare coupling into expression of C_2 — (16) we find:

$$C_2 = \frac{k_R q^2}{2} + k_R^3 \left(\frac{k_R^2}{\epsilon} \left(b_1 q^4 - \frac{a_2}{2} q^2 \Lambda^2 \right) + k_R^2 \tilde{b}_2 \right) +$$

$$
\begin{aligned}
&+ \quad k_R\left(\frac{k_R^2}{\epsilon}\right)^2\left(b_4q^6 - 3a_1b_1q^4\Lambda^2 - \frac{3a_1^2-a_4}{2}q^2\Lambda^4\right) + \\
&+ \quad k_R^3\frac{k_R^2}{\epsilon}\left(b_5q^6 - 3(b_1a_2+b_2a_1)q^4\Lambda^2 + \frac{6a_1a_2-a_5}{2}q^2\Lambda^4\right) + \cdots
\end{aligned}
$$

Consider extension of function C_2 to two parameter dependence:

$$
\begin{aligned}
C_2^*(q^2) \quad &= \quad k_R\frac{q^2}{2} + k_R\left(\frac{x}{\epsilon}\left(b_1q^2 - \frac{a_1}{2}q^2\Lambda^2\right) + k_R^2\tilde{b}_2\right) + \\
&+ \quad k_R\left(\frac{x}{\epsilon}\right)^2\left(b_4q^2 - 3a_1b_1q^2\Lambda^2 + \frac{3a_1^2-a_4}{2}q^2\Lambda^4\right) + \\
&+ \quad k_R^3\frac{x}{\epsilon}\left(b_5q^6 - 3\left(b_1a_2+b_2a_1\right)q^4\Lambda^2 + \frac{6a_1a_2-a_5}{2}q^2\Lambda^4\right) + \\
&+ \quad k_R^5\tilde{b}_6 + \cdots
\end{aligned}
\tag{19}
$$

Of course, taking $x = k_R^2$ we recover C_2 from C_2^*. Let us denote the sum of diverging terms in (19) by $\tilde{\alpha}$ (according to our assumption $\tilde{\alpha}$ is nonperturbatively finite).

$$
\begin{aligned}
\tilde{\alpha}(q^2) \quad &= \quad k_R\frac{x}{\epsilon}\left(b_1q^4 - \frac{a_1}{2}q^2\Lambda^2\right) + \\
&+ \quad k_R\left(\frac{x}{\epsilon}\right)^2\left(b_4q^6 - 3q^4\Lambda^2a_1b_1 + \frac{3a_1^2-a_4}{2}q^2\Lambda^4\right) + \\
&+ \quad k_R^3\frac{x}{\epsilon}\left(b_5q^6 - 3\left(b_1a_2+b_2a_1\right)q^4\Lambda^2 + \frac{6a_1a_2-a_5}{2}q^2\Lambda^4\right) + \cdots = \\
&= \quad C_2^* - k_R\frac{q^2}{2} - k_R^3\tilde{b}_2 - k_R^5\tilde{b}_6 + \cdots
\end{aligned}
\tag{20}
$$

Now solve (20) iteratively for x as series in

$$
\alpha(\Lambda^2) = \frac{\tilde{\alpha}(\Lambda^2)}{k_R\left(b_1\Lambda^4 - \frac{a_1}{2}\Lambda^4\right)}
\tag{21}
$$

$$
\begin{aligned}
x \quad &= \quad \epsilon\alpha(\Lambda^2) - \epsilon k_R\alpha^2(\Lambda^2)\frac{b_4 - 3a_1b_1 + \frac{3a_1^2-a_4}{2}}{b_1 - \frac{a_1}{2}}\Lambda^2 - \\
&- \quad \epsilon k_R^3\alpha(\Lambda^2)\frac{b_5 - 3(b_1a_2 + b_2a_1 + \frac{6a_1a_2-a_5}{2}}{b_1 - \frac{a_1}{2}}\Lambda^2 + \cdots
\end{aligned}
\tag{22}
$$

(Of course we could solve (20) at some other $\Lambda_1 \neq \Lambda$). The defined effective coupling constants (17) and (21) will enable us to make finite coefficients of any Green's function.

In this manner we can express *any* Green's function as finite coefficient series of two effective parameters (these can be defined from some quantities other then formfactors $C_{1,2}$). We do not present explicit calculations of particular processes here.

5 Conclusions

So we have described a method of 'renormalization' of perturbative series in non-renormalizable theories. For renormalizable ones it just coincides with the usual renormalization procedure. The method is based on suitable introduction of an extra effective expansion parameter (which is not independent). We are unable to find relation between effective couplings within the framework of perturbation theory, so for numeric analysis we need one more experimental value then the number of bare couplings and masses.

The method can be applied to any theory. (E.g. for standard model plus gravity one may use suitably adjusted extra effective expansion parameter defined in previous section.)

Unfortunately in full analogy to the conventional renormalization procedure for renormalizable theories, suggested method is insensitive to the consistency of the theory — it will produce order by order finite series even for inconsistent theories. So establishing of asymtotic character or final series in given theory is desirable, but unfortunately this problem is too complicated (e.g. it is not completely solved even for so 'well explored' theory as QED). Although we hope that quantum gravity based on Einstein's lagrangian is nonperturbatively finite and calculations using suggested method are meaningful.

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