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Delta Excitations in Neutrino-Nucleus Scattering

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Abstract

We derive the contribution of Δ-h excitations to quasielastic charged-current neutrino-nucleus scattering in the framework of relativistic mean-field theory. We discuss the effect of Δ production on the determination of the axial mass M_A in neutrino scattering experiments.

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I. INTRODUCTION

The interpretation of neutrino-nucleus scattering data relies on accurate knowledge of the νA cross section. Above the quasielastic peak higher resonances increasingly contribute to the cross section. Neutrino experiments generally measure integrated yields which include contributions from quasielastic nucleon knock-out as well as from higher resonance production. We discuss, in the following, the effect of Δ production in neutrino scattering. It has already been shown in the case of electron scattering that a qualitative description of the data can only be achieved when Δ-h excitations are included [1,2]. Charged-current neutrino-nucleus scattering has been used as tool to investigate axial-vector form factor of the nucleon [3]. For low momentum transfers the Q-dependence of the form factor can be parameterized by a dipole mass M_A, Eq. (A19). We will consider the effect of Δ excitations on the experimental extraction of M_A. Kim *et al.* [4] have examined nuclear structure corrections to the extraction of M_A but do not consider the Δ. Singh and Oset [5] have included Δ-h but calculated in nonrelativistic formalism.

The article is organized as follows. First, in Section II we introduce the relativistic mean-field formalism of the nucleus including the Δ resonance, deriving the appropriate set of nuclear response functions. In Section III we discuss the results for cross sections and yields as modified by the inclusion of the Δ channel.

II. FORMALISM

In this section, we derive the inclusive cross section for quasielastic charged-current neutrino scattering including Δ-h excitations in the nucleus. We consider a neutrino with four-momentum k=$(E_\nu, \mathbf{k})$ which scatters from a nucleus via $W^\pm$ boson exchange producing a charged lepton with four-momentum k'=$(E_{\mathbf{k}'},\mathbf{k}')$. Using an impulse approximation and a Fermi gas description of the nucleus the formula for the double differential scattering cross section for mass number A is given by (we assume a symmetric $N = Z$ nucleus):

$$\frac{d^3\sigma}{d^2\Omega_{\mathbf{k}'} dE_{\mathbf{k}'}} = -\frac{AG_F^2 \cos^2\theta_c\, |\mathbf{k}'|}{32\pi^3 \rho E_\nu} \,\mathrm{Im}\,(L_{\mu\nu}\Pi_A^{\mu\nu}) \,, \tag{1}$$

where $\rho = 2k_F^3/3\pi^2$ is the baryon density with Fermi momentum k_F. G_F denotes the Fermi constant and θ_c is the Cabibbo mixing angle. The leptonic tensor $L_{\mu\nu}$ is defined as

$$L_{\mu\nu} = 8\left(k_\mu k'_\nu + k_\nu k'_\mu - k\cdot k' g_{\mu\nu} \mp i\epsilon_{\alpha\beta\mu\nu}k^\alpha k'^\beta\right) \,, \tag{2}$$

with the minus (plus) sign denoting neutrino (anti-neutrino) scattering. $\Pi_A^{\mu\nu}$ is the polarization tensor of the target nucleus for the charged weak current. Here we consider p-h, $\Pi_{ph}^{\mu\nu}$, and Δ-h, $\Pi_{\Delta h}^{\mu\nu}$, contributions to the polarization:

$$\Pi_A^{\mu\nu} = \Pi_{ph}^{\mu\nu} + \Pi_{\Delta h}^{\mu\nu} \,. \tag{3}$$

The expressions for the p-h polarizations have been derived in previous publications [4,6]. The weak interaction contains vector current (v) and axial-vector current (a) contributions. Therefore we split $\Pi_{\Delta h}^{\mu\nu}$ into:

$$\Pi_{\Delta h}^{\mu\nu} = (\Pi_{\Delta h}^{vv})^{\mu\nu} + (\Pi_{\Delta h}^{aa})^{\mu\nu} + (\Pi_{\Delta h}^{va})^{\mu\nu} + (\Pi_{\Delta h}^{av})^{\mu\nu} \,. \tag{4}$$

$(\Pi_{\Delta h}^{va})^{\mu\nu}$ and $(\Pi_{\Delta h}^{av})^{\mu\nu}$ are interference terms of the vector and axial-vector currents. In a Hartree approximation the polarization tensor can be written in the form

$$\begin{aligned}(\Pi_{\Delta h}^{ij})_{\mu\nu} = &\; -i\sum_{n,p}\int \frac{d^4p}{(2\pi)^4}\,\mathrm{Tr}[\Gamma^i_{\beta\mu}(-q,-p)\; S^{\beta\alpha}(p)\; \Gamma^j_{\alpha\nu}(q,p)\; G(p-q)\;] \\ &+ (q_\mu \to -q_\mu) \qquad\qquad (i,j) = (a,v) \,. \end{aligned} \tag{5}$$

We rewrite the interference term as

$$(\Pi_{\Delta h}^{va})^{\mu\nu} = (\Pi_{\Delta h}^{av})^{\mu\nu} = i\epsilon^{\mu\nu\alpha 0} q_\alpha \Pi_{\Delta h}^{va} \,. \tag{6}$$

$S^{\mu\nu}(p)$ is the Rarita-Schwinger form of the free spin 3/2 propagator with momentum p [1]:

$$S^{\mu\nu}(p) = -\frac{\not p + M_\Delta}{p^2 - M_\Delta^2 + i\epsilon}\left[g^{\mu\nu} - \frac{1}{3}\gamma^\mu\gamma^\nu - \frac{2}{3}\frac{p^\mu p^\nu}{M_\Delta^2} + \frac{p^\mu\gamma^\nu - p^\nu\gamma^\mu}{3M_\Delta}\right] , \tag{7}$$

Note that this expression is not unique and other forms of the Δ propagator have been considered (see Ref. [7]). The differences enter the off-shell behavior of the propagator which do not affect the following calculations. The vector part of the nucleon-delta vertex has been studied in the case of the $\gamma N\Delta$ transition [1],

$$\Gamma^v_{\mu\nu}(q,p)=\sqrt{2}F_\Delta T^\pm\left[(-q_\mu\gamma_\nu+g_{\mu\nu}\not{q})M_\Delta\gamma_5+(q_\mu p_\nu-q\cdot p g_{\mu\nu})\gamma_5\right] . \tag{8}$$

The isospin raising (lowering) operator originating from W^+ (W^-) exchange is defined through

$$T^\pm=\frac{1}{\sqrt{2}}(T_1\pm iT_2) \tag{9}$$

where 2×4 isospin matrices T^i satisfy [8]

$$T^i(T^\dagger)^j=\delta^{ij}-\frac{1}{3}\tau^i\tau^j . \tag{10}$$

The form factor F_Δ is defined in the appendix [Eq. (A18)].

The vertex for the axial $N\Delta$ transition is given by [8,9]

$$\Gamma^a_{\mu\nu}=-r_{N\Delta}G_A\frac{T^\pm}{\sqrt{2}}g_{\mu\nu} , \tag{11}$$

with the axial form factor G_A, Eq. (A19). The parameter $r_{N\Delta}$ indicates the strength of axial $N\Delta$ transition and will be discussed later. In the noninteracting limit the nucleon propagator $G(p)$ reduces to the free fermion propagator $G^o(p)$ for a relativistic Fermi gas with Fermi momentum k_F. We consider only the density-dependent part G^o_F as vacuum contributions do not enter at the Hartree level. In the rest frame of the nucleus one obtains

$$G^o_F(p)=(\not{p}+M)\frac{i\pi}{E_{\mathbf{p}}}\delta(p_0-E_{\mathbf{p}})\theta(k_F-|\mathbf{p}|) . \tag{12}$$

Using an impulse approximation, the imaginary parts of $(\Pi^{ij}_{\Delta h})$ enter the cross section Eq. (1). As long as the Δ is assumed to be stable, the imaginary parts can be calculated analytically. The resulting expressions are given in the appendix.

In a relativistic mean field description of the nucleus, nucleons and Δs interact with the background of scalar (σ) and vector (ω) meson mean fields. The interactions are assumed to be analogous to those of nucleons with possible new couplings, g^s_Δ and g^v_Δ. The values of the couplings can be constrained somewhat by fitting the quasielastic peak of Δ production in electron scattering [1,2].

In a relativistic mean-field approximation (MFA), the noninteracting nucleon propagator $G_F^o(p)$ is replaced by

$$G_F^*(p) = (\not{p}^* + M^*) \frac{i\pi}{E_{\mathbf{p}}^*} \delta(p_0 - E_{\mathbf{p}}) \theta(k_F - |\mathbf{p}|) \tag{13}$$

where

$$M^* = M - S_N \ , \quad E_{\mathbf{p}}^* = \sqrt{\mathbf{p^2} + M^{*2}} \ , \quad p^{*\mu} = p^\mu - V_N g^{\mu 0} \ . \tag{14}$$

The scalar (S_N) and the vector (V_N) self-energies can be obtained for a given k_F [10]. Analogously, the mass and the momentum in the Δ propagator $S^{\mu\nu}$ are replaced by [1]

$$M_\Delta \rightarrow M_\Delta^* = M_\Delta - S_\Delta \ , \quad t^\mu \rightarrow t^{*\mu} = t^\mu - V_\Delta g^{\mu 0} \ . \tag{15}$$

The calculation proceeds in the same way as in the case of the free Δ. The expressions for the imaginary parts are given in the appendix. For simplicity we assume the Δ self-energies S_Δ, V_Δ to be the same as nucleon self-energies S_N, V_N.

In free space, a Δ decays into πN with a width $\Gamma = 115$ MeV. In the medium the situation is more complicated. The πN decay channel is partially suppressed because of Pauli blocking, i.e., the phase space available to the nucleon produced in the Δ decay is reduced by the Fermi sea. However, the Δ in the medium has additional channels of decay and obtains a "spreading" width with the main decay mechanism from $\Delta + N \rightarrow N + N$. These two competing effects cancel each other partially. In our calculation, lacking better theoretical and experimental knowledge of the Δ width in nuclear matter, we assume a value identical to the free width $\Gamma = 115$ MeV [1,11].

As we are mainly interested in integrated cross sections, the results do not depend strongly on the way the width of the Δ is treated. We adopt a simple method to include the decay width by averaging the nuclear response over the Δ mass with a Breit-Wigner distribution [1,2]. The averaged cross section follows as

$$\left\langle \frac{d^3\sigma}{d^2\Omega_{\mathbf{k}'} dE_{\mathbf{k}'}} \right\rangle = \int_{M^2}^{\infty} d\mu^2 \frac{d^3\sigma}{d^2\Omega_{\mathbf{k}'} dE_{\mathbf{k}'}}(\mu) f(\mu^2) \ / \ \int_{M^2}^{\infty} d\mu^2 f(\mu^2) \ , \tag{16}$$

$$f(\mu^2) = \frac{M_\Delta \Gamma}{(M_\Delta^2 - \mu^2)^2 + M_\Delta^2 \Gamma^2} \tag{17}$$

integrating from threshold to infinity.

III. RESULTS

In this section we present result for the Δ-h calculations of charged-current neutrino interactions. We discuss the case of muon neutrinos but the general features of the results hold for electron neutrinos as well. In addition to a relativistic Fermi gas calculation we consider the effects of the mean field and include the decay width of the Δ. The target nucleus is assumed to be ^{16}O with a Fermi momentum $k_F = 225\,\mathrm{MeV}$. For the strength of the axial $N\Delta$ transition a simple argument using the Δ decay width suggests a value $r_{N\Delta} \sim 2.2$ [8] whereas constituent quark models give a somewhat smaller value of $r_{N\Delta} = \frac{6\sqrt{2}}{\sqrt{5}} \sim 1.7$. We choose an intermediate value $r_{N\Delta} = 2$ for our numerical calculations.

Figure 1 shows the double differential cross section for measuring an outgoing muon produced by an incoming neutrino with energy $E_\nu = 1$ GeV and three-momentum transfer $|\mathbf{q}| = 0.5$ GeV. First, note that the curve neglecting the decay width of the Δ has a peak around $q_0 = 0.37$ GeV which agrees with the expected elastic Δ peak at

$$(q_0)_{el} = \sqrt{\mathbf{q}^2 + M_\Delta^2} - M \ , \tag{18}$$

assuming the initial nucleon at rest. The Δ cross section is similar to the p-h cross section. Therefore, measurements of integrated quantities cannot neglect Δ production. Including a finite delta width reduces the peak height by about 30 to 35 percent but does not significantly reduce the total integrated strength.

Nuclear matter effects are included using a mean-field approximation (MFA). Here we use the same scalar and vector couplings for nucleon and Δ (known as "universal couplings"). For $k_F = 225$ MeV, a self-consistent nuclear-matter calculation yields the effective masses

$M^*_\Delta = 931$ MeV, $M^* = 638$ MeV, and the vector self-energy $V_N = V_\Delta = 239$ MeV. Mean-field results are also shown in Fig. 1. p-h and Δ-h contributions are reduced by about 30 percent. Both peaks are shifted to higher energies due to the smaller effective masses.

As we have seen that Δ-h excitations can contribute substantially to the charged-current cross sections, it is interesting to study the effect of Δ production in neutrino-nucleus scattering experiments. Experiments using muon neutrino beams have measured

$$\left(d\sigma/dQ^2\right)_{exp} \equiv \int \frac{d\sigma}{dQ^2}(E_\nu)f(E_\nu)dE_\nu \quad . \tag{19}$$

$f(E_\nu)$ denotes the spectrum of the neutrino beam. $d\sigma/dQ^2$ is given by

$$\frac{d\sigma}{dQ^2} = \int_0^{Q_c} \frac{\pi}{E_\nu|\mathbf{k}'|}\frac{d^3\sigma}{dE_{\mathbf{k}'}d^2\Omega_{\mathbf{k}'}}dq_0 \tag{20}$$

where $Q^2 = -q^2 = \mathbf{q}^2 - q_0^2$ and the cut off for the energy transfer Q_c reads

$$Q_c = E_\nu + \frac{q^2 - m_\mu^2}{4E_\nu} + \frac{E_\nu m_\mu^2}{q^2 - m_\mu^2} \quad . \tag{21}$$

Using the neutrino spectrum from the charged-current experiment at BNL [3] the resulting cross sections are shown in Fig. 2 where we used an axial mass $M_A = 1.09\,\mathrm{GeV}$. At larger momentum transfers the contribution from Δ-h excitations is as large as the nucleon knock-out.

We now discuss the influence of deltas on the extraction of the axial mass from quasielastic data. Different kinds of experiments are possible. If only a charged lepton is detected, all Δ events will be included on an equal footing with p-h excitations. Alternatively, an experiment could detect pions and thereby separate Δ events producing real pions from p-h excitations. However, a significant fraction of Δ excitations lead to two-particle two-hole excitations without a real pion. A Δ in a nucleus can decay via $\Delta + N \to N + N$. This is related to either pion absorption or weak meson exchange currents (involving an intermediate Δ). It may be difficult to separate two-particle two-hole from one-particle and one-hole final states. The axial mass is often fit to reproduce the Q^2 dependence of observed events. This cancels some errors from unknown flux normalizations. Therefore it is interesting to consider the Q^2 dependence of (a) p-h excitations only (b) p-h plus that fraction (see below) of Δ-h excitations leads to 2p-2h (c) p-h plus all Δ-h excitations. An incorrect value of M_A could be extracted if one assumes only p-h excitations while the data is "contaminated" by significant Δ-h excitations.

We leave it to the experimental groups to analyze their data in detail. For example, Ref. [3] analyzed their data assuming only p-h excitations and extracted a value of $M_A = 1.09$ GeV with a very small statistical error of ± 30 MeV. To estimate the uncertainty in this extracted value of M_A from Δ-h excitations we try and fit the Q^2 dependence from 0.3 – 1 GeV^2 of our full calculation (p-h plus some fraction of Δ-h) with a p-h only model. Thus the full calculation assumes some value of M_A (which is essentially arbitrary) and the p-h calculation attempts to reproduce this result by using a possibly different value of M_A. The important quantity is the difference between the assumed and extracted M_A. This may represent some of the systematic error (from Δ-h excitations) in a p-h only analysis of data.

Figure 3 (a) shows a likelihood function for reproducing our theoretical results assuming a p-h only free Fermi gas with different values of M_A. (Note, all of the theoretical calculations used $M_A = 1.09$ GeV.) For theoretical calculations assuming only a p-h response (solid line), the input $M_A = 1.09$ GeV is of course reproduced in the fit. However, for theoretical calculations including either all of the Δ-h (dashed line) or half of the Δ-h events (dots), M_A is underestimated by 70 to 90 MeV.

This factor of half represents a very crude estimate of the $\Delta + N \rightarrow N + N$ to $\Delta \rightarrow N\pi$ and $\Delta + N \rightarrow N + N$ branching ratio. Theoretical results [12,13] are consistent with this factor. However, there could be both important Q^2 and model dependence in this branching ratio. Further theoretical work on the branching ratio would be very useful. Alternatively one could try and measure it in coincidence electro-excitation experiments.

Finally in figure 3 (b), we fit theoretical calculations including scalar and vector mean fields (assumed independent of momentum) as described in Ref. [14] with a free p-h calculation without mean fields. Again large shifts in the extracted M_A are found. Δ events tend to increase the effective cross section at high Q^2 which might be fit with a smaller M_A.

Alternatively, mean field effects tend to reduce the cross section at high Q^2 which can be fit using a larger M_A. Thus there is some cancellation between the two effects. However this cancellation is unlikely to be perfect and the theoretical uncertainties are large. We conclude that the theoretical uncertainty on an extracted M_A could be of order 0.1 GeV and thus large compared to the claimed experimental error of ± 0.03(stat)± 0.02(syst) GeV [3].

IV. SUMMARY AND OUTLOOK

We have calculated charged-current neutrino cross sections including Δ-h excitations of the target nucleus. The calculation was done for free deltas as well as including the effects of relativistic scalar and vector mean fields in the nucleus. Δ-h excitations are found to give significant corrections to quasielastic nucleon knock-out processes in experiments measuring neutrinos in the GeV range. In extracting the axial form factor of the nucleon from neutrino scattering data the Δ-h channel enters with similar strength as p-h contributions. This may introduce significant error in the extracted nucleon axial form factor.

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APPENDIX

The imaginary parts of the Δ-h polarizations are evaluated analytically and listed below. Note that the second term of the polarization, Eq. (5), with $(q_\mu \to -q_\mu)$ vanishes for on-shell Δ:

$$\mathrm{Im}(\Pi^{vv}_{\Delta h})^{\mu}{}_{\mu} = \frac{\alpha}{9\pi|\mathbf{q}|}F_\Delta^2[-q^4 + 2q^2(M^2+M_\Delta^2) - (M^2-M_\Delta^2)^2]E_1^\Delta \ , \tag{A1}$$

$$\begin{aligned}\mathrm{Im}(\Pi^{vv}_{\Delta h})^{00} = &-\frac{\alpha}{18\pi|\mathbf{q}|}F_\Delta^2\Big[4q^2E_3^\Delta + 4q_0\beta E_2^\Delta \\ &+[q^4 - 2q^2(M^2+M_\Delta^2) + (M^2-M_\Delta^2)^2 + 4M^2q_0^2]E_1^\Delta\Big] \ ,\end{aligned} \tag{A2}$$

$$\mathrm{Im}(\Pi^{aa}_{\Delta h})^{\mu}{}_{\mu} = \frac{2\alpha}{3\pi|\mathbf{q}|}G_A^2E_1^\Delta \ , \tag{A3}$$

$$\mathrm{Im}(\Pi^{aa}_{\Delta h})^{00} = -\frac{2\alpha}{9\pi|\mathbf{q}|}G_A^2\Big[\frac{E_3^\Delta + 2q_0E_2^\Delta + q_0^2E_1^\Delta}{M_\Delta^2} - E_1^\Delta\Big] \ , \tag{A4}$$

$$\mathrm{Im}(\Pi^{aa}_{\Delta h})^{01} = -\frac{\alpha}{9\pi\mathbf{q}^2M_\Delta^2}G_A^2[2q_0E_3^\Delta + (3q_0^2 + \mathbf{q}^2 + M^2 - M_\Delta^2)E_2^\Delta + q_0(\beta+2\mathbf{q}^2)E_1^\Delta] \ , \tag{A5}$$

$$\mathrm{Im}(\Pi^{aa}_{\Delta h})^{11} = -\frac{2\alpha}{9\pi\mathbf{q}^3M_\Delta^2}G_A^2[q_0^2E_3^\Delta + q_0(\beta+2\mathbf{q}^2)E_2^\Delta + (\frac{\beta^2}{4} + q_0^2 + M^2)E_1^\Delta] \ , \tag{A6}$$

$$\mathrm{Im}(\Pi^{va}_{\Delta h}) = -\frac{\alpha}{9\pi\mathbf{q}^2}G_AF_\Delta[2q^2E_2^\Delta + q_0\beta E_1^\Delta] \ , \tag{A7}$$

where $\alpha = [(M+M_\Delta)^2 - q^2]$ and $\beta = q^2 + M^2 - M_\Delta^2$. Also

$$E_n^\Delta = \frac{E_F^n - E_-^{\Delta n}}{n} \qquad (n=1,2,3) \ , \tag{A8}$$

$$E_-^\Delta = \mathrm{Min}(E_F, E_{\Delta max}) \ , \tag{A9}$$

$$E_{\Delta max} = -\frac{\beta q_0 + |\mathbf{q}|\sqrt{\beta^2 - 4M^2q^2}}{2q^2} \ . \tag{A10}$$

$\mathrm{Im}(\Pi^{aa}_{\Delta h})^{22}$ is obtained from the relation,

$$\Pi^{22} = \frac{\Pi^{00} - \Pi^{11} - \Pi^{\mu}{}_{\mu}}{2} \ . \tag{A11}$$

In the mean field approximation, the Δ and nucleon masses in the propagators are shifted by strong scalar fields. Since we take the same interaction as in free space, the polarizations involve complicated traces. After a little algebra, the polarizations are written as

$$(\Pi^{*vv}_{\Delta h})^{\mu\nu} = -\frac{8}{3}F_\Delta^2\int_{M^*}^{E_F}dE_\mathbf{p}\int_{-1}^{1}d\chi\,\frac{|\mathbf{p}|}{8\pi^2}\frac{T^{\mu\nu}_{vv}}{(p+q)^2 - M_\Delta^2 + i\epsilon} \ , \tag{A12}$$

$$(\Pi^{*aa}_{\Delta h})^{\mu\nu} = -\frac{8}{3}G_A^2\int_{M^*}^{E_F} dE_{\mathbf{p}}\int_{-1}^{1} d\chi\,\frac{|\mathbf{p}|}{8\pi^2}\frac{T^{\mu\nu}_{aa}}{(p+q)^2-M_\Delta^2+i\epsilon}\ , \tag{A13}$$

$$(\Pi^{*va}_{\Delta h})^{\mu\nu} = -\frac{8}{3}G_A F_\Delta\int_{M^*}^{E_F} dE_{\mathbf{p}}\int_{-1}^{1} d\chi\,\frac{|\mathbf{p}|}{8\pi^2}\frac{i\epsilon^{\mu\nu\alpha 0}q_\alpha T_{va}}{(p+q)^2-M_\Delta^2+i\epsilon}\ , \tag{A14}$$

where $T^{\mu\nu}_{vv}$, $T^{\mu\nu}_{aa}$ and T_{va} result from evaluating the traces. These can be determined straightforwardly as

$$\begin{aligned}
T^{\mu\nu}_{vv} = \frac{1}{3M_\Delta^2}\Big\{ & (p^*\cdot t^* - MM^*)\ \Big[-t\cdot qM_\Delta M^*_\Delta(q^\mu t^{*\nu}+q^\nu t^{*\mu}) \\
& +(2M^*_\Delta t\cdot q - M_\Delta t^*\cdot q)(t^\mu q^\nu + p^\nu q^\mu)M^*_\Delta \\
& +(t^\mu t^{*\nu}+t^\nu t^{*\mu})(M^*_\Delta M_\Delta q^2 - 2t\cdot qt^*\cdot q) \\
& -2(t\cdot q)^2(g^{\mu\nu}M^{*2}_\Delta - t^{*\mu}t^{*\nu}) + 2t^\mu t^\nu[(t^*\cdot q)^2 - q^2M^{*2}_\Delta]\Big] \\
& + (q^\mu t^{*\nu}+q^\nu t^{*\mu})\ M_\Delta\ [M_\Delta(M^{*2}_\Delta p^*\cdot q + 2t^*\cdot p^* t^*\cdot q) + t\cdot qM^{*2}_\Delta M\,] \\
& + M^{*2}_\Delta M_\Delta[(t^\mu q^\nu + t^\nu q^\mu)(t^*\cdot qM - p^*\cdot qM^*_\Delta) + (q^\mu p^{*\nu} \\
& + q^\nu p^{*\mu})(t^*\cdot qM_\Delta - t\cdot qM^*_\Delta)] \\
& + M^*_\Delta M_\Delta[(t^\mu t^{*\nu} + t^\nu t^{*\mu})(p^*\cdot qt^*\cdot q - MM^*_\Delta q^2) \\
& +(p^{*\mu}t^{*\nu}+p^{*\nu}t^{*\mu})(t\cdot qt^*\cdot q - q^2M_\Delta M^*_\Delta)] \\
& + 2(g^{\mu\nu}q^2 - q^\mu q^\nu)M^2_\Delta M^{*2}_\Delta(t^*\cdot p^* + M^*_\Delta M) \\
& + 2t\cdot qp\cdot qM_\Delta M^*_\Delta\ (g^{\mu\nu}M^{*2}_\Delta - p^{*\mu}p^{*\nu}) \\
& - 2t^*\cdot p^* M^2_\Delta(g^{\mu\nu}(t^*\cdot q)^2 + q^2t^{*\mu}t^{*\nu}) \\
& + 2M_\Delta M^*_\Delta t^*\cdot qg^{\mu\nu}\ (t^*\cdot p^* t\cdot q - M_\Delta M^*_\Delta p^*\cdot q - 2t\cdot qMM^*_\Delta) \\
& + M_\Delta M^*_\Delta(q^2M^{*2}_\Delta - (t^*\cdot q)^2)(t^\mu p^{*\nu} + t^\nu p^{*\nu})\Big\}\ ,
\end{aligned} \tag{A15}$$

$$\begin{aligned}
T^{\mu\nu}_{aa} = \frac{1}{6M^{*2}_\Delta}&[3(t^{*2}-M^{*2}_\Delta)(p^{*\mu}t^{*\nu}+p^{*\nu}t^{*\mu}) - 16t^{*\mu}t^{*\nu}(p^*\cdot t^* + M^*M^*_\Delta) \\
&+g^{\mu\nu}t^{*2}(p^*\cdot t^* - 2M^*M^*_\Delta) + 15g^{\mu\nu}p^*\cdot t^*M^{*2}_\Delta + 18g^{\mu\nu}M^*M^{*3}_\Delta]\ ,
\end{aligned} \tag{A16}$$

$$\begin{aligned}
T_{va} = -\frac{2}{3M^*_\Delta}(E^*_{\mathbf{p}} - \frac{|\mathbf{p}|q_0}{|\mathbf{q}|}\chi)&[2p\cdot qM^*_\Delta - M^{*2}M_\Delta - 4M^*M^*_\Delta M_\Delta + q^2M_\Delta \\
&-3M^{*2}_\Delta M_\Delta]\ .
\end{aligned} \tag{A17}$$

The vector form factor of the $N\Delta$ vertex is given by

$$F_\Delta = \frac{-(M_\Delta + M)}{M((M_\Delta + M)^2 - q^2)} \, \frac{9}{2} \left(1 - \frac{q^2}{0.71\mathrm{GeV}^2}\right)^{-2} \left(1 - \frac{q^2}{3.5\mathrm{GeV}^2}\right)^{-1/2} \tag{A18}$$

which has dipole form including some phenomenological corrections [15]. For the axial vertex we use the nucleon axial form factor

$$G_A = \frac{1.26}{(1 - q^2/M_A^2)^2} \ . \tag{A19}$$

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FIGURES

FIG. 1. Double differential cross section for (ν, μ) scattering from ^{16}O showing p-h and Δ-h contributions separately. Results are shown neglecting (solid line) and including the width of the delta (dashed line). The thin lines denote cross sections in the mean-field approximation.

FIG. 2. $d\sigma/dQ^2$ averaged over BNL antineutrino spectrum are shown for Fermi gas (solid line) and mean field approximation (dashed line). p-h and Δ-h results are shown separately.

FIG. 3. Likelihood function (normalized) of fitting experimental antineutrino-scattering results varying the extracted axial mass, assuming a free particle-hole (p-h) only Fermi gas response. Parts (a) and (b) show results for Fermi-gas and mean-field approximations, respectively. Curves are shown for p-h events (solid line), the sum of p-h and Δ-h events (dashed line), and p-h plus half of the Δ-h events (dots). Note, all theoretical calculations assumed $M_A = 1.09$ GeV.