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STRINGY UNIFICATION HELPS SEE-SAW MECHANISM

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ABSTRACT

In this paper we explore the possibility of intermediate scale physics in the context of superstring models with *higher Kac-Moody levels*, by focusing on left-right and Pati-Salam symmetries. We find that the left-right scale may lie in the range $10^{10} - 10^{12}$ GeV which is favored by neutrino physics, while the Pati-Salam scale is at most two or three orders of magnitude below the unification scale M_X. We also show that the scale of $B-L$ breaking can be as low as 1 TeV or so, providing protection against too rapid proton decay in supersymmetry. Our results allow a natural value for the scale $M_X \sim 10^{18}$ GeV and the agreement with the experiment requires the value of $\sin^2\theta_w$ at M_X to be in general very different from the usually assumed 3/8.

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1. Introduction

One of the main reasons to study supersymmetric theories is that they could alleviate the problem of the gauge hierarchy. The minimal supersymmetric extension of the standard model (MSSM) leads to a remarkable prediction [1]: the gauge couplings α_3, α_2, α_1 of $SU(3)_c$, $SU(2)_L$ and $U(1)_Y$ respectively, are unified at a scale $M_{\rm GUT} \simeq 2 \times 10^{16}$GeV [2] through the relation :

$$\alpha_3(M_{\rm GUT}) = \alpha_2(M_{\rm GUT}) = \frac{5}{3}\alpha_1(M_{\rm GUT}). \tag{1}$$

While very exciting, this result rests, however, on the hypothesis of the "Big Desert Scenario",which states that "nothing" happens between the scale of supersymmetry breaking ($\sim$ TeV) and the scale of unification ($\sim M_{\rm GUT}$). At higher energies the normalization factor $\frac{5}{3}$ of $U(1)_Y$ allows to embbedd $SU(3)_c \times SU(2)_L \times U(1)_Y$ in a Grand Unified Theory (GUT) group $SU(5), SO(10), \cdots$.

There are obvious reasons to look beyond this Big Desert Scenario. The main one is the possibility of detecting some new particles at future colliders. This needs the intermediate scale physics to be of the order of TeV, which is usually hard to obtain naturally. This is the case for example of extra gauge bosons or more exotic such as an extra-dimension.

Another important motivation for intermediate scales is the question of neutrino masses. In the MSSM neutrino masses are made to vanish by hand, through the requirement of the absence of their right handed partners. If the latter are present, the small values of neutrino masses could naturally come from the see-saw mechanism [3,4]. If right handed neutrinos get majorana masses at some scale M_{ν_R}, one expects a mass matrix of the form:

$$\begin{pmatrix} 0 & m_D \\ m_D & M_{\nu_R} \end{pmatrix} \tag{2}$$

where m_D are the Dirac masses of the neutrino. If $m_D << M_{\nu_R}$ then the masses of the right and left handed neutrinos are M_{ν_R} and $\frac{m_D^2}{M_{\nu_R}} << M_{\nu_R}$, respectively. For the see-saw mechanism to give us the neutrinos masses, the large scale M_{ν_R} should be predicted. Normally, one associates this scale with the breaking of some (gauge) symmetry. For example, this can be naturally implemented if at some intermediate scale M_I, the symmetry is enhanced to a left-right group [5][1].

[1] Hereafter, for us M_I will denote any intermediate scale: $M_W << M_I << M_X$ where M_X is the unification scale.

In the standard model, left handed quarks and leptons are doublets of $SU(2)_L$, while their right partners are singlets. In left-right models, the explicit violation of Parity is replaced with a spontaneous one, rendering our world more symmetric. In fact right handed quarks and leptons (and thus neutrinos too) appear now also in doublet representations, but under another gauge group $SU(2)_R$. To relate these models to the MSSM, one has to introduce two scales: the scale of parity breaking M_{Parity} and the scale of $SU(2)_R$ breaking M_R, with obviously $M_R \leq M_{Parity}$. It then natural to relate these scales as $M_{\nu_R} \sim M_R$ and $M_{Parity} \sim M_R$ or $M_{Parity} \sim M_X$.

The simplest realization of this idea needs the introduction of a Higgs triplet of $SU(2)_R$, usually denoted Δ_R that gets a vacuum expectation value M_R. Anomaly cancellation imposes the presence of another field $\bar{\Delta}_R$. If one wants to have M_{Parity} of the order of M_R, triplets $\Delta_L + \bar{\Delta_L}$ of $SU(2)_L$ must also be introduced. A natural question to ask is what happens to the gauge coupling unification when these new states are present.

In all previous analysis, the unification of these models was supposed to appear inside an $SO(10)$ gauge group, and the above triplets arise from **126** representations. It is easy to see that unification constraints then imply that $M_R \sim M_{GUT}$, and thus preventing the existence of such an intermediate scale. Thus these models were studied without refering to unification. Moreover, new *ad-hoc* and more complicated states are often introduced with the only purpose to lead to $SO(10)$ unification.

In this paper, we want to study the possibility of string unification with arbitrary Kac-Moody levels for these models. In fact, these simple models with triplets of $SU(2)_R$ are the first phenomenologically interesting ones, well motivated by the see-saw mechanism, that would *need stringy unification*. It is well known that this type of unification does not require the existence of a GUT group (see below). We will see below that, in contrast to all previous analysis, values of $\sin^2\theta_w$ at the unification scale very different from 3/8 could be in agreement with the experimental data. We will also use this opportunity to make some useful comments on the models.

In section 2 we review the problem of gauge couplings in string theory. This will also allow us to define our notations and strategy. In section 3 we will discuss the models and then determine some sets of values for Kac-Moody levels leading to gauge couplings unification. Comments of the results and conclusions are given in section 4.

2. Stringy unification of gauge couplings

2.1 STRING UNIFICATION AND MSSM

Superstring theory is considered today as a good candidate for the unification of all known interactions, as we hope that it contains a finite theory of quantum gravity. It is thus always important to re-address the question of unification of gauge couplings in its context.

In heterotic string theories, all the gauge (and the gravitational) couplings are given by the expectation value of a particular field: the dilaton. Moreover, the four dimensional space-time gauge symmetries are associated with an appropriate Kac-Moody algebra on the two dimensional string worldsheet [6]. To a Kac-Moody algebra corresponds its level, a positive parameter k_i (integer for non-abelian groups), which determines the corresponding tree-level gauge coupling constant α_i in terms of the four-dimensional string coupling α_{st}, $\alpha_i = \alpha_{st}/k_i$. The values of the levels also constrain the allowed unitary representations present in the chiral massless spectrum [7].

A natural question to ask then is: what are the values of the levels k_3, k_2, k_1 associated with the standard model gauge groups $SU(3)_c\, SU(2)_L$, and $U(1)_Y$ respectively? In most of the models built up to now $k_3 = k_2 = 1$, while $k_1 \geq \frac{5}{3}$. These level one constructions have the nice feature to explain the presence of only singlet and fundamental chiral representations in the standard model. However, they generally suffer from the presence of fractionally charged particles in the massless spectrum at the string level. Moreover, the most popular value of $k_1 = \frac{5}{3}$ doesn't allow one to embbedd the MSSM in a GUT group because of the absence of chiral adjoint representations.

In principle arbitrary higher Kac-Moody levels ($k_3, k_2 > 1$) are allowed. However, the corresponding string models have been found very difficult to build. They also allow the presence of larger representations leading to phenomenological problems [8]. For these reasons, they have been mainly disregarded. Recently, some (still unsuccesfull) attempts have been made to build such theories, with $k_3 = k_2 = \frac{3}{5}k_1 = 2$, trying to embbedd the standard model is some GUT group and explain the *a priori arbitrary* normalization k_1 of $U(1)_Y$ [9,10].

It is worth to notice that the (field theoretical) direct unification of gauge couplings, which is the remarkable prediction of the Big Desert Scenario, takes place at a scale $M_{\rm GUT} \simeq 2 10^{16}$GeV. However, in contrast to the case in field theory, the unification scale $M_{\rm SU}$ in string models can be predicted. Within some large class of models, it was found to be $M_{\rm SU} \simeq 2\sqrt{\alpha_{st}} \times 10^{18}$GeV, nearly two orders of magnitude bigger than $M_{\rm GUT}$ [11]. Some ideas have been presented to reconcile the two scales. They fall in two categories.

In the first category, one tries to push down $M_{\rm SU}$ toward $M_{\rm GUT}$ either by invoking large string threshold corrections [12], or by arguing on the possibility of a unified evolution of the gauge couplings between these two scales. These solutions have as good feature a small ratio $\frac{M_{\rm GUT}}{M_{\rm Planck}}$, which could be associated with some explanations of the fermions mass spectrum, or with the strength of the fluctuations in the COBE observations. Unfortunately, the needed large thresholds do not seem to appear naturally.

In the second category, one tries to push $M_{\rm GUT}$ toward $M_{\rm SU}$. This involves either the modification of the hypercharge normalization (such as [2] $k_1 \simeq \frac{4}{3}$) [13], or the presence of some extra particles in some intermediate scale(s) [14]. These particles could be standard like or exotic fractionally charged ones [15]. These last possibilities are a clear abandon of the Big Desert Scenario. In fact, they seem natural solutions as string models usually contain more particles than the MSSM ones in the massless spectrum at the string level, and some of them could be lying somewhere between the TeV and the string scales.

2.2 STRINGY UNIFICATION WITH ONE INTERMEDIATE SCALE

Below, we would like to investigate the (next to minimal) situation where some intermediate scale appears corresponding to some symmetry breaking, with a minimal particle content motivated by some phenomenological reasons. We ask about possible existence of string unification for these models. This corresponds to determine if there exist a set of levels k_is compatible with it. Most of such models contain some large representations that need some high level Kac-Moody algebras [16]. With our actual knowledge of conformal field theories, building such compactifications is a challenging problem. In realistic models, we would also have to explain why and how only the wanted particles appear at the low and intermediate scales. In particular, some representations (of smaller conformal weight than the ones considered for example) could (probably would) appear and spoil our analysis.

[2] Such normalization doesn't appear in known level one constructions.

In view of the above discussion, we will not try to answer these problems, but being less ambitious, we will constrain ourselves to the analysis of the gauge coupling unification in such hypothetical [3] cases.

We restrict our analysis to the one-loop unification of gauge couplings in some particular supersymmetric models. We mainly consider the possibility of one intermediate scale M_I lying in the region between the supersymmetry breaking scale m_s and the unification scale M_X.

Below M_I, the gauge group is the standard model $SU(3)_c \times SU(2)_L \times U(1)_Y$ with corresponding Kac-Moody levels k_3, k_2,and k_1 satisfying some constraints arising from the particle content of the model below the string scale [7]. More precisely, a representation $(r_1, r_2, \cdots, r_n, q_1, \cdots, q_m)$ of $SU(N_1) \times \cdots \times SU(N_n) \times U(1)_1 \times \cdots \times U(1)_m$, of levels $k_{N_1} \cdots k_{N_n}, k_{1_1} \cdots k_{1_m}$, has a conformal weight:

$$h = \sum_{i=1}^{n} \frac{C(r_i) =}{k_{N_i} + C(N_i)} + \sum_{j=1}^{m} \frac{q_j^2 =}{k_{1_j}} \tag{3}$$

This state to be present in the string massless spectrum needs to have $h \leq 1$.

In the minimal case of MSSM content, k_3 and k_2 are positive integers while $k_1 \geq 1$. Notice that the case where all the levels go to infinity correspond to the field theory limit as the string scale goes to infinity and all the representations are allowed. Unification is meaningless in this limit.

The associated effective couplings at the supersymmetry breaking scale m_s are given, at one loop by:

$$\frac{1}{\alpha_i(m_s)} = \frac{k_i}{\alpha_{st}} + \frac{b_i}{2\pi}\ln(\frac{M_I}{m_s}) + \frac{b_i'}{2\pi}\ln(\frac{M_X}{M_I}) \tag{4}$$

where b_i and b_i' are the one-loop beta-function coefficients in the corresponding energy domains. They are given by:

$$b_i = -3C(G_i) + \sum_{\text{reps} R_i} T(R_i). \tag{5}$$

where the quadratic Casimir $C(G_i)$ of the group G_i equals N for $SU(N)$ and $N-2$ for $SO(N)$. The index $T(R_i)$ of the matter representation R_i is equal to $\frac{1}{2}$ for chiral

[3] Neither the MSSM, nor its phenomenological viable extensions (even some versions with extra chiral matter with *appropriate spectrum* needed to raise the unification scale) have been by now derived from strings.

supermultiplets in the fundamental representation of $SU(N)$, while it is given by the sum of the squares of charges in the case of $U(1)$.

The perturbative unification at the scale M_X imposes a strong constraint $\alpha_{st}/k_i < 1$, which rules out the possibility of a low M_I scale in most of our models.

We would like to have some reasonable constraints on the allowed values of k_is. We first notice that the string unification scale is predicted to be of the order of:

$$M_{\rm SU} \simeq 2\sqrt{\alpha_{st}} \times 10^{18}{\rm GeV} = 2\sqrt{\alpha_i k_i} \times 10^{18}{\rm GeV} = 2\sqrt{\alpha_3 k_3} \times 10^{18}{\rm GeV} \qquad (6)$$

We would like to keep $M_{\rm SU} \simeq 10^{18}$GeV $< M_{Planck}$. As in all our cases, $\alpha_3 \gtrsim 1/25$, this means that k_3 should not exceed a number of order 100. A stronger constraint could come if we assume the existence of an extra non-abelian group with a smaller level $k < k_3$ then $k_3 \lesssim 25k$. Moreover, another condition that could be imposed on the levels is:

$$\frac{1}{3}k_3 + \frac{1}{4}k_2 + \frac{1}{4}k_1 = \text{integer} \qquad (7)$$

which is required in order to avoid the appearance of fractionally electrically charged particles in the massless spectrum [7]. If this condition is not satisfied, these undesired particles could however still get masses of the order M_X [4]. Notice that what we mean by charge quantization is that all color singlet states have a charge which is integer multiple of the electron charge. In orbifold compactifications for instance, it has been shown that a weaker charge quantization, where the elementary charge is a fraction $1/N$ ($N \leq 12$) of the electron charge, can be imposed [172].

In (4) we have absorbed the unknown string threshold corrections (usually denoted Δ_i) in the definition of M_X which can then be different from the computed value $M_{\rm SU}$. While the *natural value of* M_X *is* $\simeq 10^{18}$GeV, for practical computations, we allow it to take values between 10^{16}GeV and (more natural value) 10^{18}GeV. The former value has the advantage of introducing naturally a small ratio in the theory and thus it is often considered as a good value in the literature.

In addition to M_X, our other inputs are of two kind:

-as experimental inputs within our one-loop approximations, the values of the strong , electromagnetic coupling constants at m_Z, $\alpha_s \equiv \alpha_3$, $\alpha_{em} = \frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}$ respectively, and the weak angle $s \equiv \sin^2\theta_w = \frac{\alpha_1}{\alpha_1+\alpha_2}$ will be taken in the range:

$$\alpha_{em} = 1/128 \qquad 0.230 \lesssim s \lesssim 0.233 \qquad \text{and} \quad 0.11 \lesssim \alpha_s \lesssim 0.13 \qquad (8)$$

[4] If they are confined at very high scale by some extra gauge factor [15, 18], or get a mass through one of the mechanisms discussed in [16]

-as theoretical inputs, we take $m_S = m_Z$, which is usually a good approximation at one loop. Below M_I, we will make the assumption that the massless spectrum is the one of the MSSM with three generations ($n_g = 3$) and two Higgs doublets ($n_H = 2$). The coefficients b_i take the values:

$$b_3 = -9+2n_g = -3 \qquad b_2 = -6+2n_g+n_H/2 = 1 \qquad b_1 = (10/3)n_g+n_H/2 = 11 \tag{9}$$

Our strategy is to solve (4) in order to get the ratios k_1/k_2 and k_3/k_2 as function of $\ln(\frac{M_X}{M_I})$:

$$\frac{k_1}{k_2} = \frac{1-s-\frac{\alpha_{em}}{2\pi}\left[b_1 \ln(\frac{M_I}{m_Z}) + (b_1'-b_1)\ln(\frac{M_X}{M_I})\right]}{s-\frac{\alpha_{em}}{2\pi}\left[b_2 \ln(\frac{M_I}{m_Z}) + (b_2'-b_2)\ln(\frac{M_X}{M_I})\right]} \tag{10}$$

$$\frac{k_2}{k_3} = \frac{\frac{s}{\alpha_{em}}-\frac{1}{2\pi}\left[b_2 \ln(\frac{M_I}{m_Z}) + (b_2'-b_2)\ln(\frac{M_X}{M_I})\right]}{\frac{1}{\alpha_s}-\frac{1}{2\pi}\left[b_3 \ln(\frac{M_I}{m_Z}) + (b_3'-b_3)\ln(\frac{M_X}{M_I})\right]} \tag{11}$$

By plotting these functions as well as the values of α_{st}/k_i which have to remain small, one can read the allowed intermediate scale and the corresponding ratios of levels.

3. Models with intermediate mass scales

3.1 THE MODELS

We want to focus on a single intermediate scale, although we shall also discuss a case with two such scales. Our analysis continue the analysis of [16] and it is parallel to the ones for $SO(10)$ unification. There are four possible rank five gauge groups at the intermediate scale, with their respective levels:

a)$SU(3)_c \times SU(2)_L \times U(1)_R \times U(1)_{B-L}$ with levels k_3, k_2, k_R and k_{B-L}. This model has two nice features, if the scale M_I lies in the TeV region. On one side it predicts the observation of a new vector boson at future colliders, and on the other hand the $B-L$ gauge symmetry forbidds the appearance of (nonrenormalizable) operators leading to fast proton decay.

b)$SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ with levels k_3, k_2, k_{2R} and k_{B-L}. In addition to the aesthetic beauty of parity as a symmetry [5], these models can explain small neutrino masses through the see-saw mechanism [4]. Two cases are a priori allowed and will be analysed below. The first case corresponds to the direct breaking to the standard model at the scale M_I. If this happens at the TeV scale, then $B-L$ symmetry protects the proton from fast decay [20]. The model predicts then the observation of extra neutral and charged gauge bosons. However, in the natural approximation that the neutrino Dirac masses are of the same order as the corresponding charged lepton masses, the see-saw mechanism leads to a spectrum of too heavy left handed neutrino masses overclosing the universe.

Namely, with $m_D \simeq m_l$ and $M_{\nu_R} \simeq 1$ TeV, we predict

$$m_{\nu_e} =\simeq 1\text{eV} \qquad m_{\nu_\mu} \simeq 10\text{keV} \qquad m_{\nu_\tau} \simeq 1-10\text{MeV} \tag{12}$$

Now, ν_τ can in principle decay through the weak currents: $\nu_\tau \rightarrow e^+e^-\nu_e$, by assuming the CKM-like matrix in the leptonic sector. The case of ν_μ is more problematic and it requires the presence of Δ_L, a left-handed analog of Δ_R [21].

On the other hand, if all ν's are lighter than 10-100 eV, then we have no problem with the overclosure of the universe, and Δ_L's are not necessarily present. In this case we have a constraint $M_{\nu_R} \geq 10^8$ GeV (assuming the relation $m_D = m_l$ as in the above).

If we wish to have the MSW explanation of the solar neutrino puzzle (through $\nu_e - \nu_\mu$ oscillations), a prefered value for the intermediate scale becomes $M_I =\simeq 10^{10}$ GeV, with

$$m_{\nu_e} =\simeq 10^{-7}\text{eV} \qquad m_{\nu_\mu} \simeq 10^{-3}\text{eV} \qquad m_{\nu_\tau} \simeq 1\text{eV} \tag{13}$$

in which case ν_τ can play a role of dark matter (or some fraction of it). Due to the uncertainties in m_D, we quote this as $M_I \simeq 10^8 - 10^{12}$ GeV. In this case, $B-L$ does not protect the proton from decaying too fast.

A spontaneous breaking of R-parity giving a vev to the sneutrino $< \tilde{\nu}_R >$ could lead to proton decay through dimension four operator in the superpotential. One then could introduce a discrete symmetry or look for models where such operators are forbidden by some string selection rules.

Another possibility to get rid of this problem is to break the group in two steps: first to $SU(3)_c \times SU(2)_L \times U(1)_R \times U(1)_{B-L}$ and then to break it again at the TeV scale to the standard model group, protecting the proton from decaying too fast. We will investigate a minimal version of this scenario too.

c)$SU(4)_c \times SU(2)_L \times U(1)_R$ with levels k_4, k_2 and k_R. This partially unified model provides a symmetry between quarks and leptons.

d)$SU(4)_c \times SU(2)_L \times SU(2)_R$ with levels k_4, k_2 and k_{2R}. This Pati-Salam partial unification is the minimal unification based on simple group of the standard group [22] and offers both left-right symmetry and quark-lepton unification.

3.2 UNIFICATION CONSTRAINTS

a) $SU(3)_c \times SU(2)_L \times U(1)_R \times U(1)_{B-L}$

The minimal content of matter is just the MSSM spectrum plus three chiral superfields with quantum numbers $\nu_R = (1, 1, -1/2, 1)$ under $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ group , which can then be identified as right handed neutrinos. The condition imposed by the presence of these states on the Kac-Moody levels is:

$$\frac{1}{4k_R} + \frac{1}{k_{B-L}} \leq 1 \tag{14}$$

The level k_1 given by:

$$k_1 = k_R + \frac{k_{B-L}}{4} \tag{15}$$

can take the standard value $k_1 = 5/3$. When one of the new fields $< \tilde{\nu}_R >$ gets a vev, it breaks one combination $U(1)'$ of $U(1)_R \times U(1)_{B-L}$ leading to an extra Z' massive vector boson at the scale M_I, while it leaves the hypercharge $U(1)_Y$ with $Y = Q_R + (B-L)/2$ unbroken, where Q_R is the generator of $U(1)_R$. How would the neutrinos get a mass in this case? One possibility is then the mechanism suggested in [19]. A see-saw mechanism is obtained through the mass matrix between the left and right neutrino and the gaugino partner of Z'. This however gives a mass only to the neutrino whose partner got a vev. The other neutrinos presumably get masses through some non-renormalizable operators.

Another possibility is to explain neutrino masses by the usual see-saw mechanism. The gauge symmetry breaking is achieved by giving vev to some extra state with gauge numbers $(1, 1, 1, -2)$ (one also introduces a $(1, 1, -1, 2)$ representation to cancel the $U(1)$ anomalies). Then one has the condition:

$$\frac{1}{k_R} + \frac{4}{k_{B-L}} \leq 1 \tag{16}$$

which implies that $k_R > 1$ and $k_{B-L} > 4$ so $k_1 > 2$. As the new state couple equally to all the neutrinos, they generate small masses to all of them through the see-saw mechanism. An important question to raise is what are the expected values for the Dirac masses. In the general case we are considering, it is not possible to make a model independent statement. As the right handed neutrinos and electrons are a priori independent, and could come from different sectors of the string compactification, having different moduli dependence, the relative Yukawa couplings could be very different. Having smaller values for m_D corresponding to ν_μ would allow a low scale M_I.

In any case the gauge couplings of $SU(3)_c \times SU(2)_L \times U(1)_Y$ are not affected and evolve in the same way as in the MSSM, with the new appropriate normalizations k_i. The scale M_I is only constrained by collider experiments, and the corresponding gauge coupling of $U(1)'$ can be now computed, because at M_X, it is equal to the one of $U(1)_Y$ ($k' = k_R + \frac{k_{B-L}}{4} = k_1$), and the associated beta-function coefficient is known: b_1 or $b_1 + 2 = 13$ in the first and second examples described above respectively. At low energies, the corresponding coupling is equal or smaller than the hypercharge one.

b) $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$

This case is more interesting because it is more restricting. The minimal matter content of the model is n'_g generations of matter representations $Q = (3, 2, 1, 1/3)$, $Q^c = (3, 1, 2, -1/3)$, $L = (1, 2, 1, -1)$, and $L^c = (1, 2, 1, 1)$ which correspond to the quarks and leptons. There is also a Higgs sector consisting in n_{22} bidoublets $(1, 2, 2, 0)$, n_{Δ_L} pairs of $SU(2)_L$ triplets $\{\Delta_L = (1, 3, 1, 2) + \bar{\Delta}_L = (1, 3, 1, -2)\}$ as well as n_{Δ_R} pairs of $SU(2)_R$ triplets $\{\Delta_R = (1, 1, 3, -2) + \bar{\Delta}_R = (1, 1, 3, 2)\}$ and a possible parity odd singlet to make $n_{\Delta_L} = 0$ [23]. The Δ_R field, when it gets a vev, breaking $SU(2)_R \times U(1)_{B-L}$ to $U(1)_Y$, gives a Majorana mass to the right handed neutrino.

The coefficients of the beta-functions of $SU(3)_c$, $SU(2)_L$, and $U(1)_Y$ above M_I are:

$$\begin{aligned} b'_3 &= -9 + 2n'_g + n_3 \qquad b_2 = -6 + 2n'_g + n_{22} + 4n_{\Delta_L} + n_2 \\ b'_1 &= -6 + (10/3)n'_g + n_{22} + 6n_{\Delta_L} + 10n_{\Delta_R} + n_1 \end{aligned} \tag{17}$$

respectively. In all of the discussions below, the numbers n_1, n_2 and n_3 will parametrize the (unknown) contributions of extra particles that could appear at this scale. Unless explicitly stated otherwise, we take $n'_g = 3$, $n_1 = n_2 = n_3 = 0$. The normalizati= on of $U(1)_Y$ is given by:

$$k_1 = k_{2R} + \frac{k_{B-L}}{4} \tag{18}$$

The minimal values of levels are $k_{2R} \geq 2$, $k_{B-L} \geq 8$ (hence $k_1 \geq 4$), $k_2 \geq 2$ if $n_{\Delta_L} \neq 0$. One can define two possible left-right symmetries: one with equal couplings for $SU(2)_L$ and $SU(2)_R$ implying $k_2 = k_{2R}$ and $k_1 \geq k_2 + 2$ and the other with different couplings $k_2 \neq k_{2R}$. In the more symmetric case, one has the constraint $\frac{k_1}{k_2} \geq 1$, the equality corresponding to $k_2 \to \infty$. This constraint is too restrictive and it leads usually to large intermediate scales. We will relax this constraint and allow for $k_2 \neq k_{2R}$. Hence, $\frac{k_1}{k_2} \leq 1$ is allowed, and the limits will come from the perturbative unification limit on α_i at the unification scale.

We have made the analysis for different particles content and different unification scales, and we present our results in tables 1, 2, 3 and the figures. In particular we studied the cases:

i) $n_{22} = 1$, $n_{\Delta_R} = 1$, and $n_{\Delta_L} = 0$. The results are plotted in figures 1 and 2. This model could be made more symmetric if one explains $n_{\Delta_L} = 0$ through the introduction = of some parity odd singlet [23]. While from the figures one can read that the intermediate scale as low as few TeV is allowed by the runnings of the couplings, the neutrino spectrum forbidds it. In fact as discussed above, one gets too heavy ν_μ which is stable because of the absence of Δ_L, thus overclosing the universe. We then have a constraint $M_I \gtrsim 10^7$ GeV as discussed above.

If one insists on the equality of left and right couplings, then for $k_2 \simeq 10$, we get $M_I \gtrsim 10^{14.5}, 10^{11}, 10^8$ GeV for $M_X = 10^{18}, 10^{17}$ and 10^{16} GeV, respectively. A possible set, for $M_X = 10^{18}$GeV, is $k_3 = 11$, $k_2 = k_{2R} = 10$ and $k_{B-L} = 8$.

ii) $n_{22} = 2$, $n_{\Delta_R} = 1$, $n_{\Delta_L} = 0$. This case is quite similar to the first one. As we are interested in large M_I ($\simeq 10^{12}$ GeV) the ratio k_1/k_2 is not sensible to the number of bidoublets as shown in figure 3. The addition of the extra bidoublet is helpful to generate a correct Cabibbo angle. We present the corresponding results in figures 4 and 5.

iii) $n_{22} = 1$, $n_{\Delta_R} = 1$, $n_{\Delta_L} = 1$. This is the minimal fully left-right symmetric model allowing for see-saw "explanation" of the neutrino masses. For this model we present our results in figures 6 and 7. From these figures, we can see that now the scale M_I can not be as low as TeV, otherwise the hypercharge coupling will blow up before the scale M_X. In order that this does not happen, we get the lower value of $M_I \simeq 10^{10}$GeV.

iv) $n_{22} = 2$, $n_{\Delta_R} = 1$, $n_{\Delta_L} = 1$. This is the most popular model. As the perturbative condition of the couplings doesn't allow small M_I, the contribution of the second bidoublet is small. The results are displayed in figures 8 and 9. The end result is again that the region $10^{10} - 10^{12}$GeV is perfectly OK.

Thus one can have a realistic left-right model in the context of strings with the MSW mechanism and ν_τ as (some portion of) the dark matter of the universe.

The tables 3 and 4 give some possible values of the Kac-Moody levels consistent with M_I of the order of $10^{10} - 10^{12}$GeV or electric charge quantization respectively.

v) We would like to investigate the possibility (from the gauge couplings unification point of view) to have a low lying $B-L$ breaking scale around the TeV, protecting the proton from decaying; and a left-right breaking scale at an intermediate scale. This two intermediate breaking scales could be achieved for example with the following set of representations (in addition to the quarks, leptons and Higgses):

Between M_I and M_X we have n_{Δ_L} pairs of $SU(2)_L$ triplets $\{\Delta_L = (1,3,1,2) + \bar{\Delta}_L = (1,3,1,-2)\}$, as well as n_{Δ_R} pairs of $SU(2)_R$ triplets $\{\Delta_R = (1,1,3,-2) + \bar{\Delta}_R = (1,1,3,2)\}$ and new triplets n_{TL} pairs of $SU(2)_L$ triplets $\{T_L = (1,3,1,0) + \bar{T}_L = (1,3,1,0)\}$ and corresponding n_{TR} pairs of $SU(2)_R$ triplets $\{T_R = (1,1,3,0) + \bar{T}_R = (1,1,3,0)\}$[5]. At M_I the neutral component of T_R gets a vev and it breaks $SU(2)_R$ to $U(1)_R$. The beta-functions coefficients are given by:

$$\begin{aligned} b'_3 &= -9 + 2n'_g + n_3 \qquad b_2 = -6 + 2n'_g + n_{22} + 4n_{\Delta_L} + 4n_{T_L} + n_2 \\ b'_1 &= -6 + (10/3)n'_g + n_{22} + 6n_{\Delta_L} + 10n_{\Delta_R} + 4n_{T_R} + n_1 \end{aligned} \tag{19}$$

For our analysis we take $n'_g = 3$, $n_1 = n_2 = n_3 = 0$, $n_{22} == 1$ or 2, $n_{\Delta_R} = 1$, $n_{\Delta_L} = 1$ and $n_{T_L} = n_{T_R} = 1$.

At M_I where the new field T_R is supposed to get a vev, some of the above particles become massive and decouple from the running of coupling constants below M_I.

In the absence of a complete analysis of the minimization of the full superpotential and lacking a known extended survival principal for these models, we take the minimal phenomenologically viable spectrum to provide the light particles. For instance, we assume that below M_I, in addition to the particle content of the standard model, only the neutral component Δ_R^0 of Δ_R, and all of Δ_L remain massless[6].

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[5] The doubling of states is dictated by the vanishing of the Fayet-Iliopoulos term when one of these fields get a vev

[6] If $\bar{\Delta}_L$ also remains massless, then $M_I \sim M_X$.

Possible values of intermediate scales and corresponding Kac-Moody levels are displayed in table 3. It is worth noticing that in the case of $M_X \simeq 10^{16}$ GeV, one has the possibility of k_1/k_2 of order 5/3 for an $M_I \simeq 10^{10}$ GeV. This allows the hope to embedd it in an $SO(10)$ GUT. However, to cure $k_3/k_2 > 1$ one has to introduce a large extra contribution n_3 for b_3, for example $n_3 = 10$ for $\alpha_s \simeq 0.13$ which thus is extremely sensible to the exact value of α_s. In short, it is possible to keep $B - L$ symmetry in the TeV region with $M_I \simeq 10^{10} - 10^{14}$GeV

c) $SU(4)_c \times SU(2)_L \times U(1)_R$

In this case, the quarks and leptons are unified in the same representations of $SU(4)_c \times SU(2)_L \times U(1)_R$ leading to predict the existence of the right neutrinos and the size of their expected Dirac masses. In addition to the quarks and leptons in the n'_g representations:

$$(4,2,1) + (4,1,-1/2) + (4,1,1/2) \tag{20}$$

and n_h electroweak Higgs:

$$(1,2,-1/2) + (1,2,1/2) \tag{21}$$

we have n_{10} pairs of representations $(10,1,-1)$ and $(\bar{10},1,1)$ necessary to break $SU(4)_c \times SU(2)_L \times U(1)_R$ to the $SU(3)_c \times SU(2)_L \times U(1)_Y$.

The standard model levels are given by:

$$k_1 = \frac{2}{3}k_4 + k_R, \qquad\qquad k_3 = k_4 \tag{22}$$

The associated beta-functions coefficients above the intermediate scale are:

$$\begin{aligned} b'_3 &= -12 + 2n'_g + 6n_{10} + n_3 \qquad b_2 = -6 + 2n'_g + n_h + n_2 \\ b'_1 &= -8 + (10/3)n'_g + n_h + 24n_{10} + n_1 \end{aligned} \tag{23}$$

We made the analysis for $n'_g = 3$, $n_1 = n_2 = n_3 = 0$, $n_h = 1$, $n_{10} = 1$. We display in figures 10 and 11 the results for $M_X \simeq 10^{18}$ GeV. In table 4, we give some values of levels allowing intermediate scale and electric charge quantization.

d) Pati-Salam group $SU(4)_c \times SU(2)_L \times SU(2)_R$

The Pati-Salam group needs large representations to get broken to the standard model one. The contribution of these particles to the running of $U(1)_Y$ would make this coupling blow up very close to M_I. In fact M_I is typically two to three orders of magnitude below M_X when the latter goes from 10^{18} to 10^{16} GeV. Hence it is not very appealing as an intermediate scale, since only for a low value of M_X it becomes interesting for neutrino physics.

4. Discussion and Conclusion

Our results are mainly qualitative, and need some comments. The maximal values of $\frac{k_1}{k_2}$ are obtained in the absence of an intermediate scale. They vary between 1.4 for $M_X \simeq 10^{18}$ GeV to $\simeq 5/3$ for $M_X \simeq 10^{16}$ GeV, confirming previous analysis for the MSSM. In the minimal models, we found that interesting values of the scale M_I are allowed. Furthermore, since renormalization group equations usually make $U(1)_Y$ coupling increase with energy faster than $SU(2)_L$ one, $\sin^2\theta_w$ at M_X is typically bigger than 3/8. Also, the necessary levels are often large, especially when one imposes the condition (7) for charge quantization. The cases of large values of k_3 could be improved if one allows the presence of an octet of $SU(3)$ for example. An increase of precision of our analysis, by improving the uncertainty on α_s and by a serious two loops analysis with the thresholds taken into account, could constraint the allowed values of the unification scale or intermediate scale for some of these simple models. For instance, in model (b i) k_3/k_2 is constant and if it takes a value like 1.02, we can not associate it with two small integers k_3 and k_2 ($k_3 \lesssim 25$).

We would like to comment about the actual status of string model building of higher Kac-Moody levels models. The actual known trick to get such models is to notice that if you take the "diagonal" part of the product of n factors of the same group G at level one, you generate a gauge group G of level n. This method cannot generate the minimal spectrum considered above. For instance, a triplet of $SU(2)$ comes from the product of two doublets and thus is always accompanied by a singlet. We have considered the effect of the additional singlets carrying $B-L$ charges on our analysis. We found a notable effect. As an example, the deviation for model (b iii) is plotted in figures 12 and 13.

We would also like to compare with the case of usual supersymmetric $SO(10)$ GUT. That case correspond to $\frac{k_1}{k_2} = \frac{5}{3}$ and $k_3 = k_2$. From the analysis of our plots, it is obvious that there cannot be an intermediate scale with the content considered in this paper. The introduction of an intermediate scale would there necessitate an *ad hoc* introduction of a set of particles necessary for *fitting* the experimental data with a possible unification in an $SO(10)$. From this point of view, the beauty of the prediction of the MSSM for a unification of couplings is totally lost.

Two kind of string constructions of these GUTs have been investigated by now, using the method sketched above. The first uses fermionic constructions [24]. In this case, it has been obtained that possible Kac-Moody levels (normalization of any *non-abelian* groups) are $k = 1, 2, 4, 8$. This allows only for ratios k_3/k_2 equal to $\frac{1}{8}, \frac{1}{4}, \frac{1}{2}, 1, 2, 4, 8$ which is not satisfied in most of our cases.

The other analysis uses orbifold constructions [10] and it indicates that some extra states, the part of the adjoint that are not eaten by the Higgs mechanism, must remain massless at the GUT scale. They are in representations:

$$(8,1,0) + (6,1,-2/3) + (6,1,2/3) + (1,3,0) + (1,3,1) + (1,3,-1) \tag{24}$$

or

$$(8,1,0) + (1,3,0) + (1,1,0) + (1,1,1) + (1,1,-1) \tag{25}$$

They should then be lying somewhere, between the GUT and the electroweak scales. We found that the addition of these particles with a unique common mass (as one intermediate scale) always destroy the GUT unification prediction.

In conclusion, we have presented an analysis of possible string unification without GUT for simple and motivated extensions of the minimal supersymmetric standard model which, in contrast to the MSSM, necessarily need a departure from the more attractive level one string constructions.

We have focused on left-right and Pati-Salam symmetries and our analysis shows that M_R can be as low as 10^5 GeV or so, and furthermore the value $M_R \simeq 10^{10} - 10^{12}$GeV interesting for neutrino physics is perfectly consistent with unification constraints. In the case of two step breaking, we can have M_R as M_I in the range $10^{14} - 10^{11}$GeV, while allowing the $B - L$ gauge symmetry to remain unbroken all the way down to TeV. Among other effects, this can save the proton from decaying too fast which is a generic problem in supersymmetric theories.

For the Pati-Salam scale, M_{PS}, we find that it has to be quite large, some two to three orders of magnitude below M_X. Only if we push M_X down to 10^{16} GeV (not so appealing to us), we can keep M_{PS} as low as 10^{13}GeV to provide an interesting intermediate scale.

As optimistic point of view would be that these models can have interesting intermediate scales, allowing a correct string unification scale and charge quantization. A pessimistic point of view is that the predicted values of the associated

Kac-Moody levels and our poor knowledge of how to build such theories make it hopeless to construct these simple models in the near future, and they can only be studied from the effective field theory point of view keeping in mind that the gauge coupling unification could be achieved through the presence of only one tree level gauge coupling in heterotic string models.

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$(n_{22},\, n_{\Delta_L},\, n_{\Delta_R})$	M_X in GeV	M_I in GeV	(k_1, k_2, k_3)
$(1, 0, 1)$	10^{18}	$10^{11.5}$	$(9, 9, 10)$
$(1, 0, 1)$	10^{17}	10^{11}	$(6, 5, 5)$
$(1, 0, 1)$	10^{16}	10^{11}	$(7, 5, 5)$
$(2, 0, 1)$	10^{18}	$10^{11.5}$	$(10, 10, 12)$
$(2, 0, 1)$	10^{17}	10^{12}	$(25/2, 10, 11)$
$(2, 0, 1)$	10^{16}	10^{12}	$(6, 4, 4)$
$(1, 1, 1)$	10^{18}	10^{12}	$(6, 8, 14)$
$(1, 1, 1)$	10^{17}	10^{11}	$(6, 6, 10)$
$(1, 1, 1)$	10^{16}	10^{10}	$(5, 4, 6)$
$(2, 1, 1)$	10^{18}	10^{12}	$(9/2, 6, 12)$
$(2, 1, 1)$	10^{17}	10^{11}	$(5, 5, 9)$
$(2, 1, 1)$	10^{16}	10^{10}	$(5, 4, 7)$

Table 1.

Examples of values of Kac-Moody levels (k_1, k_2, k_3) of $SU(3)_c \times SU(2)_L \times U(1)_Y$ allowing for $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ at an intermediate scale of order $M_I \simeq 10^{12}$ GeV.

$(n_{22},\, n_{\Delta_L},\, n_{\Delta R})$	M_X in GeV	M_I in GeV	(k_1, k_2, k_3)
$(1, 0, 1)$	10^{18}	10^{7}	$(22/3, 10, 11)$
$(1, 0, 1)$	10^{17}	10^{8}	$(12, 12, 12)$
$(1, 0, 1)$	10^{16}	$10^{8.5}$	$(19/3, 5, 5)$
$(2, 0, 1)$	10^{18}	$10^{11.5}$	$(10, 10, 12)$
$(2, 0, 1)$	10^{17}	10^{12}	$(44/5, 8, 9)$
$(2, 0, 1)$	10^{16}	10^{11}	$(44/3, 12, 13)$
$(1, 1, 1)$	10^{18}	10^{13}	$(32/3, 12, 19)$
$(1, 1, 1)$	10^{17}	10^{9}	$(8, 12, 24)$
$(1, 1, 1)$	10^{16}	$10^{8.5}$	$(12, 12, 21)$
$(2, 1, 1)$	10^{18}	10^{13}	$(32/3, 12, 22)$
$(2, 1, 1)$	10^{17}	$10^{11.5}$	$(40/3, 12, 20)$
$(2, 1, 1)$	10^{16}	$10^{11.5}$	$(52/3, 12, 17)$

Table 2.

Examples of values of Kac-Moody levels (k_1, k_2, k_3) of $SU(3)_c \times SU(2)_L \times U(1)_Y$ allowing for $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ at an intermediate scale, leading to charge quantization.

M_X in GeV	M_I in GeV	(k_1, k_2, k_3)
10^{18}	10^{15}	$(4, 6, 30)$
10^{17}	10^{13}	$(4, 4, 24)$
10^{16}	10^{11}	$(7, 4, 24)$

Table 3.

Examples of values of Kac-Moody levels (k_1, k_2, k_3) of $SU(3)_c \times SU(2)_L \times U(1)_Y$ allowing for $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ at an intermediate scale M_I and $SU(3)_c \times SU(2)_L \times U(1)_R \times U(1)_{B-L}$ at low energies.

M_X in GeV	M_I in GeV	(k_1, k_2, k_3)
10^{18}	10^{13}	$(4, 20, 18)$
10^{17}	10^{12}	$(5, 15, 12)$
10^{16}	10^{11}	$(4, 8, 6)$

Table 4.

Examples of values of Kac-Moody levels (k_1, k_2, k_3) of $SU(3)_c \times SU(2)_L \times U(1)_Y$ allowing for $SU(4)_c \times SU(2)_L \times U(1)_R$ at an intermediate scale and charge quantization.