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IS THERE A MONOPOLE PROBLEM?

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Abstract

We show that there exists a range of parameters in $SU(5)$ theory for which the GUT symmetry remains broken at high temperature, thus avoiding the phase transition that gives rise to the overproduction of monopoles. The thermal production of monopoles can be naturally suppressed, keeping their number density below the cosmological limits.

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A. Introduction

It has been known for a long time that the existence of magnetic monopoles (a single one would suffice) would lead to the quantization of electromagnetic charge. In grand unified theories based on a simple group (or their products), the electromagnetic charge is necessarily quantized and thus the magnetic monopoles are the necessary outcome of the theory. This, what should be a blessing, is however precisely what makes these theories incompatible with standard cosmology.

Namely, it is believed that at high temperature in the early universe the spontaneously broken grand unified symmetry gets restored. If so, during the subsequent phase transition the monopoles get produced via the well-known Kibble mechanism [1] whenever the original symmetry based on a simple group G gets broken down to a subgroup H which contains (at least one) $U(1)$ factor. The trouble is that the resulting monopole number density n_M would then be some ten orders of magnitude bigger than the critical density of the Universe.

The crucial assumption in the above is the existence of a phase transition that separates the broken and the symmetric phase. The aim of this Letter is precisely to address this issue, namely, to see whether symmetry nonrestoration at high temperature [2, 3] can avoid the monopole problem.

Previous approaches to the solution of these problem are well known. One is of course inflation [4]. Unfortunately, no satisfactory model of inflation resulting from a realistic particle physics theory exists at present, and in view of this it is of extreme importance to study alternative possibilities. Among "noninflationary" attempts we want to cite the one by Langacker and Pi [5] who have argued that a period of "temporarily" broken $U(1)_{em}$ in some high temperature interval may avoid the problem, due to a rapid annihilation of monopoles (produced in a phase transition at higher T) during this period.

In the present paper we want to take a more radical approach and argue that the phase transition which would produce the monopoles may not take place at all. The fact that symmetries may remain broken at High T was already noticed [2, 3], and recently [6] it was shown that this effect may avoid the domain wall problem even in the minimal schemes of physically important discrete and continuos global symmetries, such as CP or Peccei-Quinn symmetry. However, symmetry nonrestoration is not a priory enough to solve the problem, since unwanted defects can be produced by thermal fluctuations. In the case of domain walls and global axionic strings, it was shown [6] that thermal production can be naturally suppressed for a wide range of parameters. However, there is a crucial difference in the case of monopoles: domain walls (or axionic strings) are global defects and can be produced by gauge singlet fields, therefore there is a rather large choice of parameters for the suppression of their production rate. The scenario for monopoles turns out to be dramatically different and more restrictive, since it is controlled by the value of the gauge couplings.

The important question for us is whether or not (and under which conditions), the symmetry gets restored in the minimal realistic GUTs. Here we analyze the usual prototype grand unified theory based on the $SU(5)$ gauge group in its canonical form. The heavy Higgs field responsible for the $SU(5)$ breaking, is taken to be in the **24** -dimensional adjoint representation H_{24}, and the light Higgs fields that break the standard model symmetry must belong to the **5** and **45** -dimensional representations Φ_5 and Ψ_{45}. The minimal model is normally taken to consist of Φ_5 only; whereas the minimal realistic theory of fermion masses is believed to require the existence of Ψ_{45} too.

What is crucial for the monopole problem is whether or not the vev of H_{24} vanishes at high temperatures. In the minimal model case we find that $< H_{24} > \neq 0$ at high T seems to be in conflict with the validity of perturbation theory, whereas including Ψ_{45} we find that the symmetry nonrestoration is possible for a wide range of the parameters.

Of course, avoiding the phase transition with $SU(5)$ nonrestoration does not automatically solve the problem. At high T monopoles can be thermally produced in e^+e^- collisions. Fortunately, his analysis shows that for $m_M/T \geq 35$ or so (where m_M is the monopole mass) the relic number density of monopoles is perfectly compatible with cosmology. We have studied the impact of this constraint on the broken $SU(5)$ theory at high temperature and our analysis puts the minimal model in serious trouble, whereas once again the more realistic version with Ψ_{45} works out right.

Thus, our work seems to indicate that the monopole problem is not an inevitable consequence of grand unification, but rather a dynamical question which depends on the spectrum and the parameters of the theory.

B. $SU(5)$ theory at low and high T

Case a: the minimal model

We first study the high T behavior of the minimal $SU(5)$ theory with H_{24} and Φ_5 Higgs fields (we drop their subscripts hereafter). At $T = 0$ the Higgs potential is

$$\begin{aligned} V &= -m_H^2 TrH^2 + \lambda_1 (TrH^2)^2 + \lambda_2 TrH^4 \\ &- m_\Phi^2 \Phi^\dagger \Phi + \lambda_\Phi (\Phi^\dagger \Phi)^2 \\ &- \alpha \Phi^\dagger \Phi TrH^2 - \beta \Phi^\dagger H^2 \Phi \end{aligned} \tag{1}$$

where

$$H = \sum_{a=1}^{24} H_a \lambda_a \tag{2}$$

and $T_a = \lambda_a/2$ are the generators of $SU(5)$ for the **5** dimensional representation such as Φ. The desired symmetry breaking $SU(5) \xrightarrow{<H>} SU(3)_C \times SU(2)_L \times U(1)_Y$ with

$$< H >= v_H \,\mathrm{diag}(1, 1, 1, -3/2, -3/2) \tag{3}$$

implies the conditions

$$\lambda_2 > 0 \ \ , \quad 30\lambda_1 + 7\lambda_2 > 0 \ \ ; \quad \beta > 0 \tag{4}$$

When the final stage of symmetry breaking is turned on through $< \Phi^T >= (0, 0, 0, 0, v_\Phi)$, the minimum conditions require further

$$\lambda_\Phi > 0 \ \ , \quad (30\lambda_1 + 7\lambda_2)(40\lambda_2\lambda_\Phi - \frac{9}{2}\beta^2) - 3(10\alpha + 3\beta)^2 > 0 \tag{5}$$

The conditions (4) and (5) play a crucial role in the study of the $SU(5)$ phase diagram at high T. The computation of the effective Higgs potential at high T is rather complicated, but our task is facilitated by focusing on the leading terms of order T^2. Namely, we are interested in the high T phase diagram of $SU(5)$ for $T \gg m_H$, and then we need the form of the T^2-dependent mass terms for the H and Φ fields.

In the approximation of weak couplings, assuming the validity of perturbation theory one can use the general expression given by Weinberg [2]

$$\Delta V(T) = \frac{T^2}{24}\left[\left(\frac{\partial^2 V}{\partial\varphi_i \partial\varphi^i}\right) + 3(T_a T_a)_{ij}\,\varphi^i\varphi^j\right] \tag{6}$$

where T_a are the group generators and φ_i are the real components of the fields. For our potential this gives

$$\begin{aligned} \Delta V(T) &= \frac{T^2}{24}\Big\{(48\lambda_\Phi - 96\alpha - \frac{96}{5}\beta + \frac{36}{5}g^2)\Phi^\dagger\Phi \\ &+ (208\lambda_1 + \frac{376}{5}\lambda_2 - 20\alpha - 4\beta + \frac{15}{2}g^2)TrH^2\Big\} \\ &\equiv m_\Phi^2(T)\Phi^\dagger\Phi + m_H^2(T)TrH^2 \end{aligned} \tag{7}$$

The above form has already been given in Ref [7]. Now, since $\beta > 0$ and α too is allowed to be positive, one cannot make any a priori statements about the signs of the mass terms above. Actually, it was already noticed [7] that (7) allows for a negative mass for Φ, thus enabling the non-restoration of the $SU(2)_L \times U(1)$ symmetry. Since this is achieved at the expense of α, β being positive, it is easily seen that the coefficients in (7) make the nonrestoration of H much harder to achieve.

Notice first that the conditions (4) and (5) cannot allow both mass terms in (7) negative; but what about the coefficient of H? It turns out that the nonrestoration of $< H >$ seems to require $\lambda_\Phi > 1$ and thus invalidates the weak-coupling expression (7). To see what is going on let us look at the simplified problem with λ_2 and β small. The conditions (4) and (5) now read ($\lambda_H = \lambda_1$)

$$\lambda_H > 0\ , \quad \lambda_\Phi > 0\ , \quad 4\lambda_H\lambda_\Phi > \alpha^2 \tag{8}$$

and $m_H^2(T) < 0$ requires

$$\alpha > \frac{52}{5}\lambda_H + \frac{3}{8}g^2 \tag{9}$$

It is easy to see that (8) and (9) imply

$$\lambda_\Phi > \left(\frac{26}{5}\lambda_H + \frac{3}{16}g^2\right)^2 \frac{1}{\lambda_H} \tag{10}$$

and λ_Φ as a function of λ_H has a minimum at

$$\frac{26}{5}\lambda_H = \frac{3}{16}g^2 \tag{11}$$

Thus we have a lower limit for λ_Φ

$$\lambda_\Phi \geq \frac{39}{10}g^2 \tag{12}$$

Taking a typical value $g^2/(4\pi) \simeq 1/50$, this means

$$\lambda_\Phi \geq 1 \tag{13}$$

Clearly, the weak coupling limit of (7) ceases to be justified.

Of course, the full computation must include the couplings α and β, and this requires a numerical analysis. We have performed it, and the end result is that (13) is not modified much. The point is that the couplings λ_1, λ_Φ and α enter with the largest coefficients in (7), and thus it is more or less their role to determine whether or not the $SU(5)$ symmetry may remain broken at high T ($T \gg m_H$)

Case b: including Ψ_{45}

We have seen above that the requirement of the validity of the perturbation theory points towards the usual assumption of the restoration of the $SU(5)$ symmetry. Now, the analysis was performed for the minimal $SU(5)$ model with the light Higgs Φ being **5**-dimensional. But the minimal theory suffers from the problem of the fermionic spectrum being non realistic, namely whereas $m_b \simeq m_\tau$ can be considered a success, this relation fails badly for the first two generations. It is generally believed that the realistic $SU(5)$ theory must contain at least a **45**- dimensional multiplet (Ψ) needed to cure this problem. This prompted us to perform the above analysis for this, what should be considered a minimal realistic theory. Now, from the expression for the high T mass term in (7), it is clear (as we already remarked) that it is easier to keep the vev of the smaller representation nonrestored, since α enters in its mass term with a much larger coefficient.

The analysis with **45** parallels the one performed above, and of course it gets even more messy. For the sake of space and since it worked well above, we present the computation in the limit of λ_2 and β small (and the analogous couplings for Ψ_{45} also small), i.e. we keep only α, λ_H and λ_Ψ with λ_Ψ defined as in (1). More precisely, if we decompose Ψ into 90 real (45 complex) fields Ψ_i, we can write $V(H,\Psi)$ as in (1) with $\Phi^\dagger\Phi$ substituted by $\sum_{i=1}^{90}\Psi_i^2$.

Again, from the general form in [2], one can easily deduce the mass terms for Ψ and H at high T

$$\begin{aligned} m_\Psi^2(T) &= \left(368\lambda_\Psi - 96\alpha + \frac{96}{5}g^2\right)\frac{T^2}{24} \\ m_H^2(T) &= \left(208\lambda_H - 180\alpha + \frac{15}{2}g^2\right)\frac{T^2}{24} \end{aligned} \tag{14}$$

Our point about the dimension of the representation and the nonrestoration of its vev is manifest in (14): it is clearly much easier to keep $< H >$ nonzero at high T (than $< \Psi >$). With the condition for the boundedness of the potential

$$\lambda_H > 0\ ,\quad \lambda_\Psi > 0\ ,\quad 4\lambda_H\lambda_\Psi > \alpha^2 \tag{15}$$

we now obtain (instead of (10)

$$\lambda_\Psi > \left(\frac{26}{5}\lambda_H + \frac{3}{16}g^2\right)^2\frac{1}{81\lambda_H} \tag{16}$$

Thus we get (instead of (12))

$$\lambda_\Psi \geq \frac{13}{270}g^2 \tag{17}$$

Clearly λ_Ψ is allowed to remain small, while having $< H > \neq 0$ at $T > m_H$.

Switching on other couplings in the potential does not change the results drastically. The bottom line is that $SU(5)$ may remain broken at high T, thus avoiding the phase transition which leads to the disastrous overproduction of monopoles.

C. The monopole density

As we mentioned in **A**, the nonrestoration of symmetry, although necessary, is not sufficient to guarantee the non overabundance of monopoles. Monopoles can be thermally produced in e^+e^- (and other charged particles) collisions, and from the analysis by Turner [8] we know that their density depends crucially on m_M/T at these high temperatures. He finds out that in order to be consistent with cosmology, we need

$$\frac{m_M}{T} \geq 35 \tag{18}$$

More precisely, for $m_M/T \geq 20$, he finds out

$$\frac{n_M}{n_\gamma} \simeq 3 \times 10^3 \left(\frac{m_H}{T}\right)^3 e^{-2m_M/T} \tag{19}$$

where n_γ is the photon density; and from the upper limit $n_M/n_\gamma \leq 10^{-24}$, one obtains (18)

Now, in $SU(5)$ [9] the lightest monopoles weigh

$$m_M = \frac{4\pi}{g^2} M_X \tag{20}$$

where M_X is the mass of the superheavy gauge bosons $M_X = \sqrt{\frac{25}{8}} g v_H$, and so

$$m_M = \frac{10\pi}{\sqrt{2}g} v_H \tag{21}$$

For $g^2/(4\pi) \simeq 1/50$ or $g \simeq 1/2$, $m_M \simeq 40 v_H$, and thus the consistency with the cosmological bound (18) implies

$$\frac{v_H}{T} \geq 1 \tag{22}$$

From (1) and (7)

$$\frac{v_H^2}{T^2} = -\frac{208\lambda_1 + \frac{376}{5}\lambda_2 - 20\alpha - 4\beta + \frac{15}{2}g^2}{12(30\lambda_1 + 7\lambda_2)} \tag{23}$$

for $T \gg m_H$.

Obviously (22) and (23) will put even more restrictive conditions on the parameters of the theory (than just (12) or (17)). In any case, the analysis is straightforward and we quote the results for the simplified models with only λ_Φ (λ_Ψ), λ_H and α couplings in the Higgs potential (1).

a. Let us see first what happens for the minimal model with Φ_5. For $\lambda_1 = \lambda_H$, from (8), (22) and (23) we get

$$\alpha > \frac{142}{5}\lambda_H + \frac{3}{8}g^2 \tag{24}$$

and

$$\lambda_\Phi > \frac{213}{20}g^2 \tag{25}$$

For $g^2 \simeq 1/4$, $\lambda_\Phi \geq 2.7$ and the perturbation theory clearly fails.

b. We repeat the same for the more realistic version with the Ψ_{45} representation. As before (compare with (12) and (17)), the condition (25) relaxes by a factor of 1/81, and we get

$$\lambda_\Psi > \frac{213}{1620}g^2 \tag{26}$$

which for $g^2 \simeq 1/4$ would give

$$\lambda_\Psi > \frac{1}{30} \tag{27}$$

Thus, the largest coupling of the theory λ_Ψ is still quite small and the perturbation theory is operative.

In summary, whereas in the minimal model, at least in perturbation theory, the monopole problem persists, in the more realistic version we see that it may not be there. Since the realistic theory requires the existence of *both* Φ_5 and Ψ_{45}, the nonrestoration of $< H >$ and the non overabundance of monopoles produced becomes only easier to achieve.

Unfortunately, from the exponential nature of the monopole density in (19), it is clear that due to the uncertainty in the Higgs couplings we cannot predict precisely the monopole density.

Summary and outlook

Our results seem to indicate that the problem of monopoles may not be generic to GUTs. Whether or not there is an overabundance of monopoles is directly tied up to whether the GUT symmetry is restored or not, and our analysis shows that the symmetry nonrestoration is in general allowed, but it depends on the spectrum and the couplings of Higgs scalars.

We have studied this issue in the prototype theory of all GUTs, the $SU(5)$ model, and found out that the problem persists in its minimal version with the **5**-dimensional light Higgs, but that the more realistic variant with a **45**-dimensional Higgs included eliminates (potentially) the problem.

We wish to say a few words about the generality and the meaning of our results

i) We believe that the $SU(5)$ theory, in order to be realistic, demands the existence of Φ_5 and Ψ_{45}, first to make the fermion spectrum realistic and second to offer hope for a consistent unification of the gauge couplings. This is why we have included Ψ_{45} and we find our result as yet another indication in favor of the extended Higgs sector. Note that this is still consistent with the minimal fine-tuning and the minimal standard model (with one Higgs doublet) being the low energy description of electro-weak interactions.

ii) It should be stressed that the known solutions to the monopole problem require going beyond the minimal realistic theory. For example, inflation would be a simple way out if it could be implemented in a simple GUT scenario. Before this is achieved, it cannot be considered as a true solution. On the other hand, the idea of breaking $U(1)$ at high T requires the presence of more light Higgs doublets which amounts to additional fine-tuning in the context of GUTs.

iii) Unlike inflation, the symmetry non-restoration scenario does not result in a negligible present-day value of the monopole number density. Thus, the possibility remains open for monopoles to be the required dark matter. Whether or not the monopole density is large enough to allow for experimental detection is again related to the spectrum of the theory.

iv) The important question is what happens in the supersymmetric version of the theory, which is favored from the point of view of the hierarchy problem and the unification of couplings. Unfortunately, at the level of the leading T^2 analysis for small gauge couplings, it has been shown [10] (in the context of global supersymmetry) that internal symmetries get restored at high T. This would imply the existence of the monopole problem in SUSY GUTs. It is worth investigating, though, the generality of these results, with for example the non-leading "daisy" diagrams contributions to the high T behavior of the theory, but this is beyond the scope of this paper.

v) What about other GUTs, such as $SO(10), E(6)$...? It should be clear from our discussion that the situation will depend on the Higgs spectrum of the theory. In many popular models one assumes the existence of a large representation, such as say **126** in $SO(10)$, used to provide the mass for the right handed neutrino. Obviously, the presence of such a large number of fields will help the nonrestoration of the GUT symmetry. We leave the analysis of the extended theories (with more detail on the high T analysis) for a longer paper in preparation.

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