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Exact quantum state for $N = 1$ supergravity

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Abstract

For $N = 1$ supergravity in $3+1$ dimensions we determine the graded algebra of the quantized Lorentz generators, supersymmetry generators, and diffeomorphism and Hamiltonian generators and find that, at least formally, it closes in the chosen operator ordering. Following our recent conjecture and generalizing an ansatz for Bianchi-type models we proposed earlier we find an explicit exact quantum solution of all constraints in the metric representation.

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Supersymmetry has long been recognized not only as a possible answer to some of the outstanding problems of particle physics and cosmology, but also as a mathematical structure whose presence, in spite of its seemingly higher complexity, can actually simplify field theories in an important manner. Striking examples are the proof of the positivity of energy or the recent proof of quark-confinement in such theories [1,2]. Another example will occur in the present work, where we find that supersymmetry helps to demonstrate formal closure of the generator algebra of the constraints after quantization. In supergravity, like in pure gravity, the study of spatially homogeneous models has recently provided some insight, limited as it may be, in the non-perturbative canonical quantization and in the physical states, in the sense of Dirac [3], solving the quantum constraints. It was found recently that infinitely many physical states exist even in such strongly reduced models [4,5]. Even earlier some states could be determined explicitely: For homogeneous supersymmetric models of Bianchi-type in class A of Ellis and McCallum [6], in the case where no matter fields beyond the Rarita-Schwinger field are present, states in the empty and full fermion sectors could be found [7-11] which have been interpreted as worm-hole states [8]. More physical states near the middle of the fermion-number spectrum, between the empty and full fermion-number state, were found to have the form [4,5] $\Psi = S^\alpha S_\alpha g(h_{pq})|0\rangle$, where h_{pq} is the metric tensor on the space-like slices of space-time, S_α, $\bar{S}_{\dot\alpha}$, are the supersymmetry generators [12] and $|0\rangle$ is the vacuum which is annihilated by the gravitino field, ${\psi_p}^\alpha|0\rangle = 0$. One of these states could again be determined explicitely and has been identified as a Hartle-Hawking state [5]. So far all attempts [13] to generalize the physical states in the empty and full fermion sector of the Bianchi models to full supergravity were unsuccessful, because it could be shown [14–16] that in full supergravity physical states in the zero fermion-number or anti-fermion-number sectors, (which we shall call the bare vacua, in the following), do not exist.

However, recently we made the conjecture [4] that the above mentioned form of the physical states about half-way between the bare fermion and anti-fermion vacua has a direct counterpart in full supergravity. It is the purpose of the present paper to substantiate this conjecture by formal calculations, which would, in fact, constitute a proof if they were made rigorous by paying due attention to appropriate regularizations and their subsequent removal when passing to the limit in the final result.

The explicit result we obtain in the sector half-way between the bare fermion and anti-fermion vacua has an amplitude which reduces to the worm-hole amplitude first found in [7,8] for Bianchi-type IX and extended in [9–11] to the other Bianchi-types in class A. As the latter states reside in one of the two bare vacua they would seem to be indirect spatially homogeneous counterparts of our general state, at best. Nevertheless they turn out to be its *only* counterparts: The direct spatially homogeneous restriction of our state including the restriction to a spatially homogeneous Rarita-Schwinger field turns out to vanish identically [5] in the fermion sector half-way between the bare vacua, even though we can show that the unrestricted state is non-zero in general.

Let us begin now by first presenting our results on the graded algebra of the constraint operators. We briefly recall the steps in the derivation of the latter (see e.g. [17,18]) and use this to fix our notation. The starting point is the Lagrangean of $N=1$ supergravity [12] in the tetrad-representation from which the time-derivative of the spin-connection is eliminated by adding an appropriate 3-surface term. A (3+1)-decomposition is performed introducing an arbitrary foliation of space-time by a continuous family of space-like 3-surfaces, which we shall assume to be compact, to avoid surface terms, for simplicity. The basic variables are then the tetrad fields ${e_i}^a(\boldsymbol{x})$, and the Rarita-Schwinger field ${\psi_i}^\alpha(\boldsymbol{x})$, ${\bar\psi_i}{}^{\dot\alpha}(\boldsymbol{x})$ where the space-like Einstein indices $i=1,2,3$ are from the middle and the Lorentz indices $a=0,1,2,3$; $\alpha=1,2$; $\dot\alpha=1,2$ from the beginning of the alphabet. n^a, a function of the ${e_i}^a$, will denote the future oriented normal vector orthogonal on the space-like 3-surfaces, $n^a n_a=-1$, $n^a e_{ia}=0$. Next, canonical momenta ${\hat p^i}{}_a(\boldsymbol{x})$, ${\hat\pi^i}{}_\alpha(\boldsymbol{x})$, ${\bar{\hat\pi}^i}{}_{\dot\alpha}(\boldsymbol{x})$ and associated Poisson brackets are defined as usual. This brings out the fact that certain additional constraints exist in this theory: the Lorentz constraints $J_{\alpha\beta}\approx 0\approx \bar J_{\dot\alpha\dot\beta}$ (where $\approx$ denotes weak equality in the sense of Dirac [3]), which turn out to be first-class constraints; and second class constraints relating the Grassmannian variables and their momenta. The second-class constraints are duly eliminated by passing from Poisson brackets to Dirac brackets, and the latter are simplified by introducing new non-canonical momenta $\bar{\pi}^i{}_{\dot\alpha} = 2\bar{\bar{\pi}}^i{}_{\dot\alpha} = -\varepsilon^{ijk} e_j{}^a \psi_k{}^\alpha \sigma_{a\alpha\dot\alpha}$, $P_-{}^i{}_a = \hat{P}^i_a - \frac{1}{2}\varepsilon^{ijk} C^{\dot\beta\alpha}_{lj} \sigma_{a\alpha\dot\alpha} \bar\psi_k{}^{\dot\alpha} \bar\pi^l{}_{\dot\beta}$ with $C^{\dot\alpha\alpha}_{ij} = \frac{1}{2h^{1/2}}\left[-ih_{ij}n^a + h^{1/2}\varepsilon_{ijk}e^{ka}\right]\bar\sigma_a{}^{\dot\alpha\alpha}$. Passing to the Hamiltonian by the usual Legendre transformation and adding the first-class constraint with its Lagrange multiplier we obtain the total Hamiltonian as the usual sum of generators multiplied by their Lagrange multipliers. After canonical quantization in the $(e_p{}^a, \bar\psi_p{}^{\dot\alpha})$-representation based on the Dirac brackets where $\bar\pi^i_{\dot\alpha} = -i\hbar\delta/\delta\bar\psi_i{}^{\dot\alpha}$, $P_-{}^i{}_a = -i\hbar\delta/\delta e_i{}^a$, the constraint operators in a conveniently (but otherwise arbitrarily) chosen operator ordering [19] become

$$\begin{aligned}
S_\alpha &= \frac{i}{2} P_-{}^i{}_a \sigma^a{}_{\alpha\dot\alpha} \bar\psi_i{}^{\dot\alpha} + \varepsilon^{ijk} e_i{}^a \sigma_{a\alpha\dot\alpha} \partial_j \bar\psi_k{}^{\dot\alpha} \\
&\quad - \frac{1}{2}\varepsilon^{ijk}(\partial_i e_j{}^a)\sigma_{a\alpha\dot\alpha}\bar\psi_k{}^{\dot\alpha}, \qquad (1)\\
\bar S_{\dot\alpha} &= \frac{i}{2} P_-{}^i{}_a C^{\dot\beta\alpha}_{ji} \sigma^a{}_{\alpha\dot\alpha} \bar\pi^j{}_{\dot\beta} + \partial_i \bar\pi^i{}_{\dot\alpha} \\
&\quad - \frac{1}{2}\varepsilon^{ijk}(\partial_i e_j{}^a) C^{\dot\beta\alpha}_{lk} \sigma_{a\alpha\dot\alpha} \bar\pi^l{}_{\dot\beta}, \\
J_{\alpha\beta} &= \frac{1}{4}(\sigma^a\bar\sigma^b - \sigma^b\bar\sigma^a)_\alpha{}^\gamma \varepsilon_{\gamma\beta}\left(e_{ia}P_-{}^i{}_b + i\varepsilon^{ijk}e_{ia}\partial_j e_{kb}\right), \qquad (2)\\
J_{\dot\alpha\dot\beta} &= -\frac{1}{4}\varepsilon_{\dot\alpha\dot\gamma}(\bar\sigma^a\sigma^b - \bar\sigma^b\sigma^a)^{\dot\gamma}{}_{\dot\beta}\left(e_{ia}P_-{}^i{}_b - i\varepsilon^{ijk}e_{ia}\partial_j e_{kb}\right) \\
&\quad - \frac{1}{2}\left(\bar\pi^i{}_{\dot\alpha}\varepsilon_{\dot\beta\dot\gamma}\bar\psi_i{}^{\dot\gamma} + \bar\pi^i{}_{\dot\beta}\varepsilon_{\dot\alpha\dot\gamma}\bar\psi_i{}^{\dot\gamma}\right).
\end{aligned}$$

It remains to write down also the diffeomorphism and Hamiltonian constraints $\bar H_{\alpha\dot\alpha}$. Here a crucial simplification due to supersymmetry occurs: On the classical level we have checked explicitely that $\bar H_{\alpha\dot\alpha}(\boldsymbol{x})\delta(\boldsymbol{x}-\boldsymbol{y}) = -2i\{S_\alpha(\boldsymbol{x}), \bar S_{\dot\alpha}(\boldsymbol{y})\}^* +$ (terms proportional to $J_{\gamma\delta}$, $\bar J_{\dot\gamma\dot\delta}$), where $\{\ldots\}^*$ denotes the usual Dirac bracket for Grassmann-odd variables. Therefore, instead of $\bar H_{\alpha\dot\alpha}$ we may equally well use generators $H_{\alpha\dot\alpha}$ which quantum-mechanically are defined via the supersymmetry generators.

$$H_{\alpha\dot\alpha}(\boldsymbol{x})\delta(\boldsymbol{x}-\boldsymbol{y}) = -\frac{2}{\hbar}\left[S_\alpha(\boldsymbol{x}), \bar S_{\dot\alpha}(\boldsymbol{y})\right]_+ \qquad (3)$$

This is the well-known supersymmetric square-root of gravity provided by supergravity [20]. Then a straightforward but, unfortunately, quite tedious algebra which fortunately closely paralles the corresponding calculation for the Bianchi models [4,5] yields the following graded generator algebra

$$
\begin{aligned}
[S_\alpha(\boldsymbol{x}), S_\beta(\boldsymbol{y})]_+ &= 0 = \left[\bar{S}_{\dot\alpha}(\boldsymbol{x}), \bar{S}_{\dot\beta}(\boldsymbol{y})\right]_+ \\
[H_{\alpha\dot\alpha}(\boldsymbol{x}), S_\beta(\boldsymbol{y})]_- &= i\hbar\delta(\boldsymbol{x}-\boldsymbol{y})(-\varepsilon_{\alpha\beta})\bar{D}^{\dot\beta\dot\gamma}_{\ \ \dot\alpha}(\boldsymbol{x})\bar{J}_{\dot\beta\dot\gamma}(\boldsymbol{x}) \\
\left[H_{\alpha\dot\alpha}(\boldsymbol{x}), \bar{S}_{\dot\beta}(\boldsymbol{y})\right]_- &= i\hbar\delta(\boldsymbol{x}-\boldsymbol{y})\varepsilon_{\dot\alpha\dot\beta}\Big[D_\alpha^{\ \beta\gamma}(\boldsymbol{x})J_{\beta\gamma}(\boldsymbol{x}) \\
&\quad + i\hbar\delta(0)\left(\bar{E}_\alpha^{\dot\gamma\dot\delta}(\boldsymbol{x})\bar{J}_{\dot\gamma\dot\delta}(\boldsymbol{x})\right. \\
&\quad \left. - n^a(\boldsymbol{x})h^{-1/2}(\boldsymbol{x})\,\sigma_{a\alpha\dot\gamma}\bar{S}^{\dot\gamma}(\boldsymbol{x})\right)\Big]
\end{aligned}
\tag{4}
$$

and the usual commutators with $J_{\alpha\beta}$, $\bar{J}_{\dot\alpha\dot\beta}$ which indeed we find to generate the infinitesimal Lorentz transformation expected from the index-structure of all the generators. The coefficients $D_\alpha^{\ \beta\gamma}$, $\bar{E}_\alpha^{\ \dot\beta\dot\gamma}$ are Grassmann-odd structure functions. Their form is similar to the result for the Bianchi models (see [5]) and need not be given here as the explicit form is not required in the following. We do note the divergent $\delta(0)$-factor in the last of eqs.(4), however, which may hide an anomaly and whose presence reduces this result to a formal one. To go beyond this level one would have to introduce a regularization first which renders $\delta(0)$ finite, then compute the commutator in the regularized theory and check that the algebra still closes when passing to the limit. This we shall not attempt here. Fortunately, the last commutator in (4) is not needed in our solution of the constraints, and the $\delta(0)$-term therefore does not appear there. The only remaining (and, in pure gravity, most difficult to evaluate) commutators $[H_{\alpha\dot\alpha}(x), H_{\beta\dot\beta}(y)]$ follow immediately from eqs.(4) by Jacobi-identities and are therefore easily obtained here. Again they evaluate to a linear combination of the generators S_γ, $\bar{S}_{\dot\gamma}$, $J_{\gamma\delta}$, $\bar{J}_{\dot\gamma\dot\delta}$, $H_{\gamma\dot\gamma}$ multiplied by structure functions from the left. Therefore we have established that, formally, the graded generator algebra is closed on the physical states, annihilated by all generators.

To determine a physical state explicitely we now follow the conjecture of [4] and make the ansatz

$$
\Psi = \prod_{(\boldsymbol{x})} S^\alpha(\boldsymbol{x})S_\alpha(\boldsymbol{x})g(\{e_i^{\ a}\})
\tag{5}
$$

containing a formal product over all (suitably discretized) space-points and, a yet undetermined bosonic functional g independent of $\bar\psi_i^{\dot\alpha}$ satisfying $J_{\alpha\beta}g = 0$, $J_{\dot\alpha\dot\beta}g = 0$. In the same way as for the Bianchi models the ansatz (5) ensures that the S_β-constraint and the $J_{\alpha\beta}$, $\bar{J}_{\dot\alpha\dot\beta}$-constraints are automatically satisfied. The $\bar{S}_{\dot\beta}$-constraint, after using the generator algebra and the properties of g, is satisfied if

$$\bar{S}_{\dot\alpha} S_\alpha g(\{e_i{}^a\}) = 0\,. \tag{6}$$

It is important to note that the operators $e_p{}^a \bar\sigma_a^{\dot\alpha\alpha} \bar{S}_{\dot\alpha} S_\alpha$ and $n^a \bar\sigma_a^{\dot\alpha\alpha} \bar{S}_{\dot\alpha} S_\alpha$,are Lorentz-invariant, i.e. commute with $J_{\gamma\delta}$, $\bar{J}_{\dot\gamma\dot\delta}$. For the Bianchi models a special solution of eq. (6) is found by solving $S_\alpha g = 0$, and the solution is in this case given by the restriction of the functional $g_0(\{e_i{}^a\}) = \exp[-\frac{1}{2\hbar}\int d^3x \varepsilon^{ijk} e_i{}^a \partial_j e_{ka}]$ to the appropriate spatially homogeneous tetrad. But in the present *general* case, $S_\alpha g = 0$ has *no* solution, as shown in [14–16]. Remarkably, however, the more general equation (6) *does* have solutions also in the present spatially inhomogeneous case, one of which is, surprisingly, again given by the functional g_0. However, while $\bar{J}_{\dot\alpha\dot\beta} g_0 = 0$ is satisfied, one checks that $J_{\alpha\beta} g_0 \neq 0$. A fully Lorentz-invariant amplitude g is obtained from g_0 only after explicit symmetrization with respect to the transformations generated by the three generators $J_{\alpha\beta}$. Thus $g(\{e_i{}^a\}) = \int D\mu[\omega] \exp(i\omega^{\alpha\beta} J_{\alpha\beta}) g_0(\{e_i{}^a\})$. Here $D\mu[\omega]$ is chosen as the formal direct product of the Haar measure of the $SU(2)$-rotation matrices $\Omega_\alpha{}^\beta = [\exp(i\omega_\cdot{}^\cdot]_\alpha{}^\beta$ with $\omega_2{}^2 = (\omega_1{}^1)^*$, $\omega_2{}^1 = -(\omega_1{}^2)^*$. The symmetrized amplitude g is still a solution of (6) because, as was already noted, the operator on the left is proportional to Lorentz-invariant operators, which therefore commute with the symmetrizing rotations. Rewriting the infinite product in (5) as a Grassmannian path-integral over a Grassmann field $\varepsilon^\alpha(\boldsymbol{x})$, applying the factors $S^\alpha(\boldsymbol{x})$ explicitely on the functional g, and using the identity

$$\exp(i\omega^{\alpha\beta} J_{\alpha\beta}) g_0(\{e_i{}^a\}) = [\exp(i\omega^{\alpha\beta} J_{\alpha\beta}) g_0^2(\{e_i{}^a\})$$
$$\exp(-i\omega^{\alpha\beta} J_{\alpha\beta})][g_0(\{e_i{}^a\})]^{-1}$$

satisfied by g_0 we obtain our exact result for the physical state Ψ in the final form

$$\Psi(\{h_{ij}, \bar\psi_i{}^{\dot\alpha}\}) = \int D[\varepsilon^1] D[\varepsilon^2] \Big\{ \exp\Big[-\int d^3x \varepsilon^{ijk}$$

$$
\Big[\varepsilon^{\alpha}(\boldsymbol{x})\partial_j\sigma^a_{\alpha\dot\alpha}\bar\psi_k{}^{\dot\alpha}(\boldsymbol{x})e_{ia}(\boldsymbol{x})+\frac{1}{2\hbar}e_i{}^a(\boldsymbol{x})\partial_j e_{ka}(\boldsymbol{x})\Big]\Big]
$$
$$
\int D\mu[\omega]\exp\Big[\int d^3x\varepsilon^{ijk}\Omega_\gamma{}^\alpha(\boldsymbol{x})(\partial_j\Omega^\gamma{}_\beta(\boldsymbol{x}))
$$
$$
\sigma_{a\alpha\dot\alpha}e_i{}^a(\boldsymbol{x})\Big(\varepsilon^\beta(\boldsymbol{x})\bar\psi_k^{\dot\alpha}(\boldsymbol{x})+\frac{1}{2\hbar}\bar\sigma_b{}^{\dot\alpha\beta}e_k{}^b(\boldsymbol{x})\Big)\Big]\Big\}. \qquad (7)
$$

That the right-hand side of eq. (7) is, indeed, a function of the 3-metric h_{ij} follows from the invariance under the Lorentz generators, which makes one free, without restriction of generality, to choose $e_i{}^a$ in the argument in the form $e_i{}^0=0$, $e_i{}^{\hat a}=q_{i\hat a}$, $\hat a=1,2,3$ where $q_{ij}=q_{ji}$ denotes the positive definite, symmetric matrix square-root of $h_{ij}=\sum_{k=1}^3 q_{ik}q_{jk}$ which is uniquely determined by h_{ij}.

Let us now briefly discuss our result. First we note that it seems to be close to but not identical with a result recently obtained by Matschull [21] along quite different lines using a new representation somewhere in-between the metric representation employed here and the Ashketar representation. Like our final result Matschull's also contains functional integrals over a spatial 2-component Grassmann field and a spatial field of $SU(2)$-rotation matrices. Among the differences with our result the most obvious one is the sign in the exponent of the unsymmetrized bosonic amplitude $g_0(\{e_i{}^a\})$. This sign would also be changed in our calculation by using a different operator ordering, but actually we have no freedom in the choice of this sign, if we wish to reproduce correctly the normalizable amplitude [22] $\sim\exp[-\frac{V}{2\hbar}m^{pq}h_{pq}]$ of the spatially homogeneous Bianchi-models [4,5,7–11]. The fact that our amplitude g reduces to this form in the spatially homogeneous case indicates that (7) should be interpreted as a worm-hole state. As the state obtained in [21] does not similarly fall off for large spatially homogenous 3-geometries, it cannot be interpreted in this way. Apart from this and the already mentioned results for the Bianchi class A models there seems to be no analytical result to compare with. It appears likely, however, and would indeed be very interesting to verify, that the semiclassical wave-functional obtained for $N=1$ supergravity with non-vanishing cosmological constant λ in the Ashketar representation [23] reduces to the present result in the (quite nontrivial) limit $\lambda\to0$. At least for the Bianchi class A models this happens to be the case [24].

The bosonic amplitude g is somewhat reminiscent of the ground-state functional of quantum electrodynamics when the latter is written in terms of the transversal part of the vector potential. To obtain an exact solution of this kind for the Wheeler DeWitt equation of quantum gravity has been an outstanding goal for a long time after it was formulated by Wheeler [25]. This (seemingly modest) goal has so far eluded its attainment in the case of pure gravity. Matschull [21] recently was able to construct a solution of the quantum gravity constraints in a new representation; however he also found that the same construction gives rise to a Lorentz non-invariant quantum correction in the Wheeler DeWitt equation. It is therefore quite remarkable (and another instance of the simplifying nature of supersymmetry) that the same obstruction does not occur in the case of supergravity, where the Wheeler DeWitt operator is replaced by $n^a \bar{\sigma}_a{}^{\alpha\dot{\alpha}} \bar{S}_{\dot{\alpha}} S_\alpha$ in which the anomalous term does not occur, and where therefore Wheeler's goal is attained by eq. (7). In view of the highly nonlinear form of gravity and supergravity it is very surprising that this result (apart from the Grassmannian component) turns out to be so similar to the Gaussian form expected for the ground state of a free field like the free electromagnetic field. Perhaps there is some hope, therefore, that other exact quantum states corresponding to gravitons (in the same sector) or pairs of gravitinos (in sectors differing by an even fermion number) may be found. To see whether this hope is justified, and also to find the explicit form of a Hartle-Hawking state (see [5,26] for the spatially homogeneous case), remain interesting open problems for future work.

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