Stable Calculations for Unstable Particles: Restoring Gauge Invariance

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Abstract

We discuss theoretical and phenomenological aspects of the use of boson propagators with energy-dependent widths in predictions for high-energy scattering processes. In general, gauge invariance is violated in such calculations. We discuss several approaches to restore gauge invariance, necessary for a reliable result. The most promising method is the addition of the relevant parts of the fermionic corrections, which fulfills all Ward identities. The numerical difference between this and other approaches is studied. A number of recommendations are given for LEP2 computations.

1   Introduction

Monte Carlo calculations for scattering processes for LEP2 and higher-energy colliders are required to have a precision of better than one percent. It is obvious that under such circumstances the assumption that the W and Z vector bosons are stable particles, produced on-shell, is a gross misrepresentation of the physics. Rather, one has to describe them as resonances, with a finite width so as to avoid singularities inside the physical phase space. In field theory, such widths arise naturally from the imaginary parts of higher-order diagrams describing the boson self-energies, resummed to all orders. This procedure has been used with great success in the past: indeed, the Z^0 resonance can be described to very high numerical accuracy. However, in doing a Dyson summation of self-energy graphs, we are singling out only a very limited subset of all the possible higher-order diagrams. It is therefore not surprising that one often ends up with a result that retains some gauge dependence.

In itself, this is not necessarily a problem if one treads wearily. An example is the situation at LEP1. Here, a careful separation of the gauge-invariant subsets can be performed, leading to a result which has no significant gauge dependence. For processes that become important at LEP2, the situation is in several cases more complicated. Since gauge invariance is intimately connected with the high-energy behaviour of the theory, it is to be expected that the effects of gauge violation become worse if the scattering process under study contains a ratio of masses, or of momentum transfers, that becomes large. An example, which we shall study in this paper, is the production and hadronic decay of a single W in the process

$$e^+e^- \to e^-\bar{\nu}_e W^+ \to e^-\bar{\nu}_e u\bar{d} \ . \tag{1}$$

Here, the electron may emit a virtual photon, whose q^2 can be as small as m_e^2, where m_e is the electron mass: with a total center of mass energy of $\sqrt{s}$ available, we have a mass ratio of $s/m_e^2 = \mathcal{O}\left(10^{11}\right)$, large enough to amplify even a tiny gauge violation in a disastrous way[1]. An other, currently academic, situation, connected with SU(2) rather than U(1)$_{e.m.}$, is the gauge cancellation which prevents the cross-section for $e^+e^- \to W^+W^-$ from blowing up for high energies.

In order to arrive at phenomenologically reliable predictions, various approaches can be followed[2]. In the first place, we may try to convince ourselves that, for the particular problem under study, the situation is actually not so

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[1] This was noted already in Ref. [1], and investigated further in Ref. [2].

[2] A few were investigated in Ref. [3].

bad. For instance, this is the case in the above-mentioned LEP1 processes. In processes like

$$e^+e^- \to \gamma, Z \to \mu^+\mu^- \quad, \tag{2}$$

there is no obvious dangerous large ratio of masses at energies around the Z mass, as the relevant ratio is s/M_Z^2. One might therefore hope that, by the imposition of a cut on the electron scattering angle in the process (1), which effectively leads to a lower bound on the q^2 of the virtual photon, the effects of gauge violation can be mitigated. This is, for instance, an implicit assumption made in the Excalibur Monte Carlo [4]. Of course, such a hope has to be borne out by comparison with a gauge-invariant calculation.

One may sidestep the problem by simply performing the calculation of the matrix elements without any width, and only at the end use some ad-hoc prescription like the following [5, 2]. Let the mass and width of a boson be given by M and Γ, respectively, and its momentum by q^μ (Γ may depend on q^2). Then, if we multiply the matrix element by $(q^2 - M^2)/(q^2 - M^2 + iM\Gamma)$, the pole at $q^2 = M^2$ is softened into a resonance, at the expense of mistreating the non-resonant parts. It should be noted, that there are examples, where this 'fudge-factor scheme' leads to deviations up to 30% [6].

Another way to sidestep the problem is to use the 'fixed-width scheme', i.e., to systematically replace $1/(q^2 - M^2)$ by $1/(q^2 - M^2 + iM\Gamma)$, also for $q^2 < 0$. This gives $U(1)_{e.m.}$-current conservation, but it has no physical motivation. In perturbation theory the propagator for space-like momenta does not develop an imaginary part. Moreover, the fixed-width approximation violates the $SU(2) \times U(1)$ Ward identities. Note, however, that this does not lead to a bad high-energy behaviour in $e^+e^- \to 4$ fermions, as the unitarity cancellations do not involve the masses of the W and Z bosons. In the case of $e^+e^- \to 6$ fermions (e.g., W_L scattering) the occurrence of W-mass dependent couplings means the unitarity cancellations are violated by a fixed width.

A minimalist's approach is to make use of the fact that the residue of the amplitude at the (complex) pole is gauge-invariant [7, 8]. One can split the amplitude accordingly, and resum only this pole. In this way higher-order corrections can be included consistently [9]. However, this 'pole scheme' breaks down near thresholds, and has problems with the radiation of photons of energy $E_\gamma \approx \Gamma$.

Finally, one may determine the minimal set of Feynman diagrams that is necessary to compensate for the gauge violation caused by the self-energy graphs, and try to include these. This is obviously the theoretically most satisfying solution, but it may cause an increase in the complexity of the matrix elements and a consequent slowing down of the numerical calculations.

For the vector bosons, the lowest-order widths are given by the imaginary parts of the fermion loops in the one-loop self-energies. It is therefore natural to include the other possible fermionic one-loop corrections [10, 11]. These fermionic contributions form a gauge-independent subset and obey all Ward identities exactly, even with resummed propagators [12]. This implies that the high-energy and collinear limits are properly behaved. In contrast to all other schemes mentioned above, the scheme proposed here does not modify the theory by hand but selects an appropriate set of higher-order contributions to restore gauge invariance.

To solve the problem of gauge invariance related to the width, we only have to consider here the imaginary parts of these diagrams[3]. This scheme should work properly for all tree-level calculations involving resonant W-bosons and Z-bosons or other particles decaying exclusively into fermions. For resonating particles decaying also into bosons, such as the top quark, gauge independence is lost. For simplicity, we take all fermions in loops to be massless in the following.

The justification, including masses and the details of the proper resummation and renormalization procedure, will be given in a later publication [12]. The method has already been mentioned, and implemented in the s and t channel as a Monte-Carlo generator for the processes $e^+e^- \rightarrow \ell\nu_\ell q\bar{q}'$, in Ref. [13]. For the special case of $q\bar{q}' \rightarrow \ell\nu_\ell\gamma$, this approach was also used by Baur and Zeppenfeld [6], who found that electromagnetic current conservation was restored by fermion loops that essentially rescale the $WW\gamma$ vertex.

Although the proposed scheme is well-justified in standard perturbation theory, it should be stressed that all reparation schemes are arbitrary to a greater or lesser extent: since the Dyson summation must necessarily be taken to all orders of perturbation theory, and we are not able to compute the complete set of *all* Feynman diagrams to *all* orders, the various schemes differ even if they lead to formally gauge-invariant results. It is then a numerical question how much their predictions differ.

The outline of this paper is as follows. In the next section, we study the process of Eq. 1, with emphasis on its small-angle behaviour. We show how gauge invariance gets violated by the imposition of an energy-dependent width, leading to completely wrong results. This is repaired by the inclusion of fermionic corrections to the three-boson vertex. The electromagnetic current is conserved again, and all Ward identities are satisfied. We discuss the connection between our result and that of Ref. [6]. In section 3, we present

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[3]As the Ward identities are linear, we can separate the real and imaginary parts.

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numerical comparisons between the various reparation schemes for the process (1). We finish with a number of conclusions and recommendations.

2 Gauge cancellations in $e^- e^+ \rightarrow e^- \bar{\nu}_e u \bar{d}$

In this section, we consider the process

$$e^-(p_1)\, e^+(k_1) \quad \rightarrow \quad e^-(p_2)\, \bar{\nu}_e(k_2)\, u(p_u)\, \bar{d}(p_d) \quad , \tag{3}$$

and especially concentrate on small scattering angles θ for the electron. We keep the mass of the electron finite, but shall neglect all other fermion masses (also that of the positron), so that we shall not have to worry about diagrams with Higgs ghosts connected to the positron or quark lines. The massive case can be treated analogously; this will be covered in Ref. [12]. Under these assumptions, we have to consider the subset of four Feynman diagrams given in Fig. 1, which conserves the electromagnetic current.
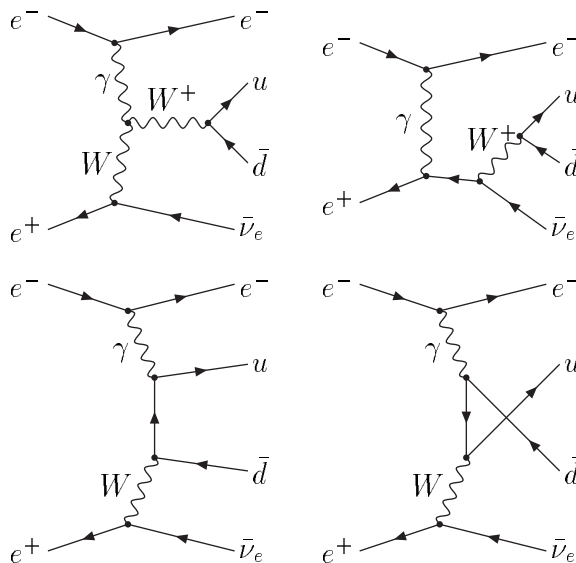

Figure 1: The four diagrams of the process $e^- e^+ \rightarrow e^- \bar{\nu}_e u \bar{d}$ which are considered here.

The matrix element $\mathcal{M}$ is given by

$$\mathcal{M} \quad = \quad \mathcal{M}^\mu\, J_\mu \quad , \quad \mathcal{M}^\mu = \sum_{i=1}^4 \mathcal{M}_i^\mu \quad ,$$

4

$$
\begin{aligned}
\mathcal{M}_1^\mu &= Q_W \, P_W(p_+^2) \, P_W(p_-^2) \, V^{\alpha\beta\mu}(p_+,-p_-,-q)\mathcal{M}_{\alpha\beta}^0 \ , \\[2mm]
\mathcal{M}_2^\mu &= \ \ 4iQ_e g_w^2 \, P_W(p_+^2) \, \bar{v}_-(k_1)\gamma^\mu \frac{\slashed{k}_1+\slashed{q}}{(k_1+q)^2}\gamma^\beta v_-(k_2) \ \bar{u}_-(p_u)\gamma_\beta v_-(p_d) \ , \\[2mm]
\mathcal{M}_3^\mu &= -4iQ_u g_w^2 \, P_W(p_-^2) \, \bar{u}_-(p_u)\gamma^\mu \frac{\slashed{p}_u-\slashed{q}}{(p_u-q)^2}\gamma^\beta v_-(p_d) \ \bar{v}_-(k_1)\gamma_\beta v_-(k_2) \ , \\[2mm]
\mathcal{M}_4^\mu &= -4iQ_d g_w^2 \, P_W(p_-^2) \, \bar{u}_-(p_u)\gamma^\beta \frac{\slashed{q}-\slashed{p}_d}{(p_d-q)^2}\gamma^\mu v_-(p_d) \ \bar{v}_-(k_1)\gamma_\beta v_-(k_2) \ , \\[2mm]
\mathcal{M}_{\alpha\beta}^0 &\equiv \ \ 4i g_w^2 \, \bar{v}_-(k_1)\gamma_\beta v_-(k_2) \ \bar{u}_-(p_u)\gamma_\alpha v_-(p_d) \ .
\end{aligned}
\tag{4}
$$

The spinors are written in a compact form, $u_-(p) \equiv \frac{1}{2}(1-\gamma^5)u(p)$ and

$$
\begin{aligned}
p_+ &= p_u + p_d \ , \quad p_- = k_1 - k_2 \ , \quad q = p_1 - p_2 \ , \\
[P_W(s)]^{-1} &= s - M_W^2 + i\gamma_W(s) \ ,
\end{aligned}
\tag{5}
$$

where M_W is the W mass and γ_W denotes the imaginary part of the inverse W propagator, which for the moment is introduced as a purely phenomenological device in order to avoid the singularities. The charged weak coupling constant g_w is given by $g_w^2 = M_W^2 G_F/\sqrt{2}$, Q_i is the electric charge of particle i, and

$$
V^{\mu_1\mu_2\mu_3}(p_1,p_2,p_3) = (p_1-p_2)^{\mu_3}g^{\mu_1\mu_2} + (p_2-p_3)^{\mu_1}g^{\mu_2\mu_3} + (p_3-p_1)^{\mu_2}g^{\mu_3\mu_1} \ . \tag{6}
$$

If we use conservation of the charged current in the massless fermion lines, we may write

$$
V^{\alpha\beta\mu}(p_+,-p_-,-q) = (2p_+ - q)^\mu g^{\alpha\beta} + 2q^\alpha g^{\beta\mu} - 2p_+^\beta g^{\mu\alpha} \ . \tag{7}
$$

The photon source is given by

$$
J^\mu = \frac{Q_e}{q^2}\bar{u}(p_2)\gamma^\mu u(p_1) \ . \tag{8}
$$

Note that the electrons can have two spin states each, but the massless fermions only contribute when they are left-handed.

The matrix element, squared and averaged over the spins of the incoming fermions, reads

$$
\begin{aligned}
\left\langle |\mathcal{M}|^2 \right\rangle &= H^{\mu\nu}\mathcal{M}_\mu \overline{\mathcal{M}}_\nu \ , \\[2mm]
H^{\mu\nu} &= \frac{1}{4}\sum_{\text{spins}} J^\mu \bar{J}^\nu = \frac{Q_e^2}{q^4}\left[p_1^\mu p_2^\nu + p_1^\nu p_2^\mu + \frac{q^2}{2}g^{\mu\nu} \right] \ .
\end{aligned}
\tag{9}
$$

Note the occurrence of q^{-4}: we may approximate

$$
|q^2| \sim \frac{m_e^2 R^4}{S(S-R^2)} + \frac{S-R^2}{2}(1-\cos\theta) \ , \tag{10}
$$

where $\sqrt{S}$ is the total energy, m_e the electron mass, θ the electron scattering angle, and $R^2 = (p_u + p_d + k_2)^2$: therefore, $|q^2|$ can be as small as $\mathcal{O}\left(m_e^2\right)$. Let us now consider the numerical behaviour of $H^{\mu\nu}$. Using the old approach of Ref. [14], we define

$$r^\mu = p_1^\mu - \beta p_2^\mu \quad , \quad \beta = p_1^0/p_2^0 \quad , \tag{11}$$

so that $r^0 = 0$ and $(r)^2 = (1-\beta)^2 m_e^2 + \beta q^2$; therefore, as $|q^2|$ becomes small, each individual component of r^μ also becomes small. We may now write

$$H^{\mu\nu} = \frac{Q_e^2}{q^4(1-\beta)^2} \left[2r^\mu r^\nu + 2\beta q^\mu q^\nu - (1+\beta)(r^\mu q^\nu + r^\nu q^\mu) + \frac{1}{2}(1-\beta)^2 q^2 g^{\mu\nu} \right]. \tag{12}$$

The unwanted q^{-4} behaviour of the cross-section will be mitigated to a q^{-2} behaviour, *provided*

$$q^\mu \mathcal{M}_\mu = 0 \quad . \tag{13}$$

This conservation of electromagnetic current is seen to be extremely important here: any small violation of it will be punished by a huge factor $\mathcal{O}\left(S/m_e^2\right)$. Multiplying q^μ into the four diagrams of Eq. 4, we obtain

$$\begin{aligned} W &\equiv q^\mu \mathcal{M}_\mu \\ &= \mathcal{M}_0 \left\{ (p_+^2 - p_-^2) Q_W \, P_W\left(p_+^2\right) P_W\left(p_-^2\right) \right. \\ &\qquad \left. + Q_e \, P_W\left(p_+^2\right) - (Q_d - Q_u) \, P_W\left(p_-^2\right) \right\} \quad , \\ \mathcal{M}_0 &\equiv \mathcal{M}_{\alpha\beta}^0 \, g^{\alpha\beta} \quad . \end{aligned} \tag{14}$$

By taking $\gamma_W = 0$, and considering the two poles at $p_+^2 = M_W^2$ and at $p_-^2 = M_W^2$, we get from the condition Eq. 13 that

$$Q_W = Q_e = Q_d - Q_u \quad , \tag{15}$$

the obvious condition of charge conservation. Therefore, we have

$$W = -i \, Q_e \mathcal{M}_0 \, P_W\left(p_+^2\right) P_W\left(p_-^2\right) \left(\gamma_W\left(p_+^2\right) - \gamma_W\left(p_-^2\right) \right) \quad . \tag{16}$$

Current conservation is therefore violated unless $\gamma_W\left(p_+^2\right) = \gamma_W\left(p_-^2\right)$. The most naive treatment of a Breit-Wigner resonance uses a *fixed width* approximation, with

$$\gamma_W(s)_{\text{fixed width}} = M_W \Gamma_W \quad . \tag{17}$$

The nominal width of an on-shell W is given by

$$\Gamma_W = \sum_{\text{doublets}} N_f \, \frac{G_F M_W^3}{6\pi\sqrt{2}} \quad , \tag{18}$$
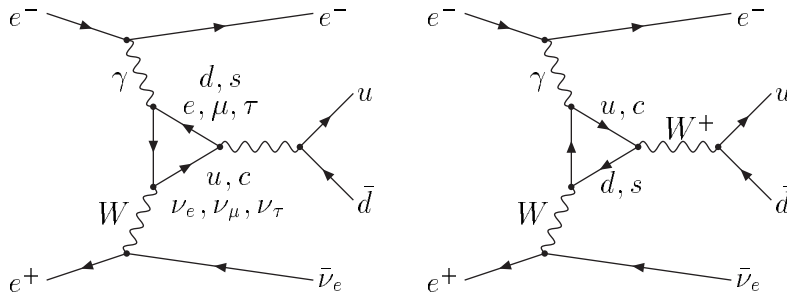
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Figure 2: The extra fermionic diagrams needed to cancel the gauge-breaking terms.

involving a sum over all massless fermion doublets with N_f (=1 or 3) colours. In this approximation, there is evidently no violation of electromagnetic current conservation.

The difficulty with the fixed-width approximation is that it cannot be justified from field theory. Indeed, in field theory the propagator only develops a complex pole off the real axis if we perform a Dyson summation of the self-energy graphs to all orders. This self-energy is inherently energy-dependent: to a good approximation[4], we may write

$$\gamma_W(s) = \frac{\Gamma_W}{M_W}s \quad , \quad s \geq 0 \quad ,$$
$$\gamma_W(t) = 0 \quad , \quad t < 0 \quad . \tag{19}$$

Consequently, propagators with space-like momenta cannot acquire finite widths in contradiction to the fixed-width scheme.

The theoretically most satisfying way to restore gauge-invariance seems to be the addition of one-loop vertex-corrections, which cancel the imaginary part in the Ward identities. In the process above, this boils down to adding the imaginary parts of the diagrams of Fig. 2. These are given by

$$\mathcal{M}_5^\mu = \frac{i}{16\pi}\mathcal{M}_{\alpha\beta}^0 \, P_W(p_+^2) \, P_W(p_-^2) \, g_w^2 \sum_{\text{doublets}} N_f \, (Q_d - Q_u) \, Z^{\alpha\beta\mu} \quad , \tag{20}$$

where we included the appropriate colour factor for the doublet, N_f. Using

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[4]Eq. 19 exactly takes into account the contributions of massless fermions, but it should be noted that above the W mass there is a contribution from the $W\gamma$ self-energy diagram, which has to be treated perturbatively.

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cutting rules, we calculate

$$Z^{\alpha\beta\mu} = \frac{1}{2\pi} \int d\Omega \ \mathrm{Tr} \left[\not{r}_1 \gamma^\mu \frac{\not{r}_1 - \not{q}}{(r_1 - q)^2} \gamma^\beta \not{r}_2 \gamma^\alpha \right] \quad , \tag{21}$$

which is the imaginary part of the triangle insertions. The momenta r_1 and r_2 are the momenta of the cut fermion lines with $p_+ = r_1 + r_2$. The expression $Z^{\alpha\beta\mu}$ satisfies the following three Ward identities

$$
\begin{aligned}
Z^{\alpha\beta\mu} q_\mu &= -\frac{8}{3} \left(p_+^\alpha p_+^\beta - p_+^2 g^{\alpha\beta} \right) \quad , \\
Z^{\alpha\beta\mu} p_\alpha^+ &= 0 \quad , \\
Z^{\alpha\beta\mu} p_\beta^- &= +\frac{8}{3} \left(p_+^\mu p_+^\alpha - p_+^2 g^{\mu\alpha} \right) \quad .
\end{aligned}
\tag{22}
$$

Because of the anomaly cancellation we have no explicit contributions from the part containing γ^5. Possible effects due to a top quark remain to be studied. Attaching the photon momentum q_μ to the sum of the diagrams $\mathcal{M}_5^\mu$ gives

$$
\begin{aligned}
W_{\mathrm{add}} &\equiv q_\mu \mathcal{M}_5^\mu \\
&= i \, Q_e \mathcal{M}_0 \, P_W(p_+^2) \, P_W(p_-^2) \, g_w^2 \sum_{\mathrm{doublets}} N_f \frac{p_+^2}{6\pi} \\
&= i \, Q_e \mathcal{M}_0 \, P_W(p_+^2) \, P_W(p_-^2) \, \Gamma_W \frac{p_+^2}{M_W} \\
&= i \, Q_e \mathcal{M}_0 \, P_W(p_+^2) \, P_W(p_-^2) \, \gamma_W(p_+^2) \quad ,
\end{aligned}
\tag{23}
$$

where we used the Ward identity of Eq. 22, the definition of the nominal W width, Eq. 18, as well as the fact that the external charged currents are conserved for massless fermions. It is clear that the extra diagrams exactly cancel the imaginary part in Eq. 16. Hence electromagnetic current conservation is restored.

Using the fact that all external fermionic currents in process (3) are conserved one finds for the compensating correction (Eq. 21)

$$
\begin{aligned}
Z^{\alpha\beta\mu} &= p_+^\mu q^\beta q^\alpha c_0 + g^{\mu\beta} q^\alpha c_1 + g^{\mu\alpha} q^\beta c_2 + g^{\alpha\beta} p_+^\mu c_3 \quad , \\
c_0 &= 32 \frac{p_+^2 p_-^2 q^2}{\lambda^2} \left\{ \left[10 \frac{p_+^2 p_-^2 q^2}{\lambda} + p_+^2 + p_-^2 + q^2 \right] f_0 \right. \\
&\qquad\qquad \left. + 20 p_+^2 \frac{p_- \cdot q}{\lambda} - \frac{8}{3} + \frac{2}{3}(p_-^2 + q^2) \frac{p_- \cdot q}{p_-^2 q^2} \right\} \quad , \\
c_1 &= -8 \frac{p_+^2 p_-^2 q^2}{\lambda} \left\{ \left[2 p_+^2 \frac{p_- \cdot q}{\lambda} - 1 \right] f_0 + 4 \frac{p_+^2}{\lambda} - 2 \frac{p_- \cdot q}{p_-^2 q^2} + \frac{1}{3} \frac{p_+^2}{p_-^2 q^2} \right\} \quad ,
\end{aligned}
$$

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$$c_2 = -8 \frac{p_+^2 p_-^2 q^2}{\lambda} \left\{ \left[2p_-^2 \frac{p_+ \cdot q}{\lambda} + 1 \right] f_0 + 4 \frac{p_+ \cdot p_-}{\lambda} + \frac{4}{3} \frac{p_+ \cdot p_-}{p_-^2 q^2} - \frac{1}{q^2} \right\} \quad ,$$

$$c_3 = +8 \frac{p_+^2 p_-^2 q^2}{\lambda} \left\{ \left[2q^2 \frac{p_+ \cdot p_-}{\lambda} + 1 \right] f_0 + 4 \frac{p_+ \cdot q}{\lambda} + \frac{4}{3} \frac{p_+ \cdot q}{p_-^2 q^2} - \frac{1}{p_-^2} \right\} \quad ,$$

$$f_0 = -\frac{2}{\sqrt{\lambda}} \ln \left(\frac{2(p_- \cdot q) + \sqrt{\lambda}}{2(p_- \cdot q) - \sqrt{\lambda}} \right) \quad , \quad \lambda \equiv 4(p_- \cdot q)^2 - 4p_-^2 q^2 \quad . \tag{24}$$

This expression, inserted in Eq. 20, gives the correction to the $WW\gamma$ vertex to be used in explicit calculations.

Now we discuss the result of Ref. [6]. They computed the process $q\bar{q}' \to \ell\nu_\ell\gamma$ with dressed propagators for the W's and an on-shell photon. Note, that in the process of single W production, one had $q + p_- = p_+$. In the following process, we have $p_- = q + p_+$. Hence, we have put $q \to -q$ with respect to the former definitions of the momenta. The invariant momentum squared flowing through both W's is positive and hence the running width is non-zero in both propagators. Without addition of extra diagrams, the corresponding amplitude will again not be gauge-invariant. Using the previous result in this section, it is easy to see that, with $q^2 = 0$, one gets for the two cut diagrams corresponding to the cut $p_+^2 > 0$:

$$\begin{aligned}
Z^{\alpha\beta\mu} = & \quad \frac{16}{3} \frac{p_+^2}{a} \left(g^{\alpha\beta} p_+^\mu + g^{\mu\alpha} q^\beta - g^{\mu\beta} q^\alpha \right) \\
& - \frac{16}{3} \frac{p_+^2 p_-^2}{a^3} \left(p_+^\mu q^\alpha - p_+ \cdot q \, g^{\mu\alpha} \right) p_+^\beta \\
& + \frac{16}{3} \frac{p_+^2 p_-^2}{a^3} \left(p_-^\mu q^\beta - p_- \cdot q \, g^{\mu\beta} \right) q^\alpha \tag{25}
\end{aligned}$$

with $p_+^2 > 0$, $p_-^2 > 0$, $q^2 = 0$ and $a \equiv p_+^2 - p_-^2$. Note that the first term is proportional to the tree-level $WW\gamma$ vertex. The cut diagrams corresponding to the cut $p_-^2 > 0$ are related by crossing symmetry. Adding the four cut diagrams, one ends up with:

$$\sum_{\text{cuts}} Z^{\alpha\beta\mu} = \frac{8}{3} 2 \left(g^{\alpha\beta} p_+^\mu + g^{\mu\alpha} q^\beta - g^{\mu\beta} q^\alpha \right) \quad . \tag{26}$$

Inserting the overall factor and the fermion lines, one sees that the extra diagrams amount to a scaling of the $WW\gamma$ vertex with $1 + i\Gamma_W/M_W$. It should be noted, that the factorization of the correction is not universal.

However, to get electromagnetic current conservation in the process (3), one can also effectively write the correction (24) in this form. In the limit $q^2 \to 0$, the overall factor multiplying the standard Yang-Mills vertex is then

given by

$$1 + i \frac{\gamma_W \left(p_+^2 \right)}{p_+^2 - p_-^2} \quad . \tag{27}$$

The parts in (24) transverse to q^μ are dropped since they do not play a role in restoring electromagnetic current conservation. Only if one would allow for a negative running width in the t-channel, rather than taking $\gamma_W \left(p_-^2 \right) = 0$, multiplying the standard Yang-Mills vertex with an overall factor $1 + i\Gamma_W / M_W$ would give a result that respects electromagnetic gauge invariance.

These simple, effectively factorizing prescriptions for restoring electromagnetic gauge invariance may be easier to implement in a Monte Carlo generator. However, in general they violate the full SU(2)×U(1) gauge invariance and, even more, upset the balance between the diagrams taking part in the unitarity cancellations at high energies. Hence, the validity is limited to the low energy range $\sqrt{s} = \mathcal{O} \left(M_W \right)$, e.g., LEP2. In contrast, the factorized form obtained from Eq. 26, being exact, does not have this problem.

3 Numerical results for $e^- e^+ \to e^- \bar{\nu}_e u \bar{d}$

The process $e^- e^+ \to e^- \bar{\nu}_e u \bar{d}$ has been studied numerically. The fermions are all taken to be massless, except for the electron, which has a mass m_e. The input parameters are given below,

$$
\begin{aligned}
m_e &= 0.511 \cdot 10^{-3} \text{ GeV } , \\
M_W &= 80.22 \text{ GeV } , \\
\alpha(0) &= 1/137.036 \quad , \\
G_F &= 1.16 \cdot 10^{-5} \text{ GeV}^{-2} , \\
\sqrt{s} &= 175 \text{ GeV } , \\
50 \text{ GeV} \leq \sqrt{p_+^2} &\leq 110 \text{ GeV } .
\end{aligned}
\tag{28}
$$

The fermionic width of the W boson is computed using Eq. 18. This gives $\Gamma_W = 2.02773\ldots$ GeV.

The cross-section for $e^- e^+ \to e^- \bar{\nu}_e u \bar{d}$ for the different schemes for two values of the minimum electron scattering angle $\theta_{\min}$ are given in table 1.

Note that all schemes were computed using the same sample, so the differences are much more significant than the integration error suggests. One sees that in this case, once current conservation is restored the results for the total cross-section of the different methods agree to $\mathcal{O} \left(\Gamma_W^2 / M_W^2 \right)$. From Fig. 3 it should be clear, that if we include running-width effects without

Scheme	Cross-section [pb]	
	$\theta_{\min} = 0°$	$\theta_{\min} = 10°$
Fixed width	.08887(8)	.01660(3)
Running width, no correction	60738(176)	.01713(3)
Fudge factor, with running width	.08892(8)	.01671(3)
Pole scheme, with running width	.08921(8)	.01666(3)
Running width, with Eq. 24	.08896(8)	.01661(3)
Running width, with Eq. 27	.08897(8)	.01662(3)

Table 1: Total cross-section for $e^- e^+ \rightarrow e^- \bar{\nu}_e u \bar{d}$ in different schemes.

taking into account the correction of the Yang-Mills vertex, too many events are sampled for small values of q^2.

4   Conclusion

The violations of gauge invariance associated with a naive introduction of a finite width for unstable particles can have disastrous consequences. We have indicated that, in the case of the vector bosons, this can be cured in a fully consistent way by the inclusion of appropriate fermionic corrections, e.g., to the three-vector-boson vertex. It has been shown explicitly in the case of massless fermions and the $WW\gamma$ vertex that the electromagnetic Ward identity is restored and current conservation holds. In the process $e^- e^+ \rightarrow e^- \bar{\nu}_e u \bar{d}$, in which gauge-breaking terms are amplified by $\mathcal{O}\left(10^{11}\right)$, this is shown to lead to a correct result. The differences between this scheme and other ways to obtain a gauge-invariant result have been shown to be small, much less than Γ_W / M_W in this specific example. The correction to the $WW\gamma$ vertex is given explicitly in Eq. 24 for current conserving sources, and in a simplified factorized form suitable for this process at not too high energies in Eq. 27. These functions can be incorporated in other event generators for LEP2.

Acknowledgments

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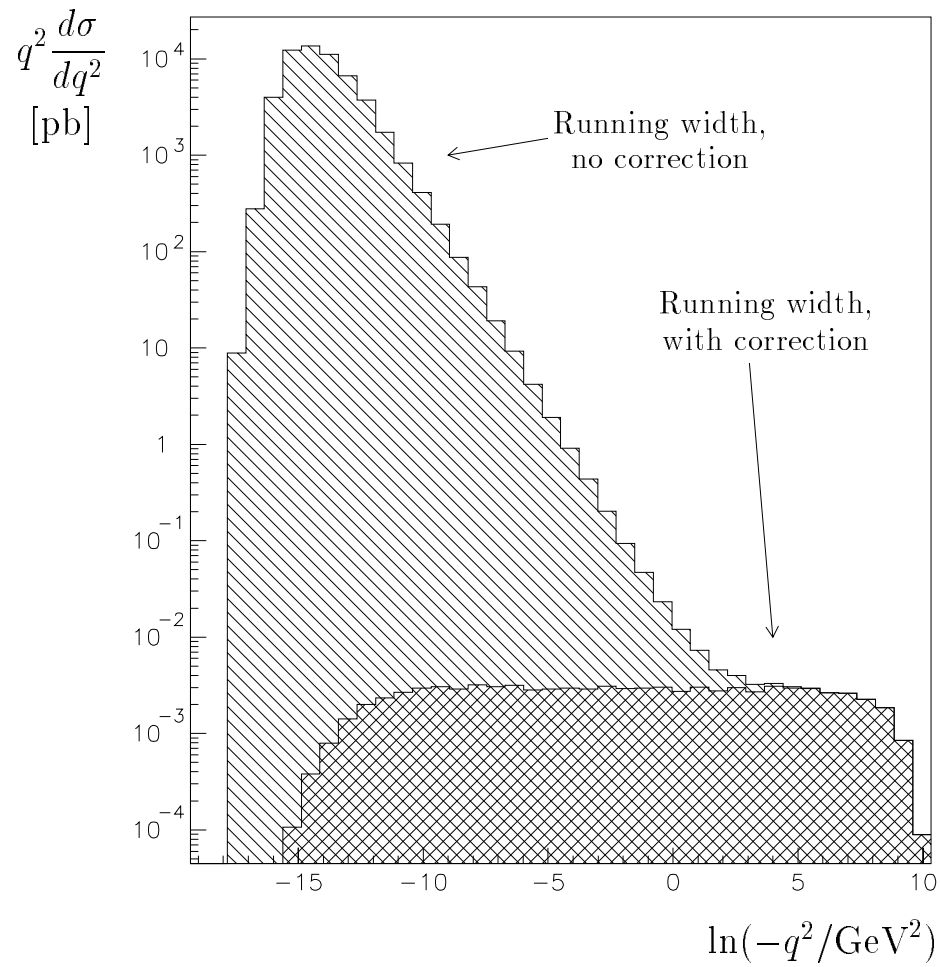[14] F. Gutbrod and Z. J. Rek, Z. Phys. **C1**, 171 (1979).

Figure 3: The effect of gauge-breaking terms in $e^- e^+ \to e^- \bar{\nu}_e u \bar{d}$ as a function of the virtuality of the photon.