TUW - 95 - 13
PITHA - 95/10
gr-qc/9507011

Explicit Global Coordinates for Schwarzschild and Reissner-Nordstrøm

THOMAS KLÖSCH*

Institut für Theoretische Physik
Technische Universität Wien
Wiedner Hauptstr. 8-10, A-1040 Vienna
Austria

THOMAS STROBL◇

Institut für Theoretische Physik
RWTH-Aachen
Sommerfeldstr. 26-28, D-52056 Aachen
Germany

Abstract

We construct coordinate systems that cover all of the Reissner-Nordstrøm solution with $m > |q|$ and $m = |q|$, respectively. This is possible by means of elementary analytical functions. The limit of vanishing charge q provides an alternative to Kruskal which, to our mind, is more explicit and simpler. The main tool for finding these global charts is the description of highly symmetrical metrics by two-dimensional actions. Careful gauge fixing yields global representatives of the two-dimensional theory that can be rewritten easily as the corresponding four-dimensional line elements.

June 1995

*e-mail: kloesch@tph.tuwien.ac.at
◇e-mail: tstrobl@pluto.physik.rwth-aachen.de

The purpose of this letter is twofold: First we want to present the Reissner-Nordstrøm (RN) and the Schwarzschild (SS) solution within one global coordinate system. It is remarkable that in both cases this is possible by means of elementary functions only. For the SS metric there certainly exist already the global Kruskal-Szekeres coordinates [1], in which

$$ds^2 = \frac{32m^3}{r} e^{-r/2m}(dT^2 - dX^2) - r^2 d\Omega^2 \tag{1}$$

where $r \equiv r(X,T)$ is defined *implicitly* by $[(r/2m) - 1]\exp(r/2m) = X^2 - T^2$ and $d\Omega^2 = (d\vartheta^2 + \sin^2\vartheta d\varphi^2)$ denotes the standard metric on the two-sphere. Our SS line element, on the other hand, has the simple explicit form (cf. also Fig. 1)

$$ds^2 = -8m\left[dxdy + \frac{y^2}{xy+2m}dx^2\right] - (xy+2m)^2 d\Omega^2 \,. \tag{2}$$

Also the transformation to the standard SS-form is straightforward in both directions (cf. Eqs. (30) and (31) below). Our line element for RN will be not that concise, but still it contains only nonsingular ratios of trigonometric functions and provides an elementary description of RN within one analytical chart (cf. also Fig. 2).

Certainly in the first place such global coordinate systems will be of educational importance. Still it is to be expected that also one or the other calculation of relevance for modern research will be facilitated by the use of simple global coordinates.

Our second interest, related to the first one as outlined below, shall be an investigation of two-dimensional gravity theories, more precisely, of (reformulated) generalized Dilaton gravities [2, 3]

$$S[g,\phi] = -\tfrac{1}{2}\int_M d^2x\sqrt{-\det g}\;[\phi R - V(\phi)]\,. \tag{3}$$

Here $g = g_{\mu\nu}dx^\mu dx^\nu$ is understood to be a Minkowskian metric on a *two*-manifold M with coordinates x^μ, $\mu = 0, 1$, R is the corresponding Levi-Civita curvature scalar, ϕ is a function on M (the Dilaton field), and V is some given smooth potential. Using an Einstein-Cartan formulation, (3) may be rewritten into

$$S[e^a,\omega,\varphi_a,\phi] = \int_M \varphi_a\, De^a + \phi\, d\omega - \tfrac{1}{2}V(\phi)\, e^+\wedge e^- \,, \tag{4}$$

where $e^a\,, a \in \{+,-\}$, is the *zweibein*, $g = 2e^+e^-$, and $\varepsilon^a{}_b\,\omega$ is the *spin*-connection[1]. (The equivalence between (4) and (3) becomes obvious when realizing that the variation with respect to the Lagrange multiplier fields φ_a yields just the torsion zero condition $De^a \equiv de^a + \varepsilon^a{}_b\omega\wedge e^b = 0$; the remaining part of the action then coincides with (3) since $R = 2 * d\omega$, where '$*$' denotes the Hodge dual).

[1]In our conventions $\varepsilon^{+-} = 1$ and, e.g., $\varphi_\pm = \varphi^\mp$.

In [4] and, more explicitly, in [5] it is shown that a model of the type (4) may be solved very efficiently when reinterpreting it as a Poisson σ-model, i.e. a σ-model where the target space N is a Poisson manifold. The particular action (4) turns out to correspond to a target space $N = I\!R^3$ carrying the Poisson bracket $\{\varphi^+, \varphi^-\} = \frac{1}{2}V(\phi)$, $\{\varphi^a, \phi\} = \varepsilon^a{}_b\varphi^b$. With the use of Poisson structure adapted coordinates on N, the equations of motion (e.o.m.) may be solved locally within some lines. It is shown here that within the more elementary approach of gauge fixation the solution to the field equations may be immediate as well. And for the purpose of constructing solutions in explicit form that have global validity *without* patching (if existent), the present method proves even preferable.

The relation between the two and the four dimensions comes in as follows. It is well-known that locally the RN-solution may be characterized by

$$ds^2 = h_{RN}(r)dt^2 - \frac{1}{h_{RN}(r)}dr^2 - r^2 d\Omega^2 \tag{5}$$

$$h_{RN}(r) = 1 - \frac{2m}{r} + \frac{q^2}{r^2}\,. \tag{6}$$

On the other hand, any local solution to the field equations of (3) is of the form [6, 7, 5]

$$g = h(r)dt^2 - \frac{1}{h(r)}dr^2\,, \qquad \phi = r\,, \tag{7}$$

with

$$h(r) = \int^r V(u)du + const. \tag{8}$$

(where the indetermination in the integration of (8) gives rise to a one-parameter family of distinct solutions). So we may conclude that (4) with the potential

$$V^{RN} = h'_{RN}(\phi) \equiv \frac{2m}{\phi^2} - \frac{2q^2}{\phi^3} \tag{9}$$

can be taken to describe RN (respectively SS for $q = 0$), if the four-dimensional metric $g_{(4)}$ is determined via

$$g_{(4)} = g - \phi^2 d\Omega^2\,. \tag{10}$$

The strategy of this letter is the following: We pick some gauge that may be enforced *globally* on space-times of a sufficiently large class so as to cover RN and SS. In this gauge we solve the e.o.m. of (3) or (4). As a first exercise we recover the local result (7), or (5), respectively. Thereafter, however, we carefully investigate the residual gauge freedom of the chosen gauge so as to obtain global charts. The representatives are then specified to SS and RN.

To break the diffeomorphism invariance we choose the light cone gauge

$$g = 2dx^0 dx^1 + k(x^0, x^1)(dx^1)^2 \,. \tag{11}$$

The region of validity of (11) can be found as follows: First we label one set of null lines on M by the x^1-coordinate. This implies $g_{00} = 0$. Then we choose the second coordinate x^0 to coincide with some affine parameter along these null extremals. This may be seen to yield g_{01} independent of x^0. As an affine parameter is determined only up to linear transformations, which in our case may be x^1-dependent, one may absorb $g_{01}(x^1)$ into x^0 by an appropriate rescaling of the affine parameter. The result is (11). Obviously this procedure works on any part of the two-dimensional space-time M that may be foliated into null-like lines. Thus in particular it will work all over (two- or four-dimensional) RN and SS space-time (cf. also Fig. 2a).

Describing the metric (11) by a *zweibein*, clearly one of the two one-forms $e^\pm$ has to have a vanishing coefficient function in front of dx^0. A globally attainable choice of the Lorentz frame then brings the *zweibein* into the form:

$$e^+ = dx^1 \,, \quad e^- = dx^0 + \tfrac{1}{2} k \, dx^1 \,. \tag{12}$$

Let us now determine the function k in the gauge (12) by means of the e.o.m. of (4). Beside torsion zero and $R = V'(\phi)$ the e.o.m. are

$$\begin{aligned} d\phi + \varepsilon_{ab} \varphi^a e^b &= 0 \\ d\varphi_a + \varepsilon_{ab} \varphi^b \omega + \tfrac{1}{2} \varepsilon_{ab} \, V(\phi) \, e^b &= 0 \,. \end{aligned} \tag{13}$$

From $De^+ = 0$ we learn at once that $\omega_0 = 0$. Then the x^0-components of (13) (with $a = -$) become $\partial_0 \varphi^+ = 0$ and $\partial_0 \phi = \varphi^+$, respectively. This yields

$$\varphi^+ = F(x^1) \,, \quad \phi = F(x^1) x^0 + G(x^1) \,, \tag{14}$$

for some functions F and G. Next, the Eqs. (13) may be seen to imply $V(\phi) d\phi = d(\varphi^2)$ where $\varphi^2 \equiv \varphi^a \varphi_a$. So

$$\varphi^2 \equiv 2 \varphi^+ \varphi^- = h(\phi) \,, \tag{15}$$

where the function h has been defined in (8) already. Writing out the one-components of the field equations (13) with $(a = -)$,

$$\partial_1 \varphi^+ + \tfrac{1}{2} V + \varphi^+ \omega_1 = 0 \,, \tag{16}$$

$$\partial_1 \phi + \varphi^- - \tfrac{1}{2} \varphi^+ k = 0 \,, \tag{17}$$

we realize that the second of these equations determines k in terms of F, G, and the integration constant of (15):

$$k = 2\frac{F'x^0 + G'}{F} + \frac{h(Fx^0 + G)}{F^2}\,. \tag{18}$$

The functions F and G are not completely unrestricted, however. From (15) and (14) we infer

$$h\left(G|_{F=0}\right) = 0\,. \tag{19}$$

Similarly (16) yields

$$F'|_{F=0} = -\tfrac{1}{2}V|_{F=0} \equiv -\tfrac{1}{2}h'\left(G|_{F=0}\right)\,, \tag{20}$$

where we used (8) for the second equality. In the case of simple zeros of F (sufficient for analyzing SS and non-extreme RN) it is straightforward to verify that these restrictions suffice to render k smooth for smooth choices of F and G. (19) and (20) are, moreover, the *only* restrictions on F and G beside smoothness. This will become obvious when analyzing the residual gauge freedom left by (11) and (12). Indeed it will be found below that, for a fixed number of (simple) zeros of F, any set of functions F and G respecting (19) and (20) are related to each other by gauge transformations.

On parts of M that may be foliated by null-extremals, (11, 18–20) provides the general solution of the field equations of (3) or (4) already (parametrized by the integration constant in (8) and the number of zeros of F). Let us mention here that the gauge (12) works similarly efficient, if one allows V in (4) to depend on φ^2, too. Such an action yields solutions with nontrivial torsion then. Also, similarly one might have used the ('Hamiltonian') gauge $e_0^- = 1, e_0^+ = 0 = \omega_0$ [8], which is attainable globally as well, if M can be foliated by null-lines. The use of a conformal gauge seems less advisible here, however.

We now turn to an analysis of the residual gauge freedom of our gauge conditions: First, (11) remains uneffected, if an x^1-dependent linear transformation of the affine parameter x^0 is compensated by a diffeomorphism in the x^1-variable:

$$\begin{aligned} x^0 &= \frac{1}{f'(\tilde{x}^1)}\tilde{x}^0 + l(\tilde{x}^1)\,, \\ x^1 &= f(\tilde{x}^1)\,, \quad f'(\tilde{x}^1) \neq 0\,. \end{aligned} \tag{21}$$

An additional Lorentz transformation

$$e^- \to f'(\tilde{x}^1)\, e^-\,, \quad e^+ \to \frac{1}{f'(\tilde{x}^1)}\, e^+ \tag{22}$$

is necessary to restore (12). For F and G in (14) this implies the following equivalence relations:

$$F(x) \sim \frac{F(f(x))}{f'(x)}\,, \quad G(x) \sim G(f(x)) + F(f(x))\, l(x)\,. \tag{23}$$

As a warm up in the study of (23) let us first consider local patches on M with $\varphi^+ = F \neq 0$ (in which case (19) and (20) are empty). On such a patch $F \sim 1$, because the differential equation $f'(x) = F(f(x))$ may be solved for a monotonous f. The subsequent choice $f(x) = x$ and $l(x) = -G(x)$ in (23) yields $G \sim 0$, furthermore. So on patches with $\varphi^+ \neq 0$ we may put $\varphi^+ = 1$ and $\phi = x^0$ by residual gauge transformations ($\leftrightarrow$ $F = 1\,,\ G = 0$). Then (18) reduces to

$$k(x^0, x^1) = h(x^0) \quad \text{(locally)}. \tag{24}$$

Thus locally the metric g takes the generalized Eddington-Finkelstein form (10, 11, 24). This form is particularly well-suited for the construction of Penrose diagrams (cf., e.g., [5]). It breaks down at zeros of φ^+. On patches restricted further by $\varphi^- \neq 0$, the diffeomorphism

$$r := x^0\ , \quad t := x^1 + \int^{x^0} \frac{dz}{h(z)} \tag{25}$$

brings g into the generalized Schwarzschild form (7) (with h unmodified), thus confirming the statement on local solutions made in the introduction. The integration constant in h is then left as the only locally meaningful parameter that cannot be gauged away. (In the case of (9) this parameter may be seen to effectively rescale m and q). The range of such patches within a Penrose diagram becomes clear when noting that according to (15) Killing horizons are labelled by zeros of $\varphi^\pm$, cf. Figs. 2.

Let us now discuss (23) in the general setting of zeros of F, restricting ourselves at a first stage, however, to the case of simple zeros of F. Maybe it is worth mentioning that solving the first relation (23) is equivalent to a classification of all C^∞-vectorfields $v := F(x)d/dx$ on a real line up to diffeomorphisms. One finds that the zeros of F may be shifted along the $x \sim x^1$-coordinate line at will, while their total number cannot be changed.[2] Also it is not difficult to verify that the linear coefficients $F'|_{F=0}$ may not be changed by the symmetry relations (23). For the case of an open interval for the variable $x \sim x^1$ this is all that remains for F: The number of its zeros together with the corresponding slopes at those points. For the case of simple zeros of F the second equivalence relation (23) may be solved readily also: G may be 'deformed' in an arbitrary way except for those points where F vanishes; there the value of G remains fixed.

All the features of F and G that may not be changed by the gauge transformations (23) are determined already by (20) and (19) (except for the number of zeros of F and

[2] Actually for $x \in I\!R$ there exist also coordinate transformations that boost zeros of F into infinity. This, however, leads to coordinate singularities at 'infinity'.

possibly a further discrete choice in solving (19)). For means of completeness let us mention here also that in the case of a closed line $x \sim x^1$, describing 2D-gravity solutions to the field equations with cylindrical topology, there remains one further unrestricted parameter in the analysis of (23). The appearance of a second continuous parameter (beside the integration constant hidden in h) for cylindrical space-time topology is important in the context of a Hamiltonian analysis of the theory (4) (or (3)). It reveals that the reduced phase space will be two-dimensional; the conservation of zeros of F indicates nontrivial topology of this phase space. More details on such aspects may be found in [5], [6].

Picking representatives F and G in the global setting, where zeros of F are not excluded, we will restrict ourselves to the SS and RN case (9) in the following. Also the integration constant in (8) shall be chosen such that h coincides with h_{RN}. The generalization to arbitrary h will be obvious, however. We begin with the simpler case $q = 0$ describing SS: Here h has one zero at $2m$. Thus (19) implies $G|_{F=0} = 2m$ and (20) $F'|_{F=0} = -1/4m < 0$. As a consequence F may have one zero only. The residual gauge transformations then allow to set

$$F(x^1) = -\frac{x^1}{4m}\,, \quad G(x^1) = 2m\,. \tag{26}$$

Insertion into (18) yields

$$k = \frac{2(x^0)^2}{x^0x^1 - 8m^2}\,, \tag{27}$$

or, as $\phi = 2m - x^1x^0/4m$, the four-dimensional line element (2) after the rescaling $x := x^1$, $y := -x^0/4m$.

Maybe it is worth mentioning that by a coordinate rescaling the mass parameter of SS (or also of RN) may be separated into an overall conformal factor. Here this implies that instead of (2) we may write also

$$ds^2 = -4m^2\left[4d\widetilde{x}d\widetilde{y} + \frac{4\widetilde{y}^2}{\widetilde{x}\widetilde{y}+1}d\widetilde{x}^2 + (\widetilde{x}\widetilde{y}+1)^2 d\Omega^2\right]. \tag{28}$$

Certainly it is not difficult to find the local coordinate transformation that maps (2) into the standard Schwarzschild form (5). First the transformation[3]

$$x^0 := \phi = xy + 2m\,, \quad x^1 := -4m\,\ln|x| \tag{29}$$

[3] It is found most easily when noting that, for $x \neq 0$, (24) is obtained from transforming F into 1 and ϕ into x^0 ($\leftrightarrow G = 0$). The first equation of (29) is then obvious and the second one results from $x = f(x^1)$ and $F(f(x^1)) := f'(x^1)$, cf. Eqs. (21, 23). (The coordinates x^μ in (29) are, of course, different from those in (26,27)!)

brings (2) into Eddington-Finkelstein form (24). The subsequent transformation (25), which becomes $r := x^0$, $t := x^1 + x^0 + 2m \ln|x^0 - 2m|$ here, yields the Schwarzschild form. Putting these transformations together, one has

$$\begin{aligned} r &= xy + 2m \\ t &= xy + 2m(1 + \ln|y/x|) \,. \end{aligned} \tag{30}$$

So, obviously not only the resulting line element (2), but also the transformation from the standard SS-form to it, has a particularly simple form. Inverting (30), one finds

$$\begin{aligned} |x| &= \sqrt{|r-2m|} \exp\left(\frac{r-t}{4m}\right) \\ |y| &= \sqrt{|r-2m|} \exp\left(\frac{t-r}{4m}\right) \,. \end{aligned} \tag{31}$$

It is straightforward to determine the radial null-directions of (2). They are $x = z_1$ and $y \exp(xy/2m + 1) = 8m\, z_2$, respectively, with z_1, z_2 constant. Clearly, if we introduced z_1 and z_2, or, more precisely, $X = (z_1 + z_2)$ and $T = (z_1 - z_2)$, as new coordinates, the metric (2) would be brought into the Kruskal-Szekeres form (1). However, the causal structure may be captured completely in the x, y-coordinates already, cf. Fig. 1.

Let us turn to the RN case $q \neq 0$ with $m > |q|$ next. In this case $h(r) = h_{RN}(r)$ has two simple zeros at

$$r_\pm = m \pm \sqrt{m^2 - q^2} \,. \tag{32}$$

Labelling the zeros of F by $x^1 = n\pi$, $n \in Z$, F has the form

$$F(x^1) = \alpha(x^1) \sin x^1 \tag{33}$$

with $\alpha \neq 0$. Then (19) becomes $G(n\pi) \in \{r_-, r_+\}$, which is respected, if we choose, e.g., $G = \frac{r_+ + r_-}{2} - \frac{r_+ - r_-}{2} \cos x^1$, i.e.

$$G(x^1) = m - \sqrt{m^2 - q^2} \cos x^1 \,. \tag{34}$$

The restriction (20), on the other hand, is one for α now: $\alpha(n\pi) = -\frac{1}{2} h'_{RN}(r_-) =: \alpha_-$ for n even and $\alpha(n\pi) = \frac{1}{2} h'_{RN}(r_+) =: \alpha_+$ for n odd. A simple possible choice for α is

$$\alpha(x^1) = C_1 + C_2 \cos x^1 \tag{35}$$

with $C_{1,2} = \frac{\alpha_- \pm \alpha_+}{2}$, i.e. with $C_1 = \sqrt{m^2 - q^2}(2m^2 - q^2)/q^4$ and $C_2 = (m^2 - q^2)\, 2m/q^4$. The full four-dimensional RN metric is then given by

$$\begin{aligned} ds^2 &= 2dx^0 dx^1 + \frac{2\partial_1 \phi + h_{RN}(\phi)/F}{F} (dx^1)^2 - \phi^2 d\Omega^2 \,, \\ \phi &= F(x^1) x^0 + G(x^1) \,, \end{aligned} \tag{36}$$

with the functions F and G as above and h_{RN} as in (6). The result is depicted in Fig. 2a.

The coefficient function k in front of the $(dx^1)^2$-term of (36) is manifestly analytic all over M, as, by construction, it is a non-singular quotient of analytical functions only. Actually, it is instructive to regard k in the vicinity of the saddle points of ϕ at $r_\pm$ (where $F = 0$). For this purpose we replace F and G in (34) and (33) by their linear approximations close to the zeros of F:

$$F = \pm\alpha_\pm x^1 \ , \quad G = r_\pm \, . \tag{37}$$

With such a choice k takes the form

$$k = \frac{2{\alpha_\pm}^2 x^0 x^1 \pm 4 r_\pm \alpha_\pm + 1}{(\pm\alpha_\pm x^0 x^1 + r_\pm)^2} (x^0)^2 \, , \tag{38}$$

generalizing the chart (2) for Schwarzschild. Indeed, $r_+|_{q=0} = 2m$, $\alpha_+|_{q=0} = -1/4m$ so that in the '+'-chart of (38) k is seen to reduce to (27) for $q = 0$.[4] The representation (38) together with $\phi = \pm\alpha_\pm x^0 x^1 + r_\pm$ may be used to describe RN within a maximal region containing one zero of $\varphi^+ = F(x^1)$, cf. Fig. 2b.

Extreme RN, characterized by $m = |q|$, does not follow from (34, 33) as a limit. One of the reasons for this is that we required the (coordinate-)distance between two adjacent simple zeros of F to be π. In the case of extreme RN F has *two*-fold zeros, however: $F'|_{F=0} = 0$, cf. Eq. (20). This may be described successfully by a limiting procedure $m \to |q|$ only, if the (coordinate-)distance between two adjacent zeros of F shrinks with $m \to |q|$, while the distance between such pairs of zeros of F remains finite. But there are also further restrictions for a finite result (cf., e.g., Eq. (39) below). Thus, for simplicity, we regard extreme RN to be treated by our gauge fixing method as an independent case by itself.

As F has a two-fold zero now, the analysis of (23) becomes more involved. Still it is straightforward to see that in such a case there are two further quantities at each zero of F which are invariant under the transformations (23), namely, the ratios between G' and F'' as well as between F''' and $(F'')^2$. Again these quantities are fixed by the equations of motion and everything else is pure gauge (except certainly, as before, $G|_{F=0}$, $F'|_{F=0}$, and the number of zeros of F). E.g. differentiation of (16) with respect to x^1 yields

[4] Certainly (27) may be obtained also directly from (34, 33) by an appropriate limiting procedure $q \to 0$. One only has to rescale x^1 and x^0 by q and $1/q$, respectively, which also leads to a multiplication of F by $1/q$ (cf. the first Eq. (23)). One obtains $\lim_{q\to 0} G(qx^1) = \lim_{q\to 0} r_+ = 2m$ and $\lim_{q\to 0} F(qx^1)/q = \lim_{q\to 0} F'(qx^1)x^1 = \lim_{q\to 0} \alpha_+ x^1 = -x^1/4m$, in coincidence with (26).

$2F''|_{F=0} = -(V'G')|_{F=0}$, or, when labelling the zeros of F again by $x^1 = n\pi$

$$F''(n\pi) = -\frac{G'(n\pi)}{m^2}\,, \tag{39}$$

where we made use of the fact that $G(n\pi) = m$ and $V'(m) = h''_{RN}(m) = 2/m^2$. For extreme RN a possible choice for F and G is

$$\begin{aligned} F(x) &= \frac{\cos x - 1}{2m}\left(\frac{\sin x}{2} - 1\right)^2 \\ G(x) &= m\left(\frac{\sin x}{2} + 1\right)\,. \end{aligned} \tag{40}$$

Extreme RN has been presented in a global chart already in [9], but again the functions involved in the description were defined implicitly only.

Concluding, it seems remarkable to us that two-dimensional gravity models proved to be powerful tools for the construction of hitherto unknown global charts for *four*-dimensional Reissner-Nordstrøm and Schwarzschild space-time. We started with an appropriate 2D Lagrangian and picked globally attainable gauge conditions. They allowed us to solve the field equations on a global level. Exploiting the residual gauge freedom on-shell, the global representatives of SS and RN, presented in this letter, just popped up.

The models (4) are studied further in their own right in [5] (but cf. also [2, 4, 6, 7, 10, 11]).

Acknowledgement

We are grateful to H. Balasin, H.D. Conradi, and W. Kummer for encouraging us to write this letter and for discussions. Also we profitted from discussions with Peter Schaller on gauge fixing within one or the other 2D gravity-model. This work has been supported in part by Fonds zur Förderung der wissenschaftlichen Forschung (Project P-10221-PHY).

References

[1] M. D. Kruskal, *Phys. Rev.* **119** (1960), 1743–1745,
G. Szekeres, *Publ. Mat. Debrecen* **7** (1960), 285–301.

[2] D. Louis-Martinez, J. Gegenberger, G. Kunstatter, *Phys. Letts.* **B321** (1994), 193.

[3] T. Banks and M. O'Loughlin, *Nucl. Phys.* **B362** (1991), 649.

[4] P. Schaller and T. Strobl, *Mod. Phys. Letts.* **A9** (1994), 3129–3136.
and preprint *Poisson Sigma Models: A Generalization of Gravity-Yang-Mills Systems in Two Dimensions*, hep-th/9411163.

[5] T. Klösch and T. Strobl, *Classical and Quantum Gravity in 1+1 Dimensions; Part* **I**: *A Unifying Treatment, Part* **II**: *All Global Solutions, Part* **III**: *The Quantum Theory*, in preparation.

[6] T. Strobl, thesis, May 1994.

[7] J. Gegenberg, G. Kunstatter and D. Louis-Martinez, preprint *Observables for Two-Dimensional Black Holes*, gr-qc/9408015.

[8] H. Grosse, W. Kummer, P. Prešnajder and D. J. Schwarz, *J. Math. Phys.* **33** (1992), 3892.

[9] B. Carter, *Phys. Lett.* **21** (1966), 423–424.

[10] J. Gegenberg and G. Kunstatter, *Phys. Rev.* **D47** (1993), R4192,
D. Louis-Martinez and G. Kunstatter, *Phys. Rev.* **D49** (1994), 5227.

[11] P. Schaller and T. Strobl, *Quantization of Field Theories Generalizing Gravity-Yang-Mills Systems on the Cylinder*, gr-qc/9406027 or in *Lecture Notes in Physics* **436**, p. 98–122, 'Integrable Models and Strings', Eds. A.Alekseev, A.Hietamaeki, K.Huitu, A.Morozov, A.Niemi, (1994).
T.Strobl, *Phys. Rev.* **D50** (1994), 7356.

Figures

Figure 1: The Schwarzschild space-time. Left coordinates (2), right the Penrose diagram. Thin lines represent null-extremals.

Figure 2a: Maximally extended Reissner-Nordstrøm space-time for $m > |q|$. Left the Penrose diagram (to be continued periodically), right the global coordinate system (33–36). In the upper halves we have drawn the null-extremals $x^1 = const$, in the lower halves the Killing-trajectories $r \equiv \phi = const$. The usefulness of the auxiliary fields ϕ and $\varphi^{\pm}$, present in the Lagrangian (4), is obvious: While ϕ coincides with the radius function r, the (Killing) horizons are marked by the zeros of $\varphi^+ = F(x^1)$ and φ^-.

Figure 2b: Range of the different coordinate systems for RN.