U(1)$_A$ Models of Fermion Masses Without a μ Problem

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Abstract

We discuss the connection between models of fermion masses and mixing involving a string-motivated flavor/generation U(1)$_A$ gauge symmetry and the μ term. We point out that in a certain class of such models the flavor physics can provide an appealing solution to the μ problem, naturally yielding a $\mu \sim O(m_w)$. A simple relationship between the $U(1)_A$ charge q_H of the μ-term and the average generational $U(1)_A$ charges of the down quark and leptonic sectors is derived. Finally, we construct an explicit model illustrating our results.

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Two of the most outstanding challenges at present in particle physics are to understand (a) the observed pattern of fermion masses and mixing; and (b) the hierarchy between the electroweak and Planck mass scales. The standard model (SM) can accomodate, but not explain these fermion masses and mixing. Renormalizable supersymmetric extensions of the standard model can stabilize the Higgs sector against large radiative corrections, but this, by itself, is not enough to explain property (b) because of the μ–problem. This problem concerns the fact that the term $\mu H_1 H_2$ in the superpotential W, where H_i, $i = 1, 2$ are the $Y = -1, 1$ Higgs chiral superfields in the minimal supersymmetric standard model (MSSM), is supersymmetric, and hence there is no apparent reason why its coefficient, μ, should have anything to do with the scale of supersymmetry breaking (which, in realistic theories is of order the electroweak scale, $v_{EW} \simeq 250$ GeV). If, as is allowed by supersymmetry itself, $|\mu| >> v_{EW}$, this would destroy the hierarchy, and, indeed, for a phenomenologically acceptable model, $|\mu|$ must be of order v_{EW}. On the other hand, one cannot forbid the μ term completely, for this would lead to a phenomenologically unacceptable massless goldstone boson.[1]

In this paper we discuss an appealing class of models yielding a unified explanation of both of these outstanding problems (a) and (b). To introduce these, we first note that recently there has been strong interest in models of fermion masses and mixing based on a string-motivated, flavor- and generation-dependent $U(1)_A$ gauge symmetry [2]–[6] in the context of a supergravity theory reducing at $E << \bar{M}_P$ to the MSSM.[2] The $U(1)_A$ symmetry has field-theoretic anomalies which are cancelled by a Green-Schwarz mechanism [7]; as a result, this symmetry is spontaneously broken, at a calculable high mass scale somewhat below the string scale [8].

When one embeds global supersymmetry in a supergravity theory, the μ–parameter has an additional contribution from the Kähler potential, K [9]

$$\tilde{\mu}(M, M^{\dagger}) H_1 H_2 \tag{1}$$

where the parenthesis indicates the dependence of $\tilde{\mu}$ on generic fields M which aquire large vacuum expectation value (vev's). Upon supersymmetry breaking, the second term can lead to a contribution to an effective superpotential μ-term of the form

$$\tilde{\mu}(< M >, < M^{\dagger} >) m_{3/2} H_1 H_2 \tag{2}$$

in W_{eff}. Here $m_{3/2}$ is the gravitino mass, which is related to the scale of supersymmetry breaking,

$$\mu \sim v_{EW} \sim m_{3/2} \tag{3}$$

If the superpotential contribution to the μ–term is zero, it becomes possible to generate a μ term of an acceptable size (3) in W_{eff}, via eq. (2), from Kähler couplings. In many string-inspired effective supergravity models the superpotential contribution to the μ-term is absent at tree level because of modular invariance, but can be generated by nonperturbative effects in the hidden sector [1, 10, 11]. Unfortunately, it is difficult to ascertain whether this nonperturbative contribution is of an acceptable size [11]. Moreover, in generic supergravity

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[1]For a recent review of the μ problem, see Ref. [1].
[2]Here, $\bar{M}_P \equiv (8\pi G_N)^{-1/2} = 2.44 \times 10^{18}$ GeV is the (reduced) Planck mass.

models there are even more serious problems with the μ term, and hence stabilization of the hierarchy, than in renormalizable supersymmetric models. This is because of nonrenormalizable couplings in K which can lead to quadratically divergent loop corrections to μ [12, 13] (see also [14]).

However, as was first pointed out in Ref. [12][3], in models with a $U(1)_A$ gauge symmetry, such a destabilization can be avoided and furthermore, the natural value of the μ-parameter is determined by the couplings allowed by the gauge group, so that in such models one can achieve a solution of the μ problem. Moreover, as we discuss here, this solution connects the physics of fermion masses and mixing with that of the hierarchy in a fundamental way. This idea was embodied in new solutions of the anomaly cancellation conditions discovered in Ref. [4] which had the property that the sum of the $U(1)_A$ charges of the Higgs, $q_H \equiv q_{H_1} + q_{H_2} \neq 0$. If and only if $q_H \neq 0$, the μ–term cannot, by itself, appear in either W or K. However, it may appear in combination with powers of various other chiral superfields such that the total $U(1)_A$ charge is zero, and in this case the natural order of the effective low energy μ–parameter, which is calculable in terms of the $U(1)_A$ charges of these various fields, can be naturally of order the electroweak scale.

We first briefly review $U(1)_A$ models of fermion masses and mixing. One of the most puzzling aspects of the known fermion masses is that if one assumes, as in the SM, that they arise from conventional, dimension-4 Yukawa operators, then the associated Yukawa couplings for all of these fermions except the top quark are all much smaller than a typical small coupling like $e = \sqrt{4\pi\alpha} \simeq 0.3$, without any explanation. In these models, this feature of the first two generations is explained in an elegant way, since they arise from higher-dimension, nonrenormalizable operators.[4] This is appealing, because these operators are generically present in the supergravity theory which forms the field-theory limit of the presumed underlying string theory for energies $E << M_{str}$ (where $M_{str} = 2(\alpha')^{-1/2} = g\bar{M}_P$ denotes the string scale) with c-number coefficients proportional to the requisite inverse powers of M_{str}. Via vacuum expectation values of the scalar components of certain chiral superfields, which we shall denote generically as v, these higher-dimension operators can yield contributions to effective dimension-4 Yukawa interactions which are suppressed by powers of the ratio $\epsilon \sim v/M_{str}$. An important aspect of this approach is that ϵ is calculable; the $U(1)_A$ symmetry-breaking scale v is given by [8] $v^2 \simeq (M_{str})^2/(192\pi^2)$, whence $\epsilon \sim \lambda^2$ (where $\lambda = |V_{us}| \approx 0.22$), so that ϵ is in the right range to explain the fermion mass hierarchies[5]

$$m_u/m_c \sim \lambda^5, \quad m_c/m_t \sim \lambda^4, \quad m_d/m_s \sim \lambda^2, \quad m_s/m_b \sim \lambda^2 \tag{4}$$

$$m_e/m_\mu \sim \lambda^4, \quad m_\mu/m_\tau \sim \lambda^2 \tag{5}$$

The $U(1)_A$ symmetry also forbids certain chiral superfield cubic couplings and hence produces zeros in some entries of the Yukawa matrices for the resultant dimension-4 Yukawa terms. This, in turn, enables one to explain why the off-diagonal elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix are small by allowing one to express them in terms of small quark

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[3]See discussion after eq. (24) in Ref. [12].

[4]An early paper noting the importance of such operators for fermion mass relations is Ref. [15].

[5]Here the running masses are used, normalized at the same scale. Note that $m_d m_s m_b = \lambda^6 m_b^3$, $m_e m_\mu m_\tau = \lambda^8 m_\tau^3$, and, since $m_b(m_b) \sim \lambda^{-2/3} m_b(v)$ with $m_b(v) \simeq m_\tau(v)$, there follows the well-known relation $\det M_d(v) \simeq \det M_L(v)$ which we shall use below.

mass ratios m_d/m_s, m_u/m_c, etc.[6]

A further puzzling property of the observed fermion mass spectrum is that, even if one restricts oneself just to the third generation, m_b and m_τ are $<< m_t$, again with no explanation. A general class of U(1)$_A$ models was discussed in Ref. [4], and a specific string construction in Ref. [17] which account for this by means of a far-reaching hypothesis, namely, that even the largest, $j, k = 3, 3$ elements of the (effective) down-quark and charged lepton Yukawa matrices Y^d and Y^L actually arise from higher-dimension operators.

We note an additional appeal of the U(1)$_A$ symmetry. One might, *a priori*, try to restrict the form of the fermion Yukawa matrices using a global symmetry. However, in addition to the unesthetic nature of global symmetries as fundamental and problems with unwanted goldstone bosons resulting from spontaneous symmetry breaking, there is also the problem that they are, in general, broken by quantum gravity, even at the semi-classical level [18].

There are at least two worthwhile ways to study U(1)$_A$–based models of low energy physics[7]: (a) one can study specific scenarios suggested by particular string constructions, as in Ref. [17, 20]; (b) alternatively, one can avoid committing oneself to any particular string constructions, and instead study general solutions to the consistency conditions implied by the Green-Schwarz mechanism. We continue to follow the second approach here, since it allows one to explore in a very general way what can and cannot be achieved with such models. As in Ref. [4], we shall study the simplest extension of the SM which may be able to account for the observed fermion masses and the μ-term, i.e. one in which the observable sector gauge group is just SU(3) × SU(2) × U(1)$_Y$ × U(1)$_A$ and the matter content includes just the usual SM fields together with some additional SM–singlet fields which can carry $U(1)_A$ charge. The model we study is a minimal supergravity extension of the SM with enough new physics built in to have the potential to explain low energy data. Our objective is to explore the implications of such a minimal model. Indeed, we find that such a model can be phenomenologically viable.[8]

As part of our analysis, we will derive a simple linear relation between q_H and the average (generational) $U(1)_A$ charges of the SM down-quark and lepton sectors. This result holds independent of (i) whether the $U(1)_A$ charges of the superpotential Yukawa terms are assumed to be symmetric in generation-space or not and (ii) the spectrum of SM–singlet $U(1)_A$ charged chiral superfields, which will be denoted generically as χ, $\tilde{\chi}$, etc. We consider the general case where the effective Yukawa matrices in the up, down, and charged lepton sectors are not necessarily symmetric in generation indices. We find that a Kähler potential origin for the effective μ-parameter may well be more natural than a superpotential origin. Indeed, if only one SM–singlet field χ gets a vev due to $U(1)_A$ breaking, then an acceptable μ_{eff} must come from K (unless $q_H = 0$) whereas if just two SM–singlet fields $\chi, \tilde{\chi}$ with opposite $U(1)_A$ charges get vev's, then the largest contribution to μ_{eff} is always from W and an acceptable value is generated only if $|q_H| \sim 11|q(\chi)|$. When all SM–singlet fields χ

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[6]The latter result was also achieved in early models of quark mass matrices [16]; however, since these used only dimension-4 Yukawa couplings, they had to put in the fermion mass hierarchy by hand.

[7]One should also note that efforts have been made to describe the origin of this pattern in terms of new physics operative at a much lower mass scale, e.g., 1-100 TeV, in theories involving dynamical electroweak symmetry breaking. See also Ref. [19]

[8]In generic string-motivated models there are extra (nonanomalous) $U(1)$ gauge symmetries, as well as discrete symmetries and nontrivial kinetic normalizations for the matter fields.

which get vev's carry the same sign $U(1)_A$ charge then an acceptable μ_{eff} may originate from either K or W, but it appears more easily from K.

If the Yukawa matrix charge assignments are symmetric in generation space, the number of free parameters reduces greatly and stronger statements are possible. In fact, in this case, it is difficult to reproduce observed fermion mass hierarchies if (i) both top and bottom masses arise from renormalizable superpotential couplings, (ii) effective Yukawa couplings are due to the vev of just one SM–neutral $U(1)_A$ charged field or (iii) effective Yukawa couplings are due to the vev's of just two SM–neutral fields with opposite (nonzero) $U(1)_A$ charges. It is possible to avoid problems associated with these three cases if the bottom mass arises from a nonrenormalizable superpotential coupling and effective Yukawa couplings are due to several SM–singlet fields which all carry the same sign (nonzero) $U(1)_A$ charge. When there are just two such fields we show that an acceptable μ-parameter cannot be achieved from W if $\tan\beta$ is not allowed to get either very large or very small.

To begin, we assign arbitrary $U(1)_A$ charges to all fields. This introduces 17 parameters for the 3 generation minimal standard model, plus additional parameters for SM singlets [4]. We use the parametrization

$$q(Q_i) = \bar{q}_Q + \alpha_i \ , q(u_i^c) = \bar{q}_{u_i^c} + \beta_i \ , \ q(d_i^c) = \bar{q}_{d^c} + \gamma_i \ , \ q(L_i) = \bar{q}_L + a_i \ , \ q(e_i^c) = \bar{q}_{e^c} + b_i \quad (6)$$

where $\bar{q}(f) = (1/3)\sum_{i=1}^{3} q(f_i)$ denotes the generational average U$(1)_A$ charge of f, and thus

$$\sum_i \alpha_i = \sum_i \beta_i = \sum_i \gamma_i = \sum_i a_i = \sum b_i = 0 \quad (7)$$

With the standard normalization of $U(1)_Y$ hypercharge, $Y(Q) = 1/3$, $Y(u) = -4/3$, $Y(d) = 2/3$, $Y(L) = -1$, $Y(e) = 2$, $Y(H_2) = 1$, and $Y(H_1) = -1$, the mixed anomaly coefficients are

$$c_1 = \mathrm{Tr}(T_a T_Y^2) = \frac{\bar{q}_Q}{2} + 4\bar{q}_{u^c} + \bar{q}_{d^c} + \frac{3}{2}\bar{q}_L + 3\bar{q}_{e^c} + \frac{q_{H_1}}{2} + \frac{q_{H_2}}{2} \quad (8)$$

$$c_2 = \mathrm{Tr}(T_a T_{SU(2)}^2) = \frac{9}{2}\bar{q}_Q + \frac{3}{2}\bar{q}_L + \frac{q_{H_1}}{2} + \frac{q_{H_2}}{2} \quad (9)$$

and

$$c_3 = \mathrm{Tr}(T_a T_{SU(3)}^2) = 3\bar{q}_Q + \frac{3}{2}\bar{q}_{u^c} + \frac{3}{2}\bar{q}_{d^c} \quad (10)$$

Since these anomaly coefficients are linear in the U$(1)_A$ charges and involve sums over all generations, they depend only on average U$(1)_A$ charges, as indicated. An immediate consequence is that

$$c_1 + c_2 - \frac{8}{3}c_3 = q_H - 3(\bar{q}_Q + \bar{q}_{d^c} - \bar{q}_L - \bar{q}_{e^c}) \quad (11)$$

The anomaly cancellation by the Green-Schwarz mechanism requires [2] $c_i/c_j = k_i/k_j$ where the k_i are the levels for the Kac-Moody algebra on the string worldsheet which determine the gauge couplings for each of the factor groups U$(1)_Y$, SU(2), and SU(3) by $g_i^{-2} = k_i\langle Re(s)\rangle$ (at M_{str}), where, in turn, s is the dilaton/axion. The unification of gauge couplings in the MSSM [9] requires $g_1^{-2} = (5/3)g_i^{-2}$, $i = 2, 3$ and hence (with $k_2 = k_3 = 1$ to avoid exotics),

$$c_1 : c_2 : c_3 = \frac{5}{3} : 1 : 1 \quad (12)$$

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[9]For a recent discussion of the current status of MSSM gauge unification, see Ref. [21].

From this it follows that

$$c_1 + c_2 - \frac{8}{3}c_3 = 0, \quad i.e., \quad q_H = 3(\bar{q}_Q + \bar{q}_{d^c} - \bar{q}_L - \bar{q}_{e^c}) \tag{13}$$

where, as before, $q_H = q_{H_1} + q_{H_2}$. Thus, the $U(1)_A$ charge q_H of the μ term $H_1 H_2$ is determined in terms of the generational-average charges of the matter fermions. Later we will give another expression for the RHS which will have important consequences for the effective μ-parameter.

A further requirement of anomaly cancellation is that

$$0 = \mathrm{Tr}(T_a^2 T_Y) = 3\bar{q}_Q^2 - 6\bar{q}_{u^c}^2 + 3\bar{q}_{d^c}^2 - 3\bar{q}_L^2 + 3\bar{q}_{e^c}^2 + q_{H_2}^2 - q_{H_1}^2 + \Delta \tag{14}$$

where Δ is quadratic in parameters $\alpha_i, \beta_i, \gamma_i, a_i, b_i$ which do not depend on average (barred) charges. In general, there is no simple solution to eq. (14); however, when $\Delta = 0$, eq. (14) imposes a simple quadratic constraint on the average $U(1)_A$ charges. Δ vanishes identically when additional symmetry requirements are imposed, e.g. when the Yukawa charge assignments are required to be symmetric in flavor space, as in [3, 4] so that $\alpha_i = \beta_i = \gamma_i, a_i = b_i$. We recall the general solutions to the full set of anomaly constraints found in Ref. [4] for the $\Delta = 0$ case:

$$\begin{array}{ccccccc}
\bar{q}_Q & \bar{q}_{u^c} & \bar{q}_{d^c} & \bar{q}_L & \bar{q}_{e^c} & q_{H_2} & q_{H_1} \\
x & x & y & y & x & z & -z \\
x & x & \frac{y}{2} - \frac{z}{2} & y & x & -\frac{3}{2}y - \frac{1}{2}z & -z \\
x + v & x + 2v & y + w & y & x & 3v + 3w + z & -z
\end{array} \tag{15}$$

The first two solutions describe two distinct 3–parameter family of solutions to the constraints. The last solution, with $v \neq 0$, describes a 4 parameter family of solutions with x given by

$$0 = -2v^2 + 2w^2 + 3v(w - x) + vz + w(y + z) \tag{16}$$

The first solution in (15) was already given in [3], while the latter two were new in Ref. [4]. These two new solutions which we discovered, and which allow $q_H = q_{H_1} + q_{H_2} \neq 0$, can play an important role in constraining the μ-parameter.

Labelling the $U(1)_A$ charges of the cubic superfield terms as

$$q(Q_3 u_3^c H_2) = \delta_t, \quad q(Q_3 d_3^c H_1) = \delta_b, \quad q(L_3 e_3^c H_1) = \delta_\tau \tag{17}$$

we have

$$\begin{aligned}
\bar{q}_Q + \bar{q}_{u^c} + q_{H_2} + \alpha_3 + \beta_3 &= \delta_t, \\
\bar{q}_Q + \bar{q}_{d^c} + q_{H_1} + \alpha_3 + \gamma_3 &= \delta_b, \\
\bar{q}_L + \bar{q}_{e^c} + q_{H_1} + a_3 + b_3 &= \delta_\tau
\end{aligned} \tag{18}$$

The $U(1)_A$ charges Yukawa coupling charge matrices are given by

$$q(Q_i u_j^c H_2) = \delta_t + \alpha_i - \alpha_3 + \beta_i - \beta_3 \tag{19}$$

$$q(Q_i d_j^c H_1) = \delta_b + \alpha_i - \alpha_3 + \gamma_i - \gamma_3 \tag{20}$$

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$$q(L_i e_j^c H_1) = \delta_\tau + a_i - a_3 + b_i - b_3. \tag{21}$$

For the symmetric charge assignment case, $\beta_i = \gamma_i = \alpha_i, b_i = a_i$, the charge matrix structures (19)-(21) imply

$$q(Q_i d_j^c H_1) = q(Q_i u_j^c H_2) + (\delta_b - \delta_\tau). \tag{22}$$

From this we deduce:

1. If $\delta_t = \delta_b$ then the up-quark $U(1)_A$ charge matrix is identical to the down-quark $U(1)_A$ charge matrix. Hence, we have $Y_{ij}^u \sim Y_{ij}^d$ which cannot fit experimental data. This includes the case where the top and bottom masses both arise from renormalizable couplings.

2. If superpotential couplings of the quark sector to a single SM–singlet field which receives a vev are responsible for all effective (low-energy) Yukawa matrices, then we again must have $Y_{ij}^u \propto Y_{ij}^d$ which is in contradiction with data.

3. If superpotential couplings of the quark sector to two SM–singlet fields which have opposite $U(1)_A$ charges and receive vev's are responsible for all (effective) Yukawa matrices, then we also must have $Y_{ij}^u \propto Y_{ij}^d$, again in disagreement with data.

Therefore if one wishes to explain low energy Yukawa data via the $U(1)_A$ symmetry, and one wants either $\delta_t = \delta_b$ *or* invokes SM fermion couplings to only a single SM–singlet field or two SM–singlet fields with opposite $U(1)_A$ charges, then one must relax the condition of symmetric charge assignments (as in [5, 6]).

However, there is no *ab initio* reason to use just a single SM–singlet field (or a pair of oppositely charged SM–singlets), and, indeed, as we previously demonstrated [4], in the context of symmetric textures, the above restrictions can be avoided if one considers, for example, two SM–singlet, $U(1)_A$-charged fields χ, χ' which do not have opposite charges. We will exhibit a new model of this type which is able to fit all low energy mass data.

Let us next define Σ as the diagonal sum of $U(1)_A$ charges in a Yukawa charge matrix. Hence,

$$\begin{aligned} \Sigma_u &= 3(\delta_t - \alpha_3 - \beta_3), \\ \Sigma_d &= 3(\delta_b - \alpha_3 - \gamma_3), \\ \Sigma_L &= 3(\delta_\tau - a_3 - b_3). \end{aligned} \tag{23}$$

Note that Σ is also the $U(1)_A$ charge of any term in the determinant of a Yukawa coupling matrix, e.g. $\Sigma_d = q(\det[Q_i d_j^c H_1])$, and similarly for Σ_L and Σ_u. From eqs. (11) and (18), we get

$$c_1 + c_2 - \frac{8}{3}c_3 = q_H + \Sigma_L - \Sigma_d \tag{24}$$

or, for the condition (12),

$$q_H = \Sigma_d - \Sigma_L. \tag{25}$$

As we will see, this form of eq. (13) makes the analysis of the relation between the effective μ-parameter and fermion mass matrices quite transparent in many cases of interest.

It is interesting to note that there is a general relation between k_1/k_3 and the Yukawa and μ-term superpotential charges. From eqs. (10), (18), and (23), we have

$$2c_3 = \Sigma_u + \Sigma_d - 3q_H. \tag{26}$$

For $k_2 = k_3$, hence $c_2 = c_3$, we have

$$c_1 - \frac{5}{3}c_3 = q_H + \Sigma_L - \Sigma_d. \tag{27}$$

Together, these last two equations yield

$$\frac{k_1}{k_3} = \frac{c_1}{c_3} = \frac{6\Sigma_L - \Sigma_d + 5\Sigma_u - 9q_H}{3\Sigma_d + 3\Sigma_u - 9q_H} \tag{28}$$

In models involving a single SM–singlet $U(1)_A$–charged field, one can derive the relation $\sin^2 \theta_w = 3/8$ if one has $q_H = 0$ and $c_2 = c_3$ [5]. (This can be seen from our eq. (28); the condition $\det Y^L \sim \det Y^d$ implies $\Sigma_L \sim \Sigma_d$, so the RHS of eq. (28) $= 5/3$ for $q_H = 0$.) However, the property that $q_H = 0$ in such models means that they do not solve the μ problem. In contrast, the relations we have derived, (13) and (25), are independent of the number of $U(1)_A$ charged fields and can be used to constrain $U(1)_A$ charges in more general situations. Our approach is to assume canonical Kac-Moody levels $k_2 = k_3 = 1$ and $k_1 = 5/3$ for which $c_1 + c_2 - \frac{8}{3}c_3 = 0$ and use eqs. (13) and (25) to analyze relations between q_H and fermion masses. The μ-term charge q_H is only zero if $\Sigma_d = \Sigma_L$, which is sufficient, but not necessary, to satisfy data on fermion masses and mixing. The point here is that when one says the i, j entry in an effective Yukawa matrix is of order λ^n, this means that the dimensionless coefficient c_{ij} multiplying λ^n lies within the range $\lambda \leq c_{ij} \leq \lambda^{-1}$. This finite range in each of the coefficients produces an intrinsic uncertainty in the actual, as opposed to formal, power of λ describing the determinant of the Yukawa matrix. Indeed, because of this uncertainty, the fermion mass data can be satisfied without requiring that $\Sigma_L = \Sigma_d$ as long as the ratio between these determinants is within the range induced by the above dimensionless coefficients. Hence, even in such a model, a small μ-parameter may be obtained by requiring q_H to be small and nonzero so that only a Kähler potential contribution to μ_{eff} is obtained.

If a model has two SM–singlet fields $\chi, \tilde{\chi}$ with opposite $U(1)_A$ charges $x, -x$, then one needs $|q_H| \sim 11|x|$ to get an acceptable value for μ_{eff}. This is because if q_H is not an integer multiple of x then no μ-term is allowed, and if q_H is an integer multiple of x then the largest contribution is always from W and is either of the form $\chi^{|q_H|/|x|}H_1H_2$ or $\tilde{\chi}^{|q_H|/|x|}H_1H_2$. Assuming vev's of $O(\lambda^2)\bar{M}_P$, we then find $\mu \sim \lambda^{2|q_H|/|x|}\bar{M}_P$.

We now study the case of two fields χ, χ' with same–sign $U(1)_A$ charges. With no loss of generality, we take

$$q[\chi] = 1, \quad q[\chi'] = \alpha > 1. \tag{29}$$

We have three possibilities:

1. $q_H = 0$. If one were to choose this value, the model would have an unresolved μ– problem, so we avoid this choice.

2. $q_H < 0$. If $q_H = -n - m\alpha$ for nonnegative integers n, m, then W contains the term

$$\frac{\chi^n(\chi')^m}{(\bar{M}_P)^{(n+m-1)}} H_1 H_2, \tag{30}$$

so that upon $U(1)_A$ breaking,

$$\mu \sim \lambda^{2(n+m)} \bar{M}_P. \tag{31}$$

In this case, unless $n + m$ is sufficiently large, one naturally has too large a μ–term. On the other hand, if $q_H < 0$ but $q_H \neq -n - m\alpha$ for nonnegative integers n, m, then the superpotential contribution to the total μ-term is zero.

3. $q_H > 0$. In this case there can be no superpotential μ–term; however the Kähler potential contains the terms

$$
\begin{aligned}
(a) \quad & (\chi^\dagger)^n (\chi'^\dagger)^m H_1 H_2 \text{ if } q_H = n + m\alpha, \\
\text{or} \quad (b) \quad & \chi(\chi'^\dagger)^m H_1 H_2 \text{ if } q_H = -1 + m\alpha, m \geq 1, \\
\text{or} \quad (c) \quad & (\chi^\dagger)^n \chi' H_1 H_2 \text{ if } q_H = n - \alpha, n \geq 2,
\end{aligned} \tag{32}
$$

for some nonnegative integers n, m. Upon SUSY and $U(1)_A$ breaking, these give contributions to the effective μ–term

$$
\begin{aligned}
(a) \quad & \mu_{eff} \sim m_{3/2} \lambda^{2(n+m)}, \\
(b) \quad & \mu_{eff} \sim m_{3/2} \lambda^{2m+2}, \\
(c) \quad & \mu_{eff} \sim m_{3/2} \lambda^{2n+2},
\end{aligned} \tag{33}
$$

If one of the conditions (a) through (c) above is not satisfied, then there is no Kähler potential contribution to μ. Otherwise, if the only contribution to the effective μ-term is from K, it will will be too small unless n and m are sufficiently small. In fact, $\mu \sim v_{EW}$ suggests that only the cases

$$q_H = 1, \quad \text{or} \quad q_H = \alpha \tag{34}$$

(corresponding to K containing $\chi^\dagger H_1 H_2$ or $\chi'^\dagger H_1 H_2$, respectively), which both give $\mu_{eff} \sim m_{3/2}\lambda^2$, can naturally yield an acceptable value of the μ-parameter.

It appears considerably more difficult to construct models where an acceptable μ–term arises from the superpotential. If we require

$$\det Y^d \sim \det Y^L \sim \lambda^{2d}, \tag{35}$$

we can derive

$$|q_H| \leq (\alpha - 1)d, \tag{36}$$

by considering upper and lower bounds on the determinants,

$$\lambda^{-2\Sigma_d} \leq \det Y^d \leq \lambda^{-2\Sigma_d/\alpha}, \quad \lambda^{-2\Sigma_L} \leq \det Y^L \leq \lambda^{-2\Sigma_L/\alpha}, \tag{37}$$

and using (25). It follows that if there is a nonzero superpotential μ-term then $q_H < 0$ and

$$\mu \geq O(\lambda^{2(\alpha-1)d}) \bar{M}_P. \tag{38}$$

Since one expects $d \sim 7$, an acceptable superpotential contribution to μ_{eff} requires $\alpha \geq 2.5$.

In models with multiple SM–singlet fields with same–sign $U(1)_A$ charges, this requires the largest magnitude $U(1)_A$ charge to be at least 2.5 times greater than the smallest magnitude $U(1)_A$ charge. To realize how restrictive this can be, we first note that (38) assumes the μ-term couples exclusively to the SM singlet with charge 1 (i.e. the smallest $U(1)_A$ charge). If, instead, it couples only to the SM singlet with charge $\alpha > 1$, then we can never produce a sufficiently small superpotential μ_{eff}. Thus, in general, we expect α to be much bigger than 2.5. Furthermore, there can be restrictions on α from data on fermion masses and mixing. For example, in the symmetric case with just two additional fields we can argue that μ_{eff} should originate from K in order to satisfy fermion mass data and $\tan \beta$ not too large (or small). To do this, we concentrate on the two most massive generations. For simplicity we assume the top mass arises from a renormalizable operator, i.e. $\delta_t = 0$. From (19) we note that in order to have $m_c/m_t \sim \lambda^4$, i.e. $Y_{22}^u Y_{33}^u - Y_{23}^u Y_{23}^u \sim \lambda^4 (Y_{33}^u)^2$,, it is necessary that $q(H_2 Q_2 u_3^c) = \alpha_1 + 2\alpha_2 = -1, -\alpha$ or $-(1+\alpha)/2$. If $\delta_b = 0$, then the mass m_b also arises from a renormalizable operator, i.e. the small value of m_b/m_t must be entirely accounted for by a large $\tan \beta$. If, however, $\delta_b = -1$ or $-\alpha$, then $Y_{33}^d/Y_{33}^u \sim \lambda^2$ and one does not need either a very large or small value of $\tan \beta$. This leads to six possible charge assignments for the up and down-quark Yukawa couplings of the the two highest generations in terms of the unkown parameter α. One can then show that the only solutions with $m_s/m_b \sim \lambda^2$ have $\alpha = 2, \frac{3}{2}, \frac{2}{3}$ or $\frac{1}{2}$. Therefore the bound $\alpha \geq 2.5$ cannot be achieved, and hence an acceptable μ_{eff} can only arise from K; if there is a contribution from W it will be too large. (We must require $q_H = 1$ or $q_H = \alpha$ in order to get a sufficiently large μ_{eff} from K.)

We next construct a model with symmetric Yukawa matrices and $q_{H_1} + q_{H_2} \neq 0$ which serves as an explicit example of how one can both solve the μ problem and account for fermion masses and mixing with the $U(1)_A$ symmetry. It incorporates and extends our results in Ref. [4]. The model assumes symmetric $U(1)_A$ charge assignments for simplicity (but could be generalized to the case of asymmetric charges). It provides a fundamental explanation for why even in the third generation, the masses m_b and m_τ are much less than the electroweak scale: these masses are generated by higher-dimension operators, in contrast to the top mass, which arises from a dimension-4 operator. Hence, Y_{33}^d and Y_{33}^L are both $\sim \lambda^2 Y_{33}^u$. We found a solution with two SM–neutral fields χ, χ' with same sign $U(1)_A$ charge. Normalizing the charge of one field, χ, to be 1, our solution is

$$q[\chi'] = \frac{3}{2}, \quad \alpha_1 = -\frac{4}{3}, \quad \alpha_2 = \frac{1}{6}, \quad \delta_t = 0, \quad \delta_b = -1 \tag{39}$$

which leads to

$$Y^u \sim \begin{pmatrix} \lambda^8 & \lambda^6 & \lambda^4 \\ \lambda^6 & \lambda^4 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix}, \quad Y^d \sim \lambda^2 \begin{pmatrix} \lambda^6 & \lambda^4 & \lambda^4 \\ \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix}, \tag{40}$$

after breaking of $U(1)_A$ with $< \chi > /\bar{M}_P \sim < \chi' > /M_P \sim \lambda^2$. As we showed in Ref. [4], these patterns can fit the data on quark masses and mixing.

We now wish to find appropriate leptonic charge assignments so that $Y_{33}^L \sim Y_{33}^d$ and $\det M_d \sim \det M_L$. This model will have $q_H = 1$ and an appropriate scale for μ_{eff}.

Requiring $Y_{33}^L \sim Y_{33}^d$ means $\delta_\tau = -1$ or $\delta_\tau = -3/2$. Here we look only at the case $\delta_\tau = -1$.

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Using eqs. (24) and (25), we deduce that

$$a_1 + a_2 = \alpha_1 + \alpha_2 - \frac{q_H}{6} = -\frac{4}{3}. \tag{41}$$

It is straightforward to classify all models which have $q_H = 1$ and the above $U(1)_A$ charge assignments. There are many such models corresponding to different values of a_1 and a_2; however, not all of these yield acceptable leptonic hierarchies. One model which gives an acceptable leptonic hierarchy has

$$a_1 = -\frac{7}{6}, \quad a_2 = -\frac{1}{6}, \tag{42}$$

so that

$$q(H_1 L_i e_j^c) = \begin{pmatrix} -6 & -5 & -3.5 \\ -5 & -4 & -2.5 \\ -3.5 & -2.5 & -1 \end{pmatrix}. \tag{43}$$

The allowed Yukawa couplings are then

$$Y_{ij}^L(\chi, \chi') \sim \begin{pmatrix} (\chi')^4 & (\chi')^2\chi^2 & \chi'\chi^2 \\ (\chi')^2\chi^2 & (\chi')^2\chi & \chi'\chi \\ \chi'\chi^2 & \chi'\chi & \chi \end{pmatrix}, \tag{44}$$

giving rise to effective Yukawa terms

$$Y^L \sim \lambda^2 \begin{pmatrix} \lambda^6 & \lambda^6 & \lambda^4 \\ \lambda^6 & \lambda^4 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix}, \tag{45}$$

There is no superpotential term of the form $H_1 H_2 \chi^n (\chi')^m$, for any $n, m \geq 0$ but there is a Kähler term

$$K \ni a\frac{\chi^\dagger}{M_P} H_1 H_2 + h.c., \tag{46}$$

which leads to

$$\mu_{eff} \sim a\lambda^2 m_{3/2}. \tag{47}$$

Once the previous charge assignments are given, eqs. (18) are three linear equations for the average generational $U(1)_A$ charges. These equations must be compatible with the solutions (15). For example, for the second set of solutions in (15) we find

$$x = -\frac{29}{15}, \quad y = -\frac{6}{5}, \quad z = \frac{8}{15}, \tag{48}$$

for the explicit model given. These charge assignments have the feature that the lepton and baryon number–violating superfield terms

$$\eta^i L_i H_2, \quad \lambda^{ijk} u_i^c d_j^c d_k^c, \quad \lambda'^{ijk} L_i Q_j d_k^c, \quad \lambda''^{ijk} L_i L_j e_k^c, \tag{49}$$

(where the coefficients are functions of χ, χ') are forbidden in perturbation theory, since one cannot render these terms invariant under $U(1)_A$ using integral powers of fields. For example,

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$q(L_1 H_2) = -5/6$, $q(L_2 H_2) = 1/6$, $q(L_3 H_2) = 5/3$, so there is no integer power of χ and/or χ' which could couple to these in a U(1)$_A$–invariant manner.

In conclusion, we have discussed a class of models of fermion masses and mixing based on a string-motivated U(1)$_A$ gauge symmetry in which there is a natural solution to the μ problem. This solution implies a profound connection between the fact that $\mu \sim v_{EW}$ and the observed pattern of fermion masses.

References

[1] C. Muñoz, in it SUSY-94, International Workshop on Supersymmetry and Unification of Fundamental Interactions, eds. C. Kolda and J. Wells (University of Michigan, 1994), p. 337.

[2] L. Ibañez, Phys. Lett. **B303** (1993) 55 (hep-ph/9205234).

[3] L. Ibañez and G. G. Ross, Phys. Lett. **B332** (1994) 100 (hep-ph/9403338).

[4] V. Jain and R. Shrock, Phys. Lett. **B352** (1995) 83 (hep-ph/9412367).

[5] P. Binétruy and P. Ramond, Phys. Lett. **350** (1995) 49 (hep-ph/9412385); P. Ramond, hep-ph/9506319 (UFIFT-HEP-95-7).

[6] E. Dudas, S. Pokorski, and C. A. Savoy, hep-ph/9504292 (SPhT T95/027, MPI-PTh 95-33).

[7] M. Green and J. Schwarz, Phys. Lett. **B149** (1984) 117.

[8] M. Dine, N. Seiberg and E. Witten, Nucl. Phys. **B289** (1987) 589; J. Atick, L. Dixon, and A. Sen, Nucl. Phys. **B292** (1987) 109; M. Dine, I. Ichinose, and N. Seiberg, Nucl. Phys. **B293** (1987) 253.

[9] G. F. Giudice and A. Masiero, Phys. Lett. **B206** (1988) 480.

[10] J. Casas and C. Muñoz, Phys. Lett. **B306** (1993) 288; E. J. Chun, J. E. Kim, and H. P. Nilles, Nucl. Phys. **B370** (1992) 105.

[11] I. Antoniadis, E. Gava, K. S. Narain, and T. R. Taylor, Nucl. Phys. Nucl. Phys. **B432** (1994) 187.

[12] V. Jain, Phys. Lett. **B351** 481 (1995) (hep-ph/9407382).

[13] J. Bagger and E. Poppitz, Phys. Rev. Lett. **71** 2380 (1993); J. Bagger, E. Poppitz and L. Randall, hep-ph/9505244.

[14] M. K. Gaillard and V. Jain, Phys. Rev. **D49** (1994) 1951; M. K. Gaillard, Phys. Lett. **B347** (1995) 284.

[15] J. Ellis and M. K. Gaillard, Phys. Lett. **B88** (1979) 315.

[16] S. Weinberg, in *A Festschrift for I. I. Rabi* (New York Academy of Sciences, 1977); H. Fritzsch, Phys. Lett. **B73** (1978) 317; Nucl. Phys. **B155** (1979) 182.

[17] A. E. Faraggi, Phys. Lett. **B274** (1992) 47; ibid., **B278** (1992) 131; ; Nucl. Phys. **B387** (1992) 239; ibid. **B403** (1993) 101.

[18] S. Hawking, Phys. Lett. **B195** (1987) 337; Phys. Rev. **D37** (1988) 904; S. Giddings and A. Strominger, Nucl. Phys. **B306** (1988) 890; S. Coleman, Nucl. Phys. **B307** (1988) 867.

[19] J. Nir, hep-ph/9504312 (WIS-95/18).

[20] S. Chaudhuri, S. W. Chung, G. Hockney, and J. Lykken, hep-ph/9501361 (FERMILAB-PUB-94/413-T).

[21] G. L. Kane, R. G. Stuart, and J. D. Wells, hep-ph/9505207 (UM-TH-94-16A).