Remarks on the formation of black holes in non-symmetric gravity

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Abstract

In a recent paper, we discussed the formation of black holes in non-symmetric gravity. That paper was then criticized by Cornish and Moffat. In the present paper, we address the arguments raised by Cornish and Moffat. In summary, we do not see any reason to doubt the validity of our former conclusions.

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gravitation, which incorporates General Relativity (GR) as a limiting case [1]. Recently, Cornish and Moffat (CM) [2] conjectured, based on a class of exact *static* spherically-symmetric solutions, that NGT was free of black holes. Later, Burko and Ori (BO) [3] found that there was no analogue in NGT to the Birkhoff theorem of GR [4], and realized that dynamics might have important consequences. In order to get some insight into the dynamical content of NGT, BO studied in Ref. [3] the evolution of small, linearized, anti-symmetric field over a symmetric GR background. The behavior of the linearized skew perturbation was found to be perfectly regular in the entire domain of dependence, and in particular at (and beyond) the event horizon. Based on this result, BO concluded that if, in a situation of gravitational collapse, the initial skew perturbation is sufficiently small, then a black hole is likely to form, just like in GR.

More recently, the analysis of Ref. [3] was criticized by CM [5]. In brief, the objection of CM is based on the observation that NGT lacks a Principle of Equivalence, and that this feature of NGT is not well captured by the linear approximation. CM thus argue that the linear analysis is misleading, and that the outcome of a gravitational collapse in NGT is more likely to be a compact object which is not a black hole.

In the present paper we briefly address the objection of CM. In brief, we find this objection to be unjustified, and we do not see any reason to doubt the validity of our former conclusions in Ref. [3].

The main arguments raised in Ref. [5] are the following:

1) The linear approximation to NGT is not valid, because it fails to capture an important aspect of the fully non-linear NGT – the absence of an Equivalence Principle.

2) The absence of the Equivalence Principle suggests that the perturbative approach will break at the horizon: With no Equivalence Principle, the evolving perturbations are allowed to "feel" the horizon, by "feeling" the non-symmetric field there.

3) Consider a static star surrounded by a static spherically-symmetric NGT field. The exterior field is described by the static spherically-symmetric NGT solution – the Wyman solution. On top of this background, consider some additional small NGT perturbations. Assume now that the star undergoes gravitational collapse. CM argue that the final state cannot be a black

stable, then, upon evolution, the small initial deviation from the Wyman solution must remain small. Therefore, the final state must be given by small perturbations on top of the Wyman solution. Such a final state cannot be a black hole (because the Wyman solution is not perturbatively close to any black-hole solution).

Here is our reply to the above arguments:

1) The absence of an Equivalence Principle in NGT is certainly an interesting issue. But this issue has nothing to do with the validity of the linear approach. For, in our analysis we did not make any use of the Equivalence Principle, or any assumption about its existence or inexistence in the theory.

Needless to say, the validity of a linear analysis has nothing to do with the issue of whether the system in question admits an Equivalence Principle or not. (The linear approach is used in countless number of systems in physics, and in most of these cases, this principle is simply irrelevant.)[1]

In addition, let us note that the most fundamental manifestation of the break-down of the Equivalence Principle occurs already at the linear level: The impossibility to find a local reference frame in which the metric tensor is locally Minkowski, and the existence of nontrivial metric-tensor scalars like $g^{[\alpha\beta]}g_{[\alpha\beta]}$, are faced already at the linear level. (As we have just pointed out, however, this issue is absolutely irrelevant to the validity of the linear approach!)

2) We agree that (in a somewhat vague sense) the absence of the Equivalence Principle allows the evolving perturbations to "feel" the non-symmetric field. But this has nothing special to do with the horizon: The antisymmetric field has a non-zero value not only at $r = 2m$, but also at greater (or smaller)

[1]Perhaps the idea behind the above argument by CM is that the usage of the linear approach should be limited to situations where the background, or the linear field, admits all the qualitative aspects of the non-linear system. This is certainly not the case: In many systems where small perturbations were successfully analyzed by the linear approach, there are important, or even crucial, non-linear phenomena which are absent at the linear level. As an example, consider sound waves in a perfect fluid [6]: It is the non-linear aspects which are responsible for the formation of shock waves, as well as turbulence and other chaotic phenomena – and yet the linear treatment is perfectly valid (in a compact neighborhood) if the initial perturbations are small. (A similar example is the linearized Einstein theory.) Another example: In a vibrating elastic body or string, coupling to higher harmonics occurs at the non-linear level only, and yet the linear approach is valid for the description of small vibrations.

does not alter the following simple fact: The event horizon is an absolutely regular location at the background, which has no local significance[2].

3) This argument of CM is based on the assumption that the initial state is, in some global sense, "close to Wyman", *i.e.*, that it can be described by a small perturbation over Wyman. This assumption is obviously wrong: If we denote the initial radius of the star by r_s, then the initial field is close to Wyman at $r > r_s$, but is *very* different from at $r < r_s$. Thus, while in the Wyman solution the antisymmetric field is typically very strong at $r = 2m$, in the initial state considered here it is presumably small there. In addition, the Wyman solution is vacuum and static, and the initial state considered here is neither of them (at $r < r_s$).[3] Therefore, the expectation that this configuration will evolve like Wyman with small perturbations has nothing to base on.

Let us examine this issue more carefully: Let us denote the section $r \geq r_s$ of the initial hypersurface by S. Then, since the initial data are close to Wyman on S, the standard stability arguments indeed suggest that the evolving field will be similar to Wyman throughout $D^+(S)$ – but not elsewhere (here it is important to recall that there is no Birkhoff theorem in NGT).

The region near $r = 2m$ is not included in $D^+(S)$, and is not even close to it. Therefore, there is no reason to expect that the evolving field near $r = 2m$ will by any means be similar to Wyman.

In fact, this last argument by CM can easily be reversed: If in the initial configuration described in argument (3) the initial skew field is sufficiently small[4] (which we assume), then the initial data can be viewed as a small NGT perturbation on top of a GR background. The same line of reasoning suggested by CM now implies that the evolving configuration will be just GR plus small skew perturbation – which means that a black hole will form!

[2]We also point out that the results of Ref. [3] are not a consequence of the pre-existing black hole (and event horizon). As it turns out, even if there is no black hole in the moment (*i.e.*, in the gravitational collapse of matter or gravitational radiation), the subsequent formation of a black hole is to be anticipated [3].

[3]This strong deviation of the initial state from Wyman considered here has nothing to do with the extra small perturbations added on top of the background; It is the *background* itself which is, in overall, very different from Wyman.

[4]This would be the case if the parameter s of the exterior Wyman solution is sufficiently small.

valid, because the initial data are close to a GR configuration in the *entire* initial slice (whereas, in contrast, they are close to Wyman at $r > r_s$ only).

In summary, we find no reason to doubt the validity of the linear approach of Ref. [3]. Therefore, although this linear analysis does not provide a strict mathematical proof for anything (a linear analysis almost never does), it should be regarded as firm and trustable – just like any other situation in physics in which linear analysis is used to study the behavior of small perturbations. We therefore insist that our conclusion in Ref. [3] is justified: In the situation of a gravitational collapse in NGT, if the initial skew perturbation is sufficiently small, one should expect a black hole to form – just like in GR.

References

[1] For a review on NGT see, *e.g.*, J.W. Moffat in *Gravitation – A Banff Summer Institute*, edited by R.B. Mann and P. Wesson (World Scientific, Singapore, 1991), p. 523.

[2] N.J. Cornish and J.W. Moffat, Phys. Lett. B **336**, 337-342 (1994); J. Math. Phys. **35**, 6628-6643 (1994).

[3] L.M. Burko and A. Ori, "On the formation of black holes in non-symmetric gravity," Technion Preprint TECHNION-PH-95-7, gr-qc/9506033 (to be published).

[4] This has been shown independently (although through a completely different precedure) by M.A. Clayton, "Massive NGT and Spherically Symmetric Systems," University of Toronto preprint UTPT-95-09, gr-qc/9505005 (unpublished), and by N.J. Cornish, J.W. Moffat and D. Tatarski, Gen. Rel. Grav., to appear (1995).

[5] N.J. Cornish and J.W. Moffat, "Comment on the formation of black holes in nonsymmetric gravity," University of Toronto preprint UTPT-95-14, gr-qc/9506055 (unpublished).

[6] See, *e.g.*, Landau and Lifshitz, *Fluid Mechanics*, second edition (Pergamon Press, Oxford, 1987).