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Limits on Kaluza-Klein Models from COBE Results

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Abstract

The large-angular-scale anisotropy of the cosmic microwave background radiation in multidimensional cosmological models (Kaluza-Klein models) is studied. Limits on parameters of the models imposed by the experimental data are obtained. It is shown that in principle there is a room for Kaluza-Klein models as possible candidates for the description of the Early Universe. However, the obtained limits are very restrictive and none of the concrete models, analyzed in the article, satisfy them.

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It was shown in the literature [1] that a certain (although quite narrow) class of four-dimensional inflationary scenarios agrees with the observational data [2] on the anisotropy of the cosmic microwave background radiation. However, many of the models which provide mechanisms of the inflationary expansion of the three-dimensional spatial part of the Universe arise from string or supergravity models (see, for example, [3]) formulated in the spacetime with more than four dimensions. As it is known the dynamics of the multidimensional Universe differs significantly from that of the four-dimensional one at the Early stage of the evolution, when the extra dimensions could have an important role to play. In particular, this is reflected in the spectrum of gravitational waves. The reason is basically due to the fact that, apparently, the scale factor of the space of extra dimensions was not static and was comparable to the scale of the three-dimensional spatial part at that epoch. This suggests that it is important to derive constraints on the models of the Early Universe within the framework of multidimensional (Kaluza-Klein) cosmology.

Our discussion in this article will be rather general in order to include a wider class of models, not just those which stem from superstring or supergravity theories. We expect that recent discovery of the angular variation in the temperature of the cosmic microwave background radiation (CMBR) by the Cosmic Background Explorer (COBE) will definitely shed light on the origin and nature of long-wavelength cosmological perturbations and in this way may give some evidence *pro* or *contra* the Kaluza-Klein hypothesis. In this letter we study gravitational waves generated quantum mechanically and calculate the temperature variation of the CMBR. Then we combine the result of this calculation, results of experimental observations, including the COBE data [2], and requirements of self-consistency of the multi-dimensional approach to obtain limits on parameters describing our cosmological scenario.

We consider Kaluza-Klein cosmological models with the spacetime given by the direct product $R \times \mathcal{M}_1^3 \times \mathcal{M}_2^d$. The manifold $R \times \mathcal{M}_1^3$ represents our four-dimensional Universe and we assume it to be of the Friedman - Lemaître - Robertson - Walker type with flat space hypersurfaces. The d-dimensional manifold $\mathcal{M}_2^d$ represents the space of extra dimensions, often called internal space, and it is assumed to be a compact symmetric homogeneous space. We restrict our considerations to the metrics of the form:

$$g = -\mathrm{d}t \otimes \mathrm{d}t + a^2(t)\tilde{g} + b^2(t)\hat{g},$$

where $\tilde{g}$ is the three-dimensional metric on $\mathcal{M}_1^3$ and $\hat{g}$ is the d-dimensional metric on the internal space $\mathcal{M}_2^d$. We consider physical gravitational waves, i.e. gravitational waves on $\mathcal{M}_1^3$. In our analysis we assume that the only spatial dependence is given by the eigentensors of the Laplacian on $\mathcal{M}_1^3$ labelled by the wavenumber n [4], i.e. we retain only the lowest (zero) mode on $\mathcal{M}_2^d$. In terms of the conformal time η the time-dependent amplitude of the wave can be expressed as $\nu_n(\eta) \equiv \mu_n(\eta)/f(\eta)$, where $f(\eta) \equiv \sqrt{a^2(\eta)b^d(\eta)}$. Then $\mu(\eta)$ obeys the following equation:

$$\mu_n''(\eta) + (n^2 - \frac{f''(\eta)}{f(\eta)})\mu_n(\eta) = 0. \tag{1}$$

This equation was derived and studied first in the four-dimensional cosmology, see, for example, [5]. In the multidimensional case Eq. (1) was considered in Refs. [4, 6, 7].

In the quantum-mechanical treatment [8] (see also [5]) the perturbation h_{ij} becomes an operator. If we require the amount of energy in each mode to be $\hbar\omega/2$, its general expression is the following:

$$h_{ij}(\eta,\tilde{x}^k)=4\sqrt{\pi}\frac{l_{Pl}b_{KK}^{d/2}}{f(\eta)}\frac{1}{(2\pi)^{3/2}}\int_{-\infty}^{+\infty}\mathrm{d}^3\mathbf{n}\sum_{s=1}^{s=2}p_{ij}^s(\mathbf{n})\frac{1}{\sqrt{2n}}(c_{\mathbf{n}}^s(\eta)e^{i\mathbf{n}\cdot\tilde{\mathbf{x}}}+c_{\mathbf{n}}^{s\dagger}(\eta)e^{-i\mathbf{n}\cdot\tilde{\mathbf{x}}}), \quad (2)$$

where $\tilde{x}^k$ are the coordinates on $\mathcal{M}_1^3$. We used the fact that the multidimensional gravitational constant $G^{(4+d)}$ is related to the four-dimensional one $G^{(4)}$ as $G^{(4+d)}=G^{(4)}V_d$ with the volume of the internal space V_d evaluated for $b=b_{KK}$, the present day value of the scale factor of the internal space. The polarization tensor $p_{ij}^s(\mathbf{n})$ satisfies the relations: $p_{ij}^s n^j=0$, $p_{ij}^s\delta^{ij}=0$, $p_{ij}^s p^{s'ij}=2\delta^{ss'}$ and $p_{ij}^s(-\mathbf{n})=p_{ij}^s(\mathbf{n})$. The time evolution of the operator $h_{ij}(\eta,\tilde{x}^k)$ is determined by the time evolution of the operators $c_{\mathbf{n}}^s$ and $c_{\mathbf{n}}^{s\dagger}$ which obey the Heisenberg equations:

$$\frac{\mathrm{d}c_{\mathbf{n}}}{\mathrm{d}\eta}=-i[c_{\mathbf{n}},H],\quad \frac{\mathrm{d}c_{\mathbf{n}}^\dagger}{\mathrm{d}\eta}=-i[c_{\mathbf{n}}^\dagger,H].$$

The Hamiltonian H, providing a description in terms of travelling waves, is given by:

$$H=nc_{\mathbf{n}}^\dagger c_{\mathbf{n}}+nc_{-\mathbf{n}}^\dagger c_{-\mathbf{n}}+2\sigma(\eta)c_{\mathbf{n}}^\dagger c_{-\mathbf{n}}^\dagger+2\sigma^*(\eta)c_{\mathbf{n}}c_{-\mathbf{n}}, \quad (3)$$

where $\sigma(\eta)\equiv if'/(2f)$. For $d=0$ the expressions of Ref. [9] are recovered. In the multidimensional case there is the second pump field $b(\eta)$ in addition to $a(\eta)$, which also appears in the four-dimensional case, and the production of gravitons will be different. The form of the Hamiltonian (3) explicitly demonstrates that, while the Universe expands, the initial vacuum state evolves into a squeezed vacuum state with characteristic statistical properties as discussed in Ref. [10]. The Heisenberg equations are resolved with the help of the standard Bogoliubov transformations: $c_{\mathbf{n}}(\eta)=u_n c_{\mathbf{n}}(\eta_0)+v_n c_{\mathbf{n}}^\dagger(\eta_0)$ and similar one for $c_{\mathbf{n}}^\dagger(\eta)$. It follows that then the function $\mu_n(\eta)\equiv u_n(\eta)+v_n^*(\eta)$ obeys the classical equation (1).

In order to derive bounds on parameters of cosmological models from COBE observational data we calculate the angular correlation function for the temperature variation of the CMBR caused by the cosmological perturbations (Sachs-Wolfe effect [11]). This function depends only on the angle δ between the unit vectors e_1 and e_2 pointing out in the directions of observation and can be expanded in terms of the Legendre polynomials P_l as follows:

$$<0|\frac{\delta T}{T}(e_1^k)\frac{\delta T}{T}(e_2^k)|0>=l_{Pl}^2\sum_{l=2}^{\infty}K_l P_l(\cos\delta), \quad (4)$$

where the multipole distributions K_l are equal to

$$K_l=(2l+1)l(l+1)[l(l+1)-2]b_{KK}^d\int_0^\infty \mathrm{d}nn^2\left|\int_0^{\eta_R-\eta_E}\mathrm{d}w\frac{J_{l+1/2}(nw)}{(nw)^{5/2}}g_n(\eta_R-w)\right|^2. \quad (5)$$

Here $\eta_E(\eta_R)$ is the time at which photons of the CMBR were emitted (received). The factors l_{Pl} and b_{KK} come from the normalization of the field operator (2). The function $g_n(\eta_R-w)$ is defined by the formula: $g_n(\eta_R-w)\equiv[\mu_n/(\sqrt{2n}f)]'$. These formulas will be used for the matter-dominated epoch, thus f will be taken equal to $f=ab_{KK}^{d/2}$ and the factor b_{KK} cancels out in the formula (5). Contributions due to extra dimensions enter through the function $\mu(\eta)$. The expressions (4), (5) for the four-dimensional case were obtained in [9, 12].

In this article we consider the evolution of the Universe which includes standard radiation dominated and matter dominated stages and an epoch during which the three-dimensional space $\mathcal{M}_1^3$ experiences inflationary expansion. We choose the following scenario for the behaviour of the scale factors:

1) Inflationary stage (I-stage): $\eta < \eta_1 < 0$

$$a(\eta) = l_0|\eta|^{1+\beta}, \quad b(\eta) = b_0|\eta|^{\gamma}. \tag{6}$$

2) Transition stage: $\eta_1 < \eta < \eta_2$

$$a(\eta) = l_0 a_e(\eta - \eta_e), \quad b(\eta) = \left\{ \frac{B\cosh[n_t(\eta - \eta_t)]}{a_e l_0(\eta - \eta_e)} \right\}^{2/d}.$$

3) Radiation-dominated stage (RD-stage): $\eta_2 < \eta < \eta_3$

$$a(\eta) = l_0 a_e(\eta - \eta_e), \quad b(\eta) = b_{KK}.$$

4) Matter-dominated stage (MD-stage): $\eta > \eta_3$

$$a(\eta) = l_0 a_m(\eta - \eta_m)^2, \quad b(\eta) = b_{KK}.$$

In order to assure the continuity of the scale factor of extra dimensions and its first derivative between the I-stage and the RD-stage, we have added a period, called transition period, at which $b(\eta)$ interpolates smoothly between its value at the end of inflation and b_{KK}. The corresponding behaviour of the scale factors at the I-stage in terms of the synchronous time is $a(t) \sim |t|^{(1+\beta)/(2+\beta)}$, $b(t) \sim |t|^{\gamma/(2+\beta)}$. All models with $1+\beta < 0$ (η must be negative in this case) describe inflationary expansion of the three-dimensional part of the Universe. It can be shown that the case $\beta = -2$ corresponds to the de Sitter expansion. The cases $\beta < -2$ correspond to power-law inflation, i.e. $a(t) \sim t^m$, $m > 1$, and the cases $-2 < \beta < -1$ correspond to super-inflation of the type $a(t) \sim |t|^m$, $m < -1$, $t < 0$. The function $b(\eta)$ is taken to be constant for $\eta > \eta_2$. This agrees with strong bounds on the time variation of the scale factor of extra dimensions obtained in [13] at the RD- and MD-stages. Its behaviour at the transition stage mimics a period of slowing down of the evolution of $b(\eta)$ in the process of compactification that appears in many Kaluza-Klein cosmological models (see, for example, [14]). At the moment we do not impose any restrictions on the parameter γ characterizing $b(\eta)$ at the I-stage. Our scenario is rather general since in most of the known models of Kaluza-Klein cosmology [15]-[18] the behaviour of the scale factors at the inflationary-compactification stage is of the same type as the one described by Eqs. (6).

To calculate the angular variation of the temperature of the CMBR we need to solve Eq. (1). The initial conditions on the wave amplitude, corresponding to the vacuum spectrum of the perturbations characterized by "a half of the quantum" in each mode, are the following: $\mu(\eta_0) = 1$, $\mu'(\eta_0) = -in$, where $\eta_0 < 0$ is such that $|\eta_0| \gg |\eta_1|$ [5]. some $\eta_0 << \eta_1 < 0$. Then the exact solution of Eq. (1) is equal to

1) I-stage:

$$\mu(\eta) = (n\eta)^{1/2} A H^{(2)}_{N+\frac{1}{2}}(n\eta),$$

where $H^{(2)}_{\nu}(z)$ is the Hankel function of the second kind, $N = \beta + (\gamma d)/2$ and $A = -i\sqrt{\pi/2}\exp[i(n\eta_0 - \pi N/2)]$.

2) Transition stage:

$$\mu(\eta) = B_1 e^{-iw(\eta-\eta_t)} + B_2 e^{iw(\eta-\eta_t)},$$

where $w^2 = n^2 - n_t^2$. If $n > n_t$, $\mu(\eta)$ is an oscillatory function whereas if $n < n_t$, μ is the sum of the exponentially growing and exponentially decreasing solutions.

3) RD-stage:

$$\mu(\eta) = C_1 e^{-in(\eta-\eta_e)} + C_2 e^{in(\eta-\eta_e)}.$$

4) MD-stage:

$$\mu(\eta) = \sqrt{\frac{\pi z}{2}}\left(D_1 J_{\frac{3}{2}}(z) + D_2 J_{-\frac{3}{2}}(z)\right), \tag{7}$$

where $z \equiv n(\eta - \eta_m)$. The coefficients B_i, C_i and D_i $(i = 1, 2)$ are determined by matching the solution and its first derivative.

To set the scale for η it is convenient to choose $\eta_R - \eta_m = 1$. All realistic cosmological models should give $a(\eta_E)/a(\eta_R) \approx 10^{-3}$, $a(\eta_3)/a(\eta_R) \approx 10^{-4}$ and $a(\eta_1)/a(\eta_R) = k$, where $3 \cdot 10^{-32} < k < 3 \cdot 10^{-12}$. The lower bound on k corresponds to the case when the radiation dominated expansion of the three-dimensional part of the Universe starts at the Planckian energy densities, whereas the upper one corresponds to the case when this process starts at the nuclear energy densities. From the continuity conditions on the scale factors $a(\eta)$ and $b(\eta)$ and its first derivatives $a'(\eta)$ and $b'(\eta)$ we obtain the following expressions for the parameters of the scenario in terms of β and k:

$$\begin{aligned}
\eta_1 &= 50k(1+\beta), \\
\eta_3 &= 50k\beta + 0.5 \cdot 10^{-2}, \quad \eta_e = 50k\beta, \quad \eta_m = -0.5 \cdot 10^{-2} + 50k\beta, \\
a_e &= -(1+\beta)(50k|1+\beta|)^{\beta}, \quad a_m = 50|1+\beta|(50k|1+\beta|)^{\beta}.
\end{aligned}$$

The characteristic scale l_0 in Eq. (6) is given by the relation

$$\frac{l_{Pl}}{l_0} = 25\left(\frac{l_{Pl}}{l_H}\right)(50k)^{\beta}|1+\beta|^{(1+\beta)}, \tag{8}$$

where $l_H \equiv a^2(\eta_R)/a'(\eta_R)$ is the present day Hubble radius. We take it to be equal to $l_H = 10^{61} l_{Pl}$.

It turns out that in order to get relations on the parameters η_2, η_t, B and n_t one has to solve the transcendental equation

$$\tanh\left(\sqrt{r^2(x^2-p^2)+1} - x + \operatorname{arg\,tanh}\frac{p}{x}\right) = \frac{1}{\sqrt{r^2(x^2-p^2)+1}}, \tag{9}$$

where $p = (N+1)/(\beta+1)$ and $r = (b(\eta_1)/b_{KK})^{d/2}$. This equation arises from the continuity conditions on $b(\eta)$ and $b'(\eta)$. If $x(r,p)$ is a solution of Eq. (9) for given r and p we get

$$n_t = \frac{1+\beta}{\eta_1}x(r,p) = \frac{x(r,p)}{50k}, \quad B = l_0 b_0^{d/2}\frac{\sqrt{x^2(r,p)-p^2}}{x(r,p)}|\eta_1|^{N+1}, \tag{10}$$

$$\eta_2 - \eta_1 = 50k\frac{\sqrt{r^2(x^2(r,p)-p^2)+1} - x(r,p)}{x(r,p)}, \quad \eta_1 - \eta_t = \frac{1}{n_t}\operatorname{arg\,tanh}\frac{p}{x(r,p)}. \tag{11}$$

From Eqs. (10) and (11) we see that physically acceptable solutions must satisfy $x(r,p) > |p|$. It can be shown that such solutions exist for $r > 1$ and $p < p_{crit}(r) < 1/r$ only. For example, for $r = 2$ $p_{crit} = 0.22$ and if we choose $p = 0.1$, the solution is $x = 0.373$. From eq. (11) it follows that $n_t(\eta_2 - \eta_1) = 0.907$ then.

Thus, the consistency of the scenario requires $p < 1$. This means that $\gamma > 0$ and the scale factor of the internal space decreases at the I-stage. The condition $r > 1$ means that this scale factor continues to decrease at the transition stage. Since we assume that the classical evolution of the multidimensional Universe may start at some value of $b(\eta_0)$ close to the Planck length, it is reasonable to limit our consideration to not very big values of r, otherwise b_{KK} appears to be much smaller than l_{Pl}. We restrict ourselves to the range $1 < r < 10$.

To impose further restrictions let us consider a condition on the size of the space of extra dimensions after the end of the compactification process. It is easy to show that

$$\frac{l_{Pl}}{b_{KK}} = \left(\frac{l_0}{b_0}\right)\frac{r^{2/d}}{4}S, \quad \text{with} \quad S = 100\left(\frac{l_{Pl}}{l_H}\right)|1+\beta|^{1+\beta-\gamma}(50k)^{\beta-\gamma}. \tag{12}$$

We make a natural supposition that at the beginning of the inflation all space dimensions in the early Universe were of the same order. In this article for the sake of simplicity we take $l_0 = b_0$.

The only experimental bound on the size of b_{KK} comes from the fact that no effects of extra dimensions are observed in high energy particle experiments. This, apparently, tells us that $\hbar c/b_{KK} > (1 \div 10)$ TeV. On the other hand the classical description of the background dynamics can be trusted only if b_{KK} is not much smaller than l_{Pl}. These arguments imply that S in eq. (12) should belong to the interval $10^{-16} < S < 1$ that gives certain restrictions on β and γ. However, we found that it is more convenient to analyze these restrictions together with the bounds coming from the COBE experiment. We are going to derive these bounds right now.

It can be shown that the main contribution to the multipole distributions K_l of the correlation function for the temperature variation of the CMBR , Eqs. (5), is given by long waves, namely by the waves with the wavelength equal or larger than the present day Hubble radius. Such waves have wavenumbers $n \ll n_H = 4\pi$. The approximate form of the solution for these wavenumbers is the following:

$$\mu_n(\eta) \approx -ie^{in\eta_0}50\frac{\Psi(N)}{r}(1+\beta)^{1+N}(50k)^N n^{1+N}(\eta-\eta_m)^2, \tag{13}$$

where $\Psi(N) \equiv \sqrt{\pi/2}\exp(i\pi N/2)[\cos(N\pi)2^{N+1/2}\Gamma(N+3/2)]^{-1}$.

It can be shown that for the separation angle $\delta = 0$ the variance of $\delta T/T$ can be approximately characterized by

$$< 0|\frac{\delta T}{T}(e^k)\frac{\delta T}{T}(e^k)|0 >\sim 10^{-5}h_H^2,$$

where h_H is the characteristic spectral component defined by $h(n;\eta) = l_{Pl}n|\mu(\eta)|/a(\eta)$ and evaluated at $n = n_H$, $\eta = \eta_R$ (see the discussion in Ref. [10]). Using Eqs. (8) and (13) we obtain that

$$h(n;\eta_R) = 25\left(\frac{l_{Pl}}{l_H}\right)\frac{|\Psi(N)|}{r}|1+\beta|^{(1+N)}(50k)^N n^{2+N}, \tag{14}$$

In this expression the limit of the four-dimensional case is achieved by putting $d = 0$. Then $N = \beta$, $r = 1$ and the analogous formula of ref. [1] is recovered. COBE experimental results give $(\delta T/T)_{exp} \approx 6 \cdot 10^{-6}$ [2], consequently h_H must be of the order 10^{-4}.

Thus, we have two conditions to be satisfied:

$$h_H = 10^{-4} \quad \text{and} \quad 10^{-16} \leq S \leq 1, \tag{15}$$

where S and $h(n_H)$ are given by Eqs. (12) and (14) respectively. Resolving these conditions we obtain

$$N = \frac{53}{3 + \lg(k/3)}, \tag{16}$$

$$\frac{d+2}{2}\gamma = -\frac{71 + 6\lg(k/3)}{(3 + \lg(k/3))(2 + \lg(k/3))} - \frac{\lg S}{2 + \lg(k/3)}. \tag{17}$$

When $k/3$ varies within the interval $10^{-32} < k/3 < 10^{-12}$, N changes within the bounds $-5.9 < N < -1.8$, which essentially coincide with the bounds on β coming from the analogous condition in the four-dimensional scenario [1]. In addition, one should check that the mean square value of the field is finite in the limit of small wave numbers n. This gives additional restriction $N > -2$, see [1]. From Eq. (17) we conclude that upper bound on this parameter is given by $(d+2)\gamma/2 < 0.14$. Recall that for the scenario to be consistent γ must be positive.

The region of allowed values of β and γ for $d = 6$ is presented in Fig. 1. For other d the shape of the region remains the same, however its area decreases when d grows. The consistent values of S within our scenario, i.e. those which admit positive values of γ, belong to the interval $10^{-4} \leq S \leq 1$. We would like to emphasize that $S = 10^{-4}$ corresponds to $\hbar c/b_{KK} = 10^{15}$ GeV, which is approximately the scale of the Grand Unification.

We are unaware of any model of Kaluza-Klein cosmology which agrees with the limits on β and γ obtained above. For example, among the models corresponding to our scenario, one finds that $\beta = -5/4$, $\gamma = 1/4$ for $d = 6$ in the perfect-fluid-dominated model [16], $\beta = -1.26$, $\gamma = 0.22$ in the $D = 4 + d = 11$ supergravity with toroidal compactification [17], $\beta = -14/11$, $\gamma = 1/11$ for $d = 22$ in the model of string-driven inflation [18]. It is easy to check that none of these models satisfy the bounds. Results of more complete analysis of Kaluza-Klein models will be presented elsewhere.

We would like to mention that using the exact solution for $\mu(\eta)$ and the formula (5) we can compute the multipole distributions K_l contributing to the angular correlation function, Eq. (4). However, it can be shown that the ratios K_l/K_2 are the same as in the four-dimensional case considered in [1] provided we replace β with N. Therefore, it seems that multidimensional cosmological models satisfying the first condition in Eq. (15) also give the values of K_l which roughly agree with the experimental data.

The transcendental equation (9) requires γ to be positive, that means that the size b of the internal space decreases at the I-stage. Therefore, though the effect of the transition period on the amplitude $\mu(\eta)$ is small, we see that the range of the variation for the parameter γ, allowed by the background model, depends on the details of the transcendental equation. It is quite possible that for other types of the behaviour of the scale factors during the transition period the case $\gamma < 0$ is permitted. Our analysis must be re-examined for that case. We will consider this possibility in a future publication.

In this article we considered the contribution to the angular variation in the temperature of the CMBR coming from the gravitational wave perturbations generated quantum-mechanically as a result of parametric interaction of the perturbations with strong variable background gravitational fields in the Early Universe. Due to universal character of this mechanism such wave perturbations have been generated inevitably, hence firmness of the limits (16) and (17) on β and γ, which are the main results of our paper. Contributions due to perturbations of other types (density and rotational perturbations, non-zero modes on $\mathcal{M}_2^d$) in the Kaluza-Klein cosmology should be analyzed as well, however they are beyond the scope of the present article.

The limits (16) and (17) (and their graphical representation in Fig. 1) show that the conditions of consistency and the recent data from the COBE experiment leave a room for multidimensional cosmological models as candidates for the description of the Early Universe. However, the limits are rather restrictive and we did not find any concrete model satisfying them. Taking into account further restrictions imposed, for example, by the pulsar-timing data or by the future LIGO experiment will allow to make the limits on multidimensional models more restrictive thus questioning the very validity of the Kaluza-Klein hypothesis. We would like to mention that the spectral energy density of the gravitational waves and some observational bounds on multidimensional cosmological models were studied in Ref. [7].

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Figure caption

Fig. 1 The region of values of the parameters β and γ given by the equations (16) and (17) for $d = 6$. The four presented curves correspond to various values of $y = \lg S$. The dashed straight line is given by $N = -2$. The hatched region is the region of values of β and γ allowed by the observational data and the consistency conditions discussed in the article.