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Many Higgs Doublet Supersymmetric Model, Flavor Changing Interactions and Spontanoeus CP Violation*

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Abstract

I study many higgs doublet supersymmetric model with spontaneous CP violation. The damaging flavor changing interactions and large CP violation are brought under control simultaneously by an approximate PQ symmetry. I relate the smallness of the CP violating parameter in the kaon system to the small $\frac{m_b}{m_t}$ ratio.

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Today the standard model is satisfactorily describing vast amount of data, but many free input parameters that the theory has makes it hardly anyone believe to be the ultimate theory. Especially interesting is the sector of CP violation and fermion masses, which carries the most of the unknown parameters. CP violation can be acommodated in the standard model through a complex phase in the Kobayashi-Maskawa (KM) matrix, but a theoretically more promising idea is spontaneous CP symmetry violation (SCPV)[1)]. SCPV has been used also in the supersymmetric extensions of the standard model (SSM). It was studied in SSM with the addition of an singlet (see for example Ref. 2).

In this talk I consider the SCPV in the extension of the SSM with more higgs doublets. The four higgs doublet SSM (at least two doublets must be added to the MSSM in order to make the theory anomaly free) occurs naturally in left-right supersymmetric models where at least two bidoublets are required in order to get realistic fermion masses and mixings. Also it was shown[3)] to be the simplest extension that has naturally large tanβ (the ratio of the vevs of the doublets coupled to up and down quark sectors) without fine tuning the theory.

Unless some terms in the lagrangian are forbidden by some additional symmetries, the many higgs doublet SSM will in principle have flavor changing interactions (FC). For vevs and masses of the new higgs scalars of the order of weak scale or so these are below or of the order of experimental limits if the Yukawa couplings are real and are small by some approximate flavor symmetry mechanism[4)]. However, allowing for complex Yukawa couplings in the Lagrangian, the amount of CP violation is by many orders of magnitude larger than it is observed[5)]. Also, allowing for large ratio of vevs the FC interactions may be too big. The purpose of this talk is to show that by using an approximate Peccei-Quinn type symmetry (PQ) we can bring both problems (too large FC and too large CP) under control.

The most general superpotential with four higgs doublets $H_i, i = 1, 2, 3, 4$ (for previous work on four higgs doublets in supersymmetry see Refs. 6 and 7) is given by

$$\begin{aligned} W &= Q(h_1 H_1 + h_3 H_3)D^c + Q(h_2 H_2 + h_4 H_4)U^c + L(h_1^e H_1 + h_3^e H_3)E^c \\ &+ \mu_{12} H_1 H_2 + \mu_{32} H_3 H_2 + \mu_{14} H_1 H_4 + \mu_{34} H_3 H_4, \end{aligned} \tag{1}$$

where H_1, H_3 have hypercharge -1, and H_2, H_4 have hypercharge $+1$. h_i are the Yukawa matrices.

The most general scalar potential is given by

$$
\begin{aligned}
V &= m_1^2 H_1^\dagger H_1 + m_2^2 H_2^\dagger H_2 + m_3^2 H_3^\dagger H_3 + m_4^2 H_4^\dagger H_4 - \\
&- (m_{12}^2 H_1 H_2 + h.c.) - (m_{32}^2 H_3 H_2 + h.c.) - \\
&- (m_{14}^2 H_1 H_4 + h.c.) - (m_{34}^2 H_3 H_4 + h.c.) - \\
&- (m_{13}^2 H_1^\dagger H_3 + h.c.) - (m_{24}^2 H_2^\dagger H_4 + h.c.) + V_D^{4HD}, \qquad (2)
\end{aligned}
$$

where V_D^{4HD} is the D-term part of the potential. Unless the parameters are suppressed by some symmetry, we will assume that all the dimensionfull paramaters are of the order of the weak scale, while all dimensionless parameters are of order one. We do not assume any higher energy scales or accidental cancellations. We will also assume that the theory is CP invariant, i.e. all the couplings are real. The neutral components of Higgs fields will acquire complex vacuum expectation values $< H_1 >= \frac{v_1}{\sqrt{2}}$, $< H_3 >= \frac{v_3 e^{i\delta_3}}{\sqrt{2}}$ and $< H_2 >= \frac{v_2 e^{i\delta_2}}{\sqrt{2}}$, $< H_4 >= \frac{v_4 e^{i\delta_4}}{\sqrt{2}}$, where we rotated away the trivial phase of $< H_1 >$. The vacuum expectation value of the scalar potential is

$$
\begin{aligned}
< V > &= \frac{1}{2} m_1^2 v_1^2 + \frac{1}{2} m_2^2 v_2^2 + \frac{1}{2} m_3^2 v_3^2 + \frac{1}{2} m_4^2 v_4^2 - m_{32}^2 v_3 v_2 \cos(\delta_3 + \delta_2) - m_{14}^2 v_1 v_4 \cos\delta_4 \\
&- m_{12}^2 v_1 v_2 \cos\delta_2 - m_{13}^2 v_1 v_3 \cos\delta_3 - m_{34}^2 v_3 v_4 \cos(\delta_3 + \delta_4) - m_{24}^2 v_2 v_4 \cos(\delta_2 - \delta_4) + \\
&+ \frac{1}{32}(g^2 + g'^2)[v_1^2 + v_3^2 - v_2^2 - v_4^2]^2. \qquad (3)
\end{aligned}
$$

The necessary phase conditions at the minimum are

$$
\begin{aligned}
\frac{\partial V}{\partial \delta_2} &= m_{12}^2 v_1 v_2 \sin\delta_2 + m_{32}^2 v_3 v_2 \sin(\delta_3 + \delta_2) + m_{24}^2 v_2 v_4 \sin(\delta_2 - \delta_4) = 0, \\
\frac{\partial V}{\partial \delta_3} &= m_{32}^2 v_3 v_2 \sin(\delta_3 + \delta_2) + m_{13}^2 v_1 v_3 \sin\delta_3 + m_{34}^2 v_3 v_4 \sin(\delta_3 + \delta_4) = 0, \\
\frac{\partial V}{\partial \delta_4} &= -m_{24}^2 v_2 v_4 \sin(\delta_2 - \delta_4) + m_{34}^2 v_3 v_4 \sin(\delta_3 + \delta_4) + m_{14}^2 v_1 v_4 \sin\delta_4 = 0. \qquad (4)
\end{aligned}
$$

It is plausible that a general solution with the values for phases different from zero or π can be found. For coefficients m of the order of weak scale or so, we expect the vevs to be of the same order, as well as the phases of the vevs to be of order one. However, in the absence of any additional symmetries, many higgs doublet models will have flavor changing interactions. This is because diagonalization of the Yukawa matrix coupled to one Higgs will not in general diagonalize the Yukawa matrix of the other Higgs (otherwise there is at least a discrete symmetry which relates the two matrices).

Yukawa couplings, FC and the Peccei-Quinn like symmetry

The limits on FC can be avoided if we introduce some symmetry (or approximate symmetry) that will decouple (or almost decouple) the second pair of higgses from fermions. In this way the FC will be proportional to the amount by which the symmetry is broken.

A simple scenario for the couplings in a four higgs doublet SSM is to impose an approximate symmetry which is very similar to Peccei-Quinn symmetry (I call it the PQ symmetry). The terms that violate the symmetry get suppressed by powers of a small factor ϵ_{PQ}. I assume following assignment of the charges: $Q(H_3) = +1$,$Q(H_4) = -1$ and $Q(D^c) = +1$. All other fields have zero PQ charge. This is similar to the model of Nelson and Randall[3)]. The only difference is that in their model D^C has charge -1.

From the assignments of the PQ charges we observe that in the superpotential (1) h_3 is suppressed by ϵ_{PQ}^2, h_1 and h_4 are suppressed by ϵ_{PQ}, while h_2 is unsuppressed and similarly for μ_{ij}. Same as in Ref. 3, we will take $\epsilon_{PQ} = \frac{1}{\tan\beta} = \frac{m_b}{m_t}$. This gives the explanation of the large ratio of t to b mass entirely in terms of the approximate PQ symmetry, while the hierarchy between the generations of the same charge is left to the flavor symmetry breaking part. The assignments of charges also tells us that in the scalar potential $m_{14}^2, m_{32}^2, m_{13}^2, m_{24}^4$ are suppressed by ϵ_{PQ}. m_{12}^2 and m_{34}^2 remain unsuppressed (order weak scale).

Depending on the choice of the parameters, the minimum of the potential will be when one and only one pair of vevs (v_1, v_2) or (v_3, v_4) is suppressed by ϵ_{PQ} (compared to the weak scale). Otherwise we get a higher minimum, or an unbounded potential. This is obvious from looking at the vev minimum conditions for the potential. We choose the pair (v_3, v_4) to be suppressed, while v_1 and v_2 remain unsuppressed. In this way light goldstone bosons are avoided since the size of breaking is of the same order as the explicit symmetry breaking terms in the Lagrangian[3)]. Notice that the whole effect of the change of the PQ assignment of D^c is that the Yukawa of H_3 is suppressed by ϵ_{PQ}^2. Since the Yukawa of H_1, which is primarily of the order of the down quark mass matrix, is down by ϵ_{PQ}, this means that FC couplings will have an additional factor of ϵ_{PQ}. Notice that the assignment of charge $+1$ to D^c was crucial. If it were -1, the yukawa couplings h_1 would not have had any PQ suppression, and the theory would have damaging FC. The authors of Ref. 3 introduce the additional assumption of a spurion field in order to avoid couplings of H_1, thus explaining the large ratio $\frac{m_t}{m_b}$ with large ratio of vevs $\tan\beta$. With our assignment of the charges the large ratio $\frac{m_t}{m_b}$ is explained by different suppressions of the Yukawa matrices. We do *not* have large $\tan\beta$, but the origin of large $\frac{m_t}{m_b}$ is the same, namely approximate PQ symmetry.

Next, we can see from (4) that only $\sin\delta_2$ must be suppressed by ϵ_{PQ}^2, while other phases can be of order one. This is our general result: by allowing for an approximate symmetry, some phases which were CP trivial (i.e. 0 or π) in the limit of exact symmetry, may become of order one. Crucial is that these phases are *not* proportional to ϵ_{PQ}.

Notice that the elements of the KM matrix will be real up to the leading order in ϵ_{PQ}, since the up and down mass matrices couple dominantly to one higgs only, namely to H_1 and H_2. The phases of the vevs of H_1 and H_2 do not enter the diagonalization matrices and thus do not enter the KM matrix. We can compute the contribution of the exchange of flavor changing scalars to ΔM_K. It will be down by ϵ_{PQ}^2 (two couplings) compared to having just couplings comparable to those responsible for quark masses. For scalars of order weak scale or so, this means that the standard model box diagram will be the dominant contribution to ΔM_K. However this contribution is real since the KM matrix is real. Thus, the dominant contribution to the CP violating parameter ϵ_{CP} will come from the flavor changing scalar exchange. Although the scalars contribute a phase of order one its amplitude is suppressed by ϵ_{PQ}^2, and this is what makes ϵ_{CP} small. The contribution of the flavor mediating scalars is of the same order as the standard model contribution when the mass of the scalars is about 1 TeV or so. Thus,

$$\epsilon_{CP} \approx \frac{Im\Delta M_K}{\Delta M_K} \approx \epsilon_{PQ}^2 (\frac{1TeV}{M})^2 \sin\phi \approx 3\times 10^{-3} (\frac{\frac{m_b}{m_t}}{\frac{1}{60}})^2 (\frac{300GeV}{M})^2 \sin\phi, \tag{5}$$

where M is the typical mass of the flavor mediating Higgs scalar, and $\sin\phi$ is a CP violating phase of order one. This actually gives the right value for $M = 300GeV$, which is the weak scale (we took $m_b = 3GeV$ at the weak scale)! We do not need scalars of order TeV, which Hall and Weinberg considered to be somewhat heavy anyway.

The suppression of CP violating effects because of the approximate PQ symmetry despite the existence of phases of order one is not a property only of the kaon system. Any flavor changing exchange is necessarily suppressed by powers of ϵ_{PQ}. The contributions to different diagrams will come either from the large vev of H_2, which has a suppressed phase, or from other doublets which either have suppressed vevs or small Yukawa couplings. For this reason we expect direct CP violating effects will also be very small, as well as the CP violating phases in the B system. However we expect the neutron electric dipole moment (NEDM) to be of the order of experimental limit. This is because naive estimates which do not include any suppression of the flavor changing couplings usually give NEDM several orders of magnitude higher than the experimental limit. This general situation is similar to the assumption of suppressed CP violating phases of Hall and Weinberg[5)]. Here we offer an explanation for the smallness of CP violation in terms of the small ratio of bottom and top quark masses and we link it to the PQ symmetry.

We also note that as far as chargino masses go we have no light charginos[3)], because the PQ numbers for higgses are the same as in Ref. 3.

Finally, no attempt was made to include this scheme into a grand unified theory. However, many attempts have higgs multiplets which have more than two doublets (for example two 10's in SO(10) or 2 bidoublets in a LR model). Whether these doublets can stay light will depend on the details of the theory, such as intermediate scales.

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Note: After the talk, we made more progress on understanding what the conditions for SCPV in many higgs SUSY model are. It appears that the tree level potential is not sufficient by itself, and an additional contribution (radiative corrections, soft CP violation or an additional singlet) is needed[8)].

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