Duality in N=1, D=10 Superspace and Supergravity with Tree Level Superstring Corrections

N.A.Saulina [a,b], **M.V.Terentiev** [a], **K.N.Zyablyuk** [a]

[a] *Institute Theoretical and Experimental Physics* [1]
[b] *Moscow Physical Technical Institute*

Abstract

The equations of motion (e.m.'s) of the N=1, D=10 anomaly free supergravity, obtained in the framework of the superspace approach, are analyzed. The formal equivalence of the usual and dual supergravities is discussed at the level of e.m.'s. The great simplicity of the dual formulation is established. The possibillity of the lagrangian formulation of the dual supergravity is pointed out. The bosonic part of the lagrangian is found.

1 Introduction

There are two versions of the same theory: 1) the D=10, N=1 supergravity [1], [2], [3] with the 3-form graviphoton field $H^{(0)}_{abc}$ as a member of the gravity supermultiplet, and 2) the dual D=10, N=1 supergravity [1], [4], [5], [6] where the 7-form graviphoton field $N_{a_1 \dots a_7}$ is used instead of $H^{(0)}_{abc}$. For further references we introduce notations $G3$ (Gravity with the 3-form H-field) and $G7$ (Gravity with the 7-form N-field) for these two versions. The $G7$ can be derived from the $G3$-theory by the dual transformation at the lagrangian level [4], [6]. Both theories are anomalous.

The connection between usual and dual versions becomes less clear if one considers $G3$ as a low energy limit of the heterotic superstring. In this case superstring corrections (anomaly cancelling) must be added to the $H^{(0)}$ field in the $G3$-theory lagrangian [7]:

$$H^{(0)}_{abc} \Rightarrow H_{abc} = H^{(0)}_{abc} + k_g \, \Omega^{(g)}_{abc} \tag{1.1}$$

where $H^{(0)} = dB$, B is the two-form potential, $\Omega^{(g)}$ is the Lorentz-group Chern-Symons (CS) three-form, k_g is a constant $\sim \alpha'$ (the string-tension constant), $d\Omega^{(g)} = trR^2$, where R is the curvature two-form, *trace* is calculated in the fundamental represention of the Lorentz $O(1.9)$- group. (We consider here the gravity sector. The incorporation of the Yang-Mills matter can be done by standard methods).

After the change (1.1) one obtains a theory which can be made anomally-free (by addition of special counter-terms at the one-loop level), but it is not supersymmetric even at the tree-level.

[1] E-mail: saulina@vxitep.itep.ru, terent@vxitep.itep.ru, zyablyuk@vxitep.itep.ru

The supersymmetric completion of such a theory has been done at the mass shell in papers [8], [9], [10] (see also [11] for more complete list of references). The complete lagrangian has not been constructed but it has become clear that it contains (being formulated in terms of physical fields) terms $\sim R^2$ and an infinite number of terms $\sim k_g^q H^p$, $q \geq 1, p \geq 3$. (Several terms of the lowest order were found in [6], [12]). For brevity we name this theory $SG3$ (from "Superstring inspired Gravity"). The important property of the $SG3$ is the scale invariance [13], which is the tree-level (classical) symmetry. (It means that only tree-level superstring corrections are taken into account).

We discuss here the (scale invariant) dual analog of the $SG3$ - theory. We name it $SG7$ for short. It is expected, that such a theory is a low energy limit of a five-brane [14]. The $SG7$ can be formulated self-consistently and we write explicitely the dual transformation from the $SG7$ to the $SG3$ - theory at the mass-shell. (The inverse transformation is much more complicated and can be defined only as a perturbative series in k_g). The connection between $SG7$ and $SG3$-theories was suggested much earlier in [15] where explicit calculations were not presented (we agree with the remarks from [15]). We use the mass-shell superspace approach to the problem. The iterative scheme for the dual transformation and for the lagrangian of $SG7$ in the component approach was discussed in [6],[16].

Equations of motion (e.m.'s) in the $SG7$ are much simpler than in the $SG3$. That makes it possible to construct a supersymmetric lagrangian for the general $k_g \neq 0$ case. We derive here the bosonic part of this lagrangian. The simplicity of the final result is in a great contrast with the enormous complexity of intermediate calculations. The dual transformation from the (relatively simple) $SG7$ to the $SG3$ lagrangian is possible only perturbatively in k_g. (That explains the complexity of the $SG3$ - theory). The fermionic and Yang-Mills matter sectors of a the $SG7$ lagrangian can be also constructed using the described procedure. (The corresponding results will be published elsewhere).

Preparatory results for this study was given in [17], [18]. Results connected with the lagrangian construction are based on papers [19], [20]. We use the computer program "GRAMA" [21] written in MATHEMATICA for analytical calculations in supergravity. Our notations correspond in general to [17] (small differences are self-evident or explained in the text).

2 Geometrical Mass-Shell Formulation

The superspace e.m.'s can be formulated universaly for the $SG3$ and the $SG7$, using relations which are valid for both theories. These relations are:

1) Geometrical Bianchi Identities (BI's) for the supertorsion $T_{BC}{}^D$:

$$D_{[A}T_{BC)}{}^D + T_{[AB}{}^Q\, T_{QC)}{}^D - \mathcal{R}_{[ABC)}{}^D = 0. \tag{2.1}$$

The nonzero torsion components in (2.1) are $T_{abc} \equiv \eta_{cd}T_{ab}{}^d$ (T_{abc} is a completely antisymmetric tensr), $T_{ab}{}^\gamma$ and:

$$T_{\alpha\beta}{}^c = \Gamma^c_{\alpha\beta}\,, \quad T_{a\beta}{}^\gamma = \frac{1}{72}(\hat{T}\Gamma_a)_\beta{}^\gamma\,, \tag{2.2}$$

where $\hat{T} \equiv T_{abc}\Gamma^{abc}$. We use the constraints from [22].

2) Commutation relations for supercovariant derivatives D_A:

$$(D_A\, D_B - (-1)^{ab} D_B\, D_A)\, V_C = -\, T_{AB}{}^{Q}\, D_Q\, V_C - \mathcal{R}_{ABC}{}^{D}\, V_D, \tag{2.3}$$

where V_C is a vector superfield, $\mathcal{R}_{ABCD}$ is a supercurvature (which differs in sign in comparison with [17]).

3) The general result for the spinorial derivative of the dilatino χ-superfield ($\chi_\alpha \equiv D_\alpha \phi$, where ϕ is the dilaton superfield):

$$D_\alpha \chi_\beta = -\frac{1}{2}\, \Gamma^b_{\alpha\beta} D_b \phi + (-\frac{1}{36}\phi T_{abc} + A_{abc})\, \Gamma^{abc}_{\alpha\beta}, \tag{2.4}$$

Here A_{abc} is a completely antisymmetric superfield, which is unambiguously determined (see below) in terms of torsion and curvature.

Some comments on the notations are helpful. We use letters from the beginning of the alphabet for the tangent superspace indices $A = (a, \alpha)$ and letters from the middle of the alphabet for the world superspace indices $M = (m, \mu)$. Here a, m are 10-dim. vector indices, α, μ - 16-dim. spinorial indices. The veilbein is defined as follows [23]:

$$E_M{}^A| = \begin{pmatrix} e_m{}^a & \psi^\alpha_m \\ 0 & \delta^\alpha_\mu \end{pmatrix}, \tag{2.5}$$

where ψ^α_m ia a gravitino superfield.

The supercovariant vector derivative $D_a \equiv E_a{}^M\, D_M$ is equal to:

$$D_a = e^m_a\, D_m - \psi^\beta_a\, D_\beta\,, \tag{2.6}$$

where $\psi_a = e^m_a\, \psi_m$ but the space-time component of the covariant derivative is:

$$D_m \lambda = \partial\, \lambda - \omega_m\, \lambda \tag{2.7}$$

where λ^γ is any spinorial superfield and $(\omega_m)^\beta{}_\gamma \equiv \frac{1}{4}\omega_m{}^{ab}(\Gamma_{ab})^\beta{}_\gamma$ is the spin-connection which is in the algebra of $O(1.9)$.

By a standard way one finds the relation between the torsion-full spin-connection in eq.(2.7) and the standard spin-connection $\omega^{(0)}_{cab}$ defined in terms of derivatives of e^a_m:

$$\omega_{cab} = \omega^{(0)}_{cab}(e) + \frac{1}{2}\, T_{cab} + C_{cab}\,, \tag{2.8}$$

where:

$$C_{cab} = \psi_a\, \Gamma_c\, \psi_b - \frac{3}{2}\psi_{[a}\, \Gamma_c\, \psi_{b]} \tag{2.8}$$

We use the notation ∇_m for a covariant derivative with the spin-connection $\omega^{(0)}_m$ ($\nabla_{[m} e^a_{n]} = 0$). We also define $\nabla_a \equiv e^m_a \nabla_m$. Using these notations one obtains the torsion-component $T_{ab}{}^\gamma = 2 e^m_a e^n_b (D_{[m} e^\gamma_{n]})$ in the form:

$$T_{ab} = 2\, \nabla_{[a}\, \psi_{b]} - \frac{1}{72}(\Gamma_{[a}\hat{T} + 3\, \hat{T}\Gamma_{[a)})\psi_{b]} + \frac{1}{2}\, (\Gamma^{cd})\, \psi_{[a}\, C_{b]cd} \tag{2.9}$$

Below we use different notations $\mathcal{R}_{\dots}$ and $R_{\dots}$ for the curvature tensor defined in terms of spin-connections ω and $\omega^{(0)}$ correspondingly ($dR = d\omega + \omega^2$). The complete set of e.m.'s for the gravity supermultiplet derived from (2.1)-(2.4) in [17] takes the form:

$$\phi L_a - D_a\chi - \frac{1}{36}\Gamma_a\hat{T}\chi - \frac{1}{24}\hat{T}\Gamma_a\chi + \frac{1}{42}\Gamma_a\Gamma^{ijk}DA_{ijk} + \frac{1}{7}\Gamma^{ijk}\Gamma_a DA_{ijk} = 0, \tag{2.10}$$

$$D_b\Gamma^b\chi + \frac{1}{9}\hat{T}\chi + \frac{1}{3}\Gamma^{ijk}DA_{ijk} = 0. \tag{2.11}$$

$$D_a^2\phi + \frac{1}{18}\phi(T^2) - 2\,(TA) - \frac{1}{24}D\Gamma^{ijk}DA_{ijk} = 0. \tag{2.12}$$

$$\begin{gathered}\phi\mathcal{R}_{ab} - L_{(a}\Gamma_{b)}\chi - \frac{1}{36}\phi\eta_{ab}(T^2) + D_{(a}D_{b)}\phi - \\ -2\,(TA)_{(ab)} + \frac{3}{28}D\Gamma^{ij}{}_{(a}DA_{b)ij} - \frac{5}{336}\eta_{ab}D\Gamma^{ijk}DA_{ijk} = 0.\end{gathered} \tag{2.13}$$

$$\begin{gathered}D_{[a}(\phi T_{bcd]}) + \frac{3}{2}T_{[ab}\Gamma_{cd]}\chi + \frac{3}{2}\phi(T^2)_{[abcd]} + \\ +\frac{1}{12}(T\epsilon A)_{abcd} + 6\,(TA)_{[abcd]} + \frac{3}{4}D\Gamma_{[ab}{}^{j}DA_{cd]j} = 0.\end{gathered} \tag{2.14}$$

$$D^aT_{abc} = 0, \tag{2.15}$$

There are constraints:

$$T_{ab}\Gamma^{ab} = 0, \tag{2.16}$$

$$\mathcal{R} - \frac{1}{3}\,(T^2) = 0, \tag{2.17}$$

where $\mathcal{R}$ is the supercurvature scalar ($\mathcal{R} \equiv \mathcal{R}_{abcd}\eta^{ac}\eta^{bd}$)

Furthemore, there are two equations for the A_{abc}-superfield. The first one [22],[17] follows from the self-consistency of equations (2.10)-(2.15), the second one follows from (2.4) [17] and means, that the 1200 IR contribution to the A-field spinorial derivative is equal to zero.

The following notations were used in (2.10)-(2.18):

$$\begin{gathered}L_a = T_{ab}\Gamma^b, \;\; T^2 = T_{ijk}T^{ijk}, \;\; TA = T_{ijk}A^{ijk}, \;\; (TA)_{ab} = T_{aij}A_b{}^{ij}, \\ (TA)_{abcd} = T_{abj}A_{cd}{}^{j}, \;\; (T\epsilon A)_{abcd} = T^{ijk}\varepsilon_{ijkabcdmns}A^{mns}\end{gathered} \tag{2.18}$$

Spinorial derivatives of the A_{abc}- superfield can be calculated in terms of torsion and curvature. After that the zero superspace components become the e.m.'s for physical fields of the $SG3$ or $SG7$ theories. (We use the same notations for physical fields and corresponding superfields expecting that it does not lead to the confusion).

Equations under discussion are not independent. Namely (2.12) follows from (2.13) after contraction of a, b indices, but (2.11) follows from (2.10) after multiplication by Γ^a matrix.

In general, neglecting Yang-Mills matter, $A_{abc} \sim k_g$ (see below). In the limiting case $A_{abc} = 0$ these equations describe the pure gravity sector of the $G3$ - theory if $T_{abc} = -(1/\phi)\, H_{abc}$. The same equations and constraints describe the $G7$ - theory if $T_{abc} = N_{abc}$, where:

$$N_{abc} \equiv \frac{1}{7!}\, \varepsilon_{abc}{}^{a_1 \ldots a_7}\, N_{a_1 \ldots a_7} \tag{2.19}$$

Now we consider in details the general $k_g \neq 0$- case starting from the $SG3$-theory.

3 Duality on the Mass-Shell

*SG*3 **theory**

The H-superfield BI's take the form:

$$D_{[A} H_{BCD)} + \frac{3}{2}\, T_{[AB}{}^{Q}\, H_{|Q|CD)} = -3\, k_g\, \mathcal{R}_{[AB}{}^{ef} \mathcal{R}_{CD)ef} \tag{3.1}$$

($DH = k_g\, tr\mathcal{R}^2$ in superform notations). Note, that $\gamma = -2\, k_g$ in [17].

The mass-shell solution of (3.1) which is compatible with (2.1)-(2.4) can be obtained using the constraint $H_{\alpha\beta\gamma} = 0$ in the standard procedure [24], [25], [8] . We find the nonzero components of the H_{ABC}-superfield in the form:

$$H_{\alpha\beta_a} = \phi\, (\Gamma_a)_{\alpha\beta} + k_g\, U^{(g)}_{\alpha\beta a}\,, \tag{3.2a}$$

$$H_{\alpha bc} = -(\Gamma_{bc}\, \chi)_\alpha + k_g\, U^{(g)}_{\alpha bc}, \tag{3.2b}$$

$$H_{abc} = -\phi\, T_{abc} + k_g\, U^{(g)}_{abc} \tag{3.2c}$$

In this place we do not need the explicit result for the $U^{(g)}_{\alpha\beta a}$ and $U^{(g)}_{\alpha bc}$ superfields (it will be presented elsewhere). The $U^{(g)}_{abc}$-superfield is equal to:

$$\begin{aligned} U^{(g)}_{abc} &= -2\, D^2_j\, T_{abc} + 4\, (T^3)_{abc} + \frac{2}{27}\, (T^2)\, T_{abc} - 6\, T_{ab}{}^{j}\, \mathcal{R}_{cj} - \\ &-6\, T_a{}^{ij}\, (\mathcal{R}_{ij,bc} - D_i\, T_{bcj} + D_b\, T_{cij}) - T_{ij} \Gamma_{abc} T^{ij} - 12\, T_{ja} \Gamma_b T_c{}^{j} - \\ &-L_j \Gamma_{abc} L^j - 12\, L_a \Gamma_b L_c + 6\, L_a T_{bc}\,, \quad [abc] \end{aligned} \tag{3.3}$$

where $[abc]$ means the antisymmetrization of the expression in corresponding indices, $(T^3)_{abc} = T_{aij} T_b{}^{jk} T_{ck}{}^{i}$. The $U^{(g)}_{abc}$ -superfield was discussed earlier in [8], [9], [25] using another parametrization (another set of constraints) .

The A-superfield in (2.4) is also determined unambiguosly from the (2,2)-component of the BI (3.1) (the (p, q)-component of a superform contains p bosonic and q fermionic indices):

$$A_{abc} = k_g\, A^{(g)}_{abc} \tag{3.4}$$

where

$$A^{(g)}_{abc} = -\frac{1}{18}\, D^2_j\, T_{abc} + \frac{5}{18}\, (T^3)_{abc} + \frac{1}{18 \cdot 12}\, (T^2)\, T_{abc} -$$

$$
-\frac{2}{9}\,T_{ab}{}^{j}\,(\mathcal{R}_{cj}+\frac{5}{8}\,(T^2)_{cj}-\frac{1}{9}\,T_a{}^{ij}\,(-\mathcal{R}_{ij,bc}+\frac{5}{4}\,D_i\,T_{bcj}+\frac{5}{4}\,D_b\,T_{cij})-
$$
$$
-\frac{1}{24\cdot 36}\,[(T\varepsilon T^2)_{abc}+\frac{2}{3}\,(T\varepsilon D\,T)_{abc}]-\frac{1}{24}\,T_{ij}\Gamma_{abc}T^{ij}+\frac{2}{9}\,T_{aj}\Gamma_b T_c{}^{j}-
$$
$$
-\frac{7}{8\cdot 18}\,L_j\Gamma_{abc}L^j+\frac{1}{18}\,L_a\Gamma_b L_c+\frac{4}{9}\,L_a T_{bc}-\frac{1}{9}L^j\Gamma_{ab}T_{cj}\,,\quad [abc] \tag{3.5}
$$

where $X\varepsilon Y_{abc}=X^{i_1\ldots i_k}\varepsilon_{i_1\ldots i_k abc j_1\ldots j_p}Y^{j_1\ldots j_p},\quad k+p+3=10.$

The A_{abc}-superfield defined by (3.4),(3.5) turns out to be a solution of eq.'s from [17]. That provides a good check of the result. [2]

Now we are ready to discuss e.m.'s (2.10)-(2.15) in the $SG3$ - theory. All spinorial derivatives can be calculated using relations from [17]. This work is in progress. The analogous calculations were done in [10] where another parametrization was used. Unfortunately we are not able to use results from [10]. One needs the expression of T_{abc} in terms of the H_{abc}- field to get the final form of equations. That may be obtained by inverting of eq. (3.2c) (it can be done only perturbatively in k_g). Then one gets a system of equations which is enormously complicated and obviously untractable [3].

Nevertheless one can interprete all the e.m.'s (2.10)-(2.15) in the $SG3$. Equations (2.10) -(2.13) are interpreted unamiguosly as gravitino, dilatino, dilaton and graviton e.m.'s, eq. (2.15) becomes the H-field e.m., but eq. (2.14) must be the H-field BI. Then eq. (2.14) must coincide with the (4,0)-component of the BI (3.1). That is really the case. Namely, substituting (3.2c) into the (4,0)-component of (3.1) we get eq. (2.14) if the following equation is satisfied:

$$
D_a\,U^{(g)}_{bcd}+\frac{3}{2}\,T_{ab}{}^{e}\,U^{(g)}_{ecd}+\frac{3}{2}T_{ab}{}^{\gamma}\,U^{(g)}_{\gamma cd}+\frac{1}{4}\,K_{abcd}=\frac{3}{2}\,(-2\,\mathcal{R}_{ab}{}^{ij}\mathcal{R}_{cdij})\,,\quad [abcd] \tag{3.6}
$$

where

$$
K_{abcd}=24(T A^{(g)})_{abcd}+\frac{1}{3}\,(T\varepsilon A^{(g)})_{abcd}+3\,D\Gamma_{ab}{}^{j}\,DA^{(g)}_{cdj}\,,\quad [abcd] \tag{3.7}
$$

We have checked, calculating spinorial derivatives, that (3.6) is satisfied identically.

Note, that (3.6) is a (4,0)-component of a general superform-identity [8]:

$$
D\,U^{(g)}+K=tr\mathcal{R}^2 \tag{3.8}
$$

where $U^{(g)}_{(0.3)}=K_{(0.4)}=K_{(1.3)}=0$. The (2,2), (1,3), (0,4) -components of (3.8) are satisfied because they are reduced to that used for *definition* of A and $U^{(g)}$- superfields.

[2]In deriving eq. (3.5) we have corrected some errors and misprints in [17]. Namely: 1) the factor $(-84\cdot 96)$ must be inserted into the l.h.s. of eq. (3.19) in [17], 2) the coefficient 2 must be changed to 4 in next to the last term in the r.h.s of eq. (3.19) in [17], and 3) the result for the Θ_{abcd}-tensor (see (3.18) in [17]) must be changed to:

$$
\theta_{abcd}=(4/3)\,D_{[a}T_{bcd]}+(64/27)\,(T^2)_{[abcd]}
$$

This change is due to the fact that the term $+\frac{1}{14}\,D\Gamma^{ef}G^{(1440)}_{e,fabcd}$ was missed in the l.h.s of eq. (3.16) in [17]. Note, that $A^{(g)}_{abc}=-2\,L_{abc}$, where L_{abc} is determined by eq. (3.19) in [17] including all the corrections, mentioned above.

[3]It is the reason why researh in this field, starting intensively in 1987, was stopped during the last few years.

One more remark is necessary. All the relations of the $SG3$ - theory are invariant under the scale transformation [13], [5]:

$$X_j \to \mu^{q_j} X_j \tag{3.9}$$

where X_j is an arbitrary field, but q_j is a numerical factor, which has a specific value for each field, μ is a common factor. It is a classical symmetry, because the lagrangian is also transformed according to (3.9) with $q = -2$.

In the Table 1 we present the transformation rules for different fields (the numerical factors in the table are values of q_j for each field):

Table 1

ϕ	-1	D_α	$-1/4$	T_{abc}	$-1/2$	T_{ab}^{γ}	$-3/4$
e_m^a	$1/2$	A_{abc}	$-3/2$	H_{abc}	$-3/2$	ψ_a^γ	$-1/4$
D_a	$-1/2$	${\mathcal{R}_{ab}}^{cd}$	-1	N_{abc}	$-1/2$	χ	$-5/4$

Now we come to consideration of the $SG7$-case.

$SG7$ **theory**

One can interpret the same equations (2.10)-(2.15) in terms of the 7-form graviphoton superfield $N_{A_1 \ldots A_7}$. The BI for such a field takes the form:

$$D_{[A_1} N_{A_2 \ldots A_8)} + \frac{7}{2} {T_{[A_1 A_2}}^{Q} N_{|Q| A_3 \ldots A_8)} = 0 \tag{3.10}$$

($DN = 0$ in superform notations). Because of the scale invariance (3.9) it is impossible to add any 8-form $\sim k_g$ constructed from curvature into the r.h.s. of (3.10) [5].

It is remarkable that the following nonzero components provide the solution of (3.10) which is self-consistent with (2.1)-(2.4):

$$N_{\alpha\beta a_1 \ldots a_5} = -(\Gamma_{a_1 \ldots a_5})_{\alpha\beta}, \tag{3.11}$$

$$N_{abc} = T_{abc}\,, \tag{3.12}$$

where N_{abc} is defined in (2.19). This solution is valid for *any* A_{abc}*-field* , in particular for that, defined by (3.4), (3.5), derived in the $SG3$-theory.

Using (3.12) in the equations (2.10)-(2.19) and defining the A_{abc}-field according to (3.4), (3.5) , we get the mass-shell description of the $SG7$-theory in a closed and relatively simple form. Eq.(2.14) becomes the $N_{a_1 \ldots a_7}$-field e.m., but eq. (2.15) is the (8,0)-component of the N-field BI. Using (3.12) in (3.2c) we get the duality relation between H_{abc} and $N_{a_1 \ldots a_7}$ fields. Now we come to the discussion of the lagrangian in the $SG7$ theory.

4 Bosonic Part of the Lagrangian

The lagrangian of the $SG7$-theory is equal to (we consider the gravity sector):

$$\mathcal{L}^{(g)} = \mathcal{L}_0^{(g)} + k_g\, \mathcal{L}_1^{(g)} \tag{4.1}$$

where $\mathcal{L}_0^{(g)}$ is the gravity part of the (anomaly full) lagrangian of the $G7$-theory, but $\mathcal{L}_1^{(g)}$ describes the anomaly compensating term [7] and other terms, generated by supersymmetry.

The $\mathcal{L}_0^{(g)}$ has a simple form [19], which follows from the linearity in ϕ and χ -fields of the e.m.'s (2.10)-(2.15):

$$\mathcal{L}_0^{(g)} = \phi\,(\mathcal{R} - \frac{1}{3}T^2)\,| + 2\chi\,\Gamma^{ab}T_{ab}\,| \tag{4.2}$$

(As usual the symbol | means the zero superspace-component of the superfields). The bosonic part of (4.2) takes the form:

$$\mathcal{L}_{bos}^{(g)} = \phi\,R - \frac{1}{12}\,\phi\,M_{abc}^2 \tag{4.3}$$

where R is the curvature scalar (see the comment after eq. (2.9)), but

$$M_{abc} \equiv \frac{1}{7!}\,\varepsilon_{abc}{}^{a_1\ldots a_7}\,(e_{a_1}{}^{m_1}\,\ldots e_{a_7}{}^{m_7}\,N_{m_1\ldots m_7})\,, \tag{4.4}$$

where $N_{m_1\ldots m_7} = 7\,\partial_{[m_1}\,M_{m_2\ldots m_7]}$, and $M_{m_1\ldots m_6}$ is the 6-form graviphoton potential of the $SG7$ - theory. Note, that

$$M_{abc} = T_{abc} - \frac{1}{2}\,\psi_f\,\Gamma^f{}_{abc}{}^d\,\psi_d \tag{4.5}$$

as it follows from (3.12), (2.19).

The explicit form of $\mathcal{L}_0^{(g)}$ with all fermionic terms is presented in [19]. (The result coincides with [4], [6] after the field redefinition). The field transformation to the set of (primed) fields with canonical kinetic terms has the form:

$$\begin{gathered} e_m^a = exp(\frac{1}{6}\phi')\,e_m^a{}', \quad \phi = exp(-\frac{4}{3}\phi'), \quad \chi = -\frac{4}{3}exp(-\frac{17}{12}\phi')\,\chi' \\ \psi_m = exp(\frac{1}{12}\phi')(\psi_m{}' - \frac{1}{6}\Gamma_m{}'\chi'), \quad N_{abc} = -2exp(-\frac{7}{6}\phi')\,N'_{abc} \end{gathered} \tag{4.6}$$

It is the Super-Weyl transformation [26] (see [18] for details).

Now we come to the discussion of $k_g\,\mathcal{L}_1^{(g)}$-term in (4.1). It is the property of our parametrization that $\mathcal{L}_1^{(g)}$ does not depend of ϕ and χ - fields. It means that the scale invariance simplifies greatly the possible structure of $\mathcal{L}_1^{(g)}$. There are 12 possible terms:

$$\mathcal{L}_{1,bos}^{(g)} = \sum_{i=1}^{12} x_i\,L_i \tag{4.7}$$

where x_i are numbers to be determined by comparison with e.m.'s (2.10)-(2.15), but L_i are presented in the Table 2.

Table 2

i	L_i	i	L_i	i	L_i
1	R^2	5	$(M^2)\,R$	9	$M^{abc;d}(M^2)_{abcd}$
2	R_{ab}^2	6	$(M^2)_{ab}\,R^{ab}$	10	$(M^2)^2$
3	R_{abcd}^2	7	$(M^2)_{abcd}R^{abcd}$	11	$(M^2)_{ab}^2$
4	$\varepsilon^{0\ldots 9}R_{01bc}R_{23}{}^{bc}M_{4\ldots 9}$	8	$M^{abc}\nabla_d\nabla^d M^{abc}$	12	$(M^2)_{abcd}\,(M^2)^{acbd}$

where $(M^2) = M_{abc}M^{abc}, \quad (M^2)_{ab} = M_a{}^{cd}M_{bcd}$ and $(M^2)_{abcd} = M_{ab}{}^f M_{cdf}$.

Now we come to the determination of x_i in (4.7). All the terms, containing the M_{abc}-field (4.4) in the lagrangian (4.7) can be easily reconstructed with the help of the simple procedure [20]. As was discussed before, equation (2.14) (which is the N-field e.m.) is equivalent to the (4,0)-component of the H-field BI. Omitting spinorial terms, introducing the standard covariant derivative ∇_a and the curvature-tensor R_{abcd} one can rewrite (4,0)-component of eq. (3.1) in the form:

$$(H_{abc} + 3\,k_g\,(2\,T_{ija}R_{bc}{}^{ij} - T_a{}^{ij}T_{bij;c} + \frac{1}{3}\,(T^3)_{abc}))_{;d} = 3\,k_g\,R_{ab}{}^{ij}R_{cdij}, \quad [abcd] \tag{4.8}$$

Then with the help of eq.'s (3.2c), (3.3) and (4.5) one can write everything in terms of the M_{abc} - field. After that the terms in the lagrangian, containing the M_{abc} - field, are reproduced immediately from the l.h.s. of (4.8) which has the desired form of a complete derivative. The term $\sim MR^2$ is reproduced from the r.h.s. of (4.8). One can not distinguish between R and $(1/12)M^2$ on the mass shell. For this reason we are able to determine by this way only $x_j,\ j = 4.6, 7, 8, 9, 11, 12$ and find one relation between $x_j,\ j = 5, 10$.

The terms in (4.7), containg the M-field, were also derived by another procedure, which makes it possible to obtain also terms $\sim R^2$. Calculating the variation of $\mathcal{L}^{(g)}$ over the graviton field one must get the e.m. (2.13). Then, contracting indices, one must get the dilaton e.m. Comparing with (2.12) the result of such a variation, (spinorial derivatives were explicitely calculated in (2.12)), we find the values of $x_i,\ i \neq 1, 5, 10$ in (4.7) and find the relation between $x_i,\ i = 1, 5, 10$. There is the complete correspondence between this calculation and the previous one, based on eq. (4.8).

The values of x_j obtained by the described procedure are presented in Table 3.

Table 3

x_1	*undetermined*	x_5	$-2/27 - 2\,x_1/12$	x_9	$1/2$
x_2	2	x_6	$-1/2$	x_{10}	$1/162 + x_1/144$
x_3	-1	x_7	0	x_{11}	0
x_4	$(2 \cdot 6!)^{-1}$	x_8	$-1/6$	x_{12}	$-1/24$

Terms containing x_1 in (4.7) appear in the combination which is the square of the constraint (2.16). That is the reason why x_1 is undetermined by comparison with e.m.'s.

To simplify the result one can make the following redefinition of the dilaton field in (4.2):

$$\phi = \tilde{\phi} - k_g\,x_1\,(\mathcal{R} - \frac{1}{3}T^2) + k_g\,\frac{2}{27}(T^2) \tag{4.9}$$

The second term in the r.h.s of (4.9) leads to the cancellation of terms $\sim x_1$ in (4.1). Such a redefinition does not change anything at the mass-shell due to the constraint (2.16) (note, that neglecting fermions: $\mathcal{R} - (1/3)T^2 = R - (1/12)M^2$). So one can put $x_1 = 0$ from the very beginning in the Table 3.

The third term in (4.9) leads to the cancellation of terms $\sim RM^2$ and $\sim M^4$ in (4.1), so one can put $x_5 = x_{10} = 0$ in the Table 3, using $\tilde{\phi}$ instead of ϕ. The third term in the r.h.s. of (4.9) leads to the obvious change in the basic equation (2.4) and to the controlable changes in other relations, discussed before.

Finally, considering $\tilde{\phi}$ as an independent variable, one can write the bosonic part of the lagrangian (4.1) in the form:

$$\mathcal{L}^{(g)}_{bos} = \tilde{\phi}\,(R - \frac{1}{12}M^2) +$$
$$+k_g\,[2\,R^2_{ab} - R^2_{abcd} + \frac{1}{2\cdot 6!}\varepsilon^{abcdf_1\ldots f_6}\,R^2_{abcd}M_{f_1\ldots f_6} - \frac{1}{2}\,R^{ab}(M^2)_{ab} -$$
$$-\frac{1}{6}\,M^{abc}\nabla_f\nabla^f M_{abc} + \frac{1}{2}\,M^{abc;d}(M^2)_{abcd} + \frac{1}{162}\,(M^2)^2 - \frac{1}{24}\,(M^2)_{abcd}(M^2)_{acbd}] \qquad (4.10)$$

Terms $\sim k_g R^2$ and $\sim k_g M^2$ in (4.10) are not free from ghosts. It is a consequence of a supersymmetry because the part of $\mathcal{L}^{(g)}$ quadratic in the gravitino field contains ghost-full terms of the type $k_g \psi_a \Gamma^{abc}(\nabla_d)^2\psi_{c;b}$. (We have not discussed them in the present paper for short). It is the ignoring of these terms in [12], [27] has led to prediction of the ghost-free term $(R^2_{abcd} - 4R^2_{ab} + R^2)$ in the lagrangian.

The lagrangian (4.10) corresponds to the SG7-theory, which must be supersymmetric by construction after including of fermions. It contains anomalies, but anomaly compensating counter-terms appear only at the 8-th order in derivatives. All such terms in the supersymmetric lagrangian can be reconstructed iteratively in β if one adds the term βX_8 to the r.h.s. of the BI (3.10) [28],[5], where $X_8 = tr\mathcal{R}^4 + (1/4)(tr\mathcal{R}^2)^2$. In the limiting case $\beta = 0$ the SG7 is the dual analog of SG3-theory, which is also anomaly full, inspite of the Green-Schwarz term in the r.h.s. of the BI (3.1). Anomaly compensating counter-terms in the SG3-theory appear at the same (8-th) order in derivatives and has never been supersymmetrized.

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