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Decay Constants and Semileptonic Decays of Heavy Mesons in Relativistic Quark Model

Dae Sung Hwang¹, C. S. Kim² and Wuk Namgung³

1: Department of Physics, Sejong University, Seoul 133-747, Korea

2: Department of Physics, Yonsei University, Seoul 120-749, Korea

3: Department of Physics, Dongguk University, Seoul 100-715, Korea

Abstract

We investigate the B and D mesons in the relativistic quark model by applying the variational method with the Gaussian wave function. We calculate the Fermi momentum parameter p_F , and obtain $p_F = 0.50 \sim 0.54$ GeV, which is almost independent of the input parameters, α_s , m_b , m_c and m_{sp} . We then calculate the ratio f_B/f_D , and obtain the result which is larger, by the factor of about 1.3, than $\sqrt{M_D/M_B}$ given by the naive nonrelativistic analogy. This result is in a good agreement with the recent Lattice calculations. We also calculate the ratio $(M_{B^*} - M_B)/(M_{D^*} - M_D)$. In these calculations the wave function at origin $\psi(0)$ is essential. We also determine p_F by comparing the theoretical prediction of the ACCMM model with the lepton energy spectrum of $B \rightarrow e\nu X$ from the recent ARGUS analysis, and find that $p_F = 0.27 \pm \frac{0.22}{0.27}$ GeV, when we use $m_c = 1.5$ GeV. However, this experimentally determined value of p_F is strongly dependent on the value of input parameter m_c .

1: e-mail: dshwang@phy.sejong.ac.kr2: e-mail: kim@cskim.yonsei.ac.kr

1. Introduction

The B meson physics is important in the present high energy physics, since it gives us the information on the CKM matrix elements, V_{cb} and V_{ub} , and it is expected to provide the CP violation phenomena. In order to extract the value of V_{ub} from the B meson decay experiments, the method of separating the $B \rightarrow X_u l \nu$ events from the $B \rightarrow X_c l \nu$ ones at the end point region of the lepton energy spectrum has been used [1, 2]. On the other hand, the method of using the hadronic invariant mass spectrum has also been suggested recently [3]. For the analysis of the inclusive semileptonic decay process, the ACCMM model [4] has been most popularly employed, where the Fermi momentum parameter p_F is introduced as the most important parameter. The value of $p_F \simeq 0.3$ GeV has been commonly used for the experimental analyses without clear experimental or theoretical supports. In reference [5] we calculated p_F in the relativistic quark model using the variational method with the Gaussian wave function, and obtained the result of $p_F=0.54$ GeV. In this paper we also study the ratios, f_B/f_D and $(M_{B^*} - M_B)/(M_{D^*} - M_D)$, using the same method.

When one treats the heavy-light meson in analogy with the nonrelativistic situation, one expects $f_B/f_D \simeq \sqrt{M_D/M_B}$, since the reduced masses of the light and heavy quark systems of the B and D mesons have similar values, and $f_P^2 M_P = 12 |\psi(0)|^2$ by the van Royen–Weisskopf formula for the pseudoscalar meson P [6], where $\psi(0)$ is the wave function at the origin of the relative motion of quarks. If one uses the relation $f_B/f_D \simeq \sqrt{M_D/M_B}$ with the supplementary relation $f_D/f_{D_s} \simeq \sqrt{m_d/m_s}$, one can obtain the value of f_B from that of f_{D_s} , which is rather well established from the branching ratio of the $\bar{B}^0 \rightarrow D^+ D_s^-$ or $D_s^+ \rightarrow \mu^+ \nu_\mu$ process [7]. However, our calculation of f_P in the relativistic quark model, which we present in this article, shows that the above consideration with nonrelativistic analogy is much deviated by the relativistic motion of the light quark in the heavy-light

pseudoscalar meson P . This deviation has also been exposed by the recent Lattice calculations, since they give rather close values for f_B and f_D [8, 9]. This situation can be understood clearly within our relativistic calculation.

The potential model has been successful for ψ and Υ families with the nonrelativistic Hamiltonian, since their heavy quarks can be treated nonrelativistically. However, for D or B meson it has been difficult to apply the potential model because of the relativistic motion of the light quark in D or B meson. In our calculation we work with the realistic Hamiltonian which is relativistic for the light quark and nonrelativistic for the heavy quark, and adopt the variational method. We take the Gaussian function as the trial wave function, and obtain the ground state energy (and the wave function) by minimizing the expectation value of the Hamiltonian. Using the Gaussian wave function calculated as above, we get the wave function at the origin $\psi(0)$, and with which we can calculate the decay constant of the heavy-light pseudoscalar meson from the van Royen-Weisskopf formula. Through this procedure we obtain the ratio f_B/f_D . We also calculate the ratio $(M_{B^*} - M_B)/(M_{D^*} - M_D)$ from the chromomagnetic hyperfine splitting formula, where the information of $\psi(0)$ is essential. Finally, we compare the value of the Fermi momentum p_F given in our calculation with the lepton energy spectrum data of the semileptonic decay process, and find that it is just outside off one σ standard deviation.

In Section 2, using variational method in relativistic quark model we calculate the value of the parameter p_F , and the ratios f_B/f_D and $(M_{B^*} - M_B)/(M_{D^*} - M_D)$. We also extract p_F by comparing the theoretical prediction of the ACCMM model with the whole region of electron energy spectrum of $B \rightarrow e\nu X$ in Section 3. Section 4 contains the conclusions.

2. Variational method in relativistic quark model, and calculations of f_B/f_D and $(M_{B^*} - M_B)/(M_{D^*} - M_D)$

For the B meson system we treat the b -quark nonrelativistically, but the u - or d -quark relativistically with the Hamiltonian

$$H = M + \frac{\mathbf{p}^2}{2M} + \sqrt{\mathbf{p}^2 + m^2} + V(r), \quad (1)$$

where $M = m_b$ or m_c is the heavy quark mass and $m = m_{sp}$ is the u - or d -quark mass, *i.e.* the spectator quark mass in the ACCMM model. We apply the variational method to the Hamiltonian (1) with the trial wave function

$$\psi(\mathbf{r}) = \left(\frac{\mu}{\sqrt{\pi}}\right)^{3/2} e^{-\mu^2 r^2/2}, \quad (2)$$

where μ is the variational parameter. The Fourier transform of $\psi(\mathbf{r})$ gives the momentum space wave function $\chi(\mathbf{p})$, which is also Gaussian,

$$\chi(\mathbf{p}) = \frac{1}{(\sqrt{\pi}\mu)^{3/2}} e^{-p^2/2\mu^2}. \quad (3)$$

The ground state is given by minimizing the expectation value of H ,

$$\langle H \rangle = \langle \psi | H | \psi \rangle = E(\mu), \quad \frac{d}{d\mu} E(\mu) = 0 \quad \text{at} \quad \mu = \bar{\mu}, \quad (4)$$

and then $\bar{E} \equiv E(\bar{\mu})$ approximates m_B , and we get $\bar{\mu} = p_F$, the Fermi momentum parameter in the ACCMM model. The value of μ or p_F corresponds to the measure of the radius of the two body bound state, as can be seen from the relation, $\langle r \rangle = \frac{2}{\sqrt{\pi}} \frac{1}{\mu}$ or $\langle r^2 \rangle^{\frac{1}{2}} = \frac{3}{2} \frac{1}{\mu}$.

In Eq. (1), we take the Cornell potential which is composed of the Coulomb and linear potentials,

$$V(r) = -\frac{\alpha_c}{r} + Kr. \quad (5)$$

For the values of the parameters α_c ($\equiv \frac{4}{3}\alpha_s$), the quark masses m_b and m_c , and K , we use the following two sets of parameters. The set {A} is the Hagiwara *et*

al.'s [10] which has been determined by the best fit of ($c\bar{c}$) and ($b\bar{b}$) bound state spectra. And the set $\{B\}$ is chosen to have the running coupling constants at the mass scales of m_B and m_D , and the quark masses m_b and m_c which were determined to give the best ψ and Υ masses for the variational ground states.

$$\{A\} \quad \alpha_c = 0.47, \quad m_b = 4.75 \text{ GeV}, \quad m_c = 1.32 \text{ GeV}, \quad K = 0.19 \text{ GeV}^2, \quad (6)$$

$$\{B\} \alpha_c^B = 0.32, \quad m_b = 4.64 \text{ GeV}; \quad \alpha_c^D = 0.48, \quad m_c = 1.33 \text{ GeV}; \quad K = 0.19 \text{ GeV}^2. \quad (7)$$

With the Gaussian trial wave function (2) or (3), the expectation value of each term of the Hamiltonian (1) is given as follows:

$$\begin{aligned} \left\langle \frac{\mathbf{p}^2}{2M} \right\rangle &= \langle \chi(\mathbf{p}) | \frac{\mathbf{p}^2}{2M} | \chi(\mathbf{p}) \rangle = \frac{3}{4M} \mu^2, \\ \langle \sqrt{\mathbf{p}^2 + m^2} \rangle &= \langle \chi(\mathbf{p}) | \sqrt{\mathbf{p}^2 + m^2} | \chi(\mathbf{p}) \rangle = \frac{4\mu}{\sqrt{\pi}} \int_0^\infty e^{-x^2} \sqrt{x^2 + (m/\mu)^2} x^2 dx, \\ \langle V(r) \rangle &= \langle \psi(\mathbf{r}) | -\frac{\alpha_c}{r} + Kr | \psi(\mathbf{r}) \rangle = \frac{2}{\sqrt{\pi}} (-\alpha_c \mu + K/\mu). \end{aligned} \quad (8)$$

Then we have

$$\begin{aligned} E(\mu) &= \langle H \rangle \\ &= M + \frac{3}{4M} \mu^2 + \frac{2}{\sqrt{\pi}} (-\alpha_c \mu + K/\mu) + \frac{4\mu}{\sqrt{\pi}} \int_0^\infty e^{-x^2} \sqrt{x^2 + (m/\mu)^2} x^2 dx. \end{aligned} \quad (9)$$

For more details on this procedure of the variational method, see Ref. [5].

With the input value of $m = m_{sp} = 0.15 \text{ GeV}$, which is the value commonly used in experimental analyses, we minimize $E(\mu)$ of (9), and then we obtain for the B meson,

$$\begin{aligned} p_F(B) = \bar{\mu} &= 0.54 \text{ GeV}, & \bar{E}(B) &= 5.54 \text{ GeV} & \text{for } \{A\}, \\ \bar{\mu} &= 0.50 \text{ GeV}, & \bar{E}(B) &= 5.52 \text{ GeV} & \text{for } \{B\}. \end{aligned} \quad (10)$$

The B meson mass is lowered from the above values if we include chromomagnetic hyperfine splitting corrections. For comparison, let us check how much sensitive our calculation of p_F is by considering the case where $m = m_{sp} = 0$. For $m_{sp} = 0$

the integral in (9) is done easily and we obtain the following values of $\bar{\mu} = p_F$,

$$\begin{aligned}\bar{\mu} &= 0.53 \text{ GeV}, & \bar{E}(B) &= 5.52 \text{ GeV} & \text{for } \{A\}, \\ \bar{\mu} &= 0.48 \text{ GeV}, & \bar{E}(B) &= 5.49 \text{ GeV} & \text{for } \{B\}.\end{aligned}\tag{11}$$

As we see in Eq. (11), the results are similar to those in (10), where $m_{sp} = 0.15 \text{ GeV}$. We expected this insensitivity of the value of p_F on m_{sp} because the value of m_{sp} , which should be small in any case, can not affect the integral in (9) significantly. We also note that the theoretically determined value of $p_F(B)$ is completely independent of the input value of m_c , as can be seen from Eq. (9). Following the same procedure, we next obtain the results for the D meson with $m_{sp} = 0.15 \text{ GeV}$,

$$\begin{aligned}p_F(D) = \bar{\mu} &= 0.45 \text{ GeV}, & \bar{E}(D) &= 2.21 \text{ GeV} & \text{for } \{A\}, \\ \bar{\mu} &= 0.46 \text{ GeV}, & \bar{E}(D) &= 2.21 \text{ GeV} & \text{for } \{B\}.\end{aligned}\tag{12}$$

The decay constant f_P of a pseudoscalar meson P is defined by the matrix element $\langle 0|A_\mu|P(q)\rangle$:

$$\langle 0|A_\mu|P(q)\rangle = iq_\mu f_P.\tag{13}$$

By considering the low energy limit of the heavy meson annihilation, we have the relation between f_P and the ground state wave function at the origin $\psi_P(0)$ from the van Royen-Weisskopf formula including the color factor [6, 7]:

$$f_P^2 = \frac{12}{M_P} |\psi_P(0)|^2,\tag{14}$$

where M_P is the heavy meson mass. From (14) we have the ratio of f_B and f_D :

$$\frac{f_B}{f_D} = \sqrt{\frac{M_D}{M_B}} \times \frac{|\psi_B(0)|}{|\psi_D(0)|}.\tag{15}$$

For the Gaussian wave function of Eq. (2), we have

$$\psi_P(0) = \left(\frac{p_F(P)}{\sqrt{\pi}}\right)^{3/2},\tag{16}$$

then using the values of p_F in (10) and (12), we obtain

$$\begin{aligned} \frac{f_B}{f_D} &= \sqrt{\frac{M_D}{M_B}} \times \left(\frac{p_F(B)}{p_F(D)} \right)^{3/2} = 0.59 \times 1.31 = 0.77 & \text{for } \{A\}, \\ &= 0.59 \times 1.13 = 0.67 & \text{for } \{B\}. \end{aligned} \quad (17)$$

From (17) we see that f_B/f_D is enhanced, compared with $\sqrt{M_D/M_B}$, by the factor 1.31 for the parameter set $\{A\}$, and by 1.13 for the set $\{B\}$, which are given by the factor of $|\psi_B(0)/\psi_D(0)|$. Sometimes this factor has been approximated to be 1, and the relation $f_B/f_D \simeq \sqrt{M_D/M_B}$ has been used, by treating it in analogy with the nonrelativistic case [7]. However, our calculation shows that this factor is indeed important and different from 1 significantly. The factor 1.31 obtained in (17) for the parameter set $\{A\}$ is in a pretty good agreement with 1.40 of Ref. [8] and 1.39 of Ref. [9] of the recent Lattice calculations.

The mass difference between the vector meson P^* and the pseudoscalar meson P is given rise to by the chromomagnetic hyperfine splitting:

$$V_{hf} = \frac{2}{3m_Q\widetilde{m}_q} \vec{s}_1 \cdot \vec{s}_2 \nabla^2 \left(-\frac{\alpha_c}{r} \right), \quad (18)$$

where m_Q is the heavy quark mass as we used before (*i.e.* m_b or m_c), and \widetilde{m}_q is the constituent quark mass of the light quark, which is the effective mass for the baryon magnetic moments [11],

$$\widetilde{m}_u = \widetilde{m}_d = 0.33 \text{ GeV}, \quad \widetilde{m}_s = 0.53 \text{ GeV}. \quad (19)$$

Then the mass difference between B^* and B mesons is given by

$$M_{B^*} - M_B = \frac{8\pi\alpha_c}{3m_b\widetilde{m}_u} |\psi_B(0)|^2. \quad (20)$$

Using the values of p_F in (10) and (12), we obtain the ratio of $M_{B^*} - M_B$ and $M_{D^*} - M_D$ as

$$\begin{aligned} \frac{M_{B^*} - M_B}{M_{D^*} - M_D} &= \frac{m_c}{m_b} \times \left(\frac{|\psi_B(0)|}{|\psi_D(0)|} \right)^2 = \frac{m_c}{m_b} \times \left(\frac{p_F(B)}{p_F(D)} \right)^3 \\ &= 0.28 \times 1.73 = 0.48 & \text{for } \{A\}, \\ &= 0.29 \times 1.28 = 0.37 & \text{for } \{B\}. \end{aligned} \quad (21)$$

The experimental value [12] is about 0.33, which is larger than the nonrelativistic value 0.28 or 0.29, but smaller than our calculated value 0.48 or 0.37. Our calculated results are at least not much worse than the nonrelativistic values, even though there exists somewhat large discrepancy compared to experimental result. And this suggests that the behind reason for this discrepancy could be more subtle than the nonrelativistic consideration.

3. Determination of p_F from the experimental spectrum

Until now, we have discussed the theoretical determination of p_F in the relativistic quark model using the variational method, and its implications to the heavy meson masses and the decay constants of the heavy mesons. Now we would like to determine the Fermi momentum parameter p_F by comparing the theoretical prediction with the experimental charged lepton energy spectrum in semileptonic decays of B meson.

As discussed, the simplest model for the semileptonic B -decay is the spectator model which considers the decaying b -quark in the B meson as a free particle. The spectator model is usually used with the inclusion of perturbative QCD radiative corrections [13]. Then the decay width of the process $B \rightarrow X_q l \nu$ is given by

$$\Gamma_B(B \rightarrow X_q l \nu) \simeq \Gamma_b(b \rightarrow q l \nu) = |V_{bq}|^2 \left(\frac{G_F^2 m_b^5}{192 \pi^3} \right) f\left(\frac{m_q}{m_b}\right) \left[1 - \frac{2}{3} \frac{\alpha_s}{\pi} g\left(\frac{m_q}{m_b}\right) \right], \quad (22)$$

where m_q is the mass of the final q -quark decayed from b -quark. As can be seen, the decay width of the spectator model depends on m_b^5 , therefore small difference of m_b would change the decay width significantly.

Altarelli *et al.* [4] proposed for the inclusive B meson semileptonic decays their ACCMM model, which incorporates the bound state effect by treating the b -quark as a virtual state particle, thus giving momentum dependence to the b -quark mass.

The virtual state b -quark mass W is given by

$$W^2(\mathbf{p}) = m_B^2 + m_{sp}^2 - 2m_B\sqrt{\mathbf{p}^2 + m_{sp}^2} \quad (23)$$

in the B meson rest frame, where m_B is the B meson mass, m_{sp} is the spectator quark mass, and \mathbf{p} is the momentum of the spectator quark inside B meson.

For the momentum distribution of the virtual b -quark, Altarelli *et al.* considered the Fermi motion inside the B meson with the Gaussian momentum distribution,

$$\phi(\mathbf{p}; p_F) = 4\pi|\chi(\mathbf{p})|^2 = \frac{4}{\sqrt{\pi}p_F^3}e^{-\mathbf{p}^2/p_F^2}, \quad (24)$$

where the Gaussian width, p_F , is treated as a free parameter. Then the lepton energy spectrum of the B meson decay is given by

$$\frac{d\Gamma_B}{dE_l}(p_F, m_{sp}, m_q, m_B) = \int_0^{p_{max}} dp p^2 \phi(\mathbf{p}; p_F) \frac{d\Gamma_b}{dE_l}(m_b = W, m_q, m_{sp}), \quad (25)$$

where p_{max} is the maximum kinematically allowed value of $p = |\mathbf{p}|$. The ACCMM model, therefore, introduces a new parameter p_F for the Gaussian momentum distribution of the b -quark inside B meson, instead of the b -quark mass of the spectator model. In this way the ACCMM model incorporates the bound state effects and reduces the strong dependence on the b -quark mass in the decay width of the spectator model. The Fermi momentum parameter p_F is the most essential parameter of the ACCMM model, as we see in the above. However, the experimental determination of its value from the lepton energy spectrum has been very ambiguous, because various parameters of ACCMM model, such as p_F , m_q and m_{sp} , are fitted all together from the limited region of end-point lepton energy spectrum, and because the perturbative QCD corrections are very sensitive in the end-point region of the spectrum.

Recently, ARGUS [14] extracted the model independent lepton energy spectrum of $B \rightarrow X_c l \nu$ for the whole region of electron energy, but with much larger uncertainties, as shown in Fig. 1. We now compare the whole region of experimental electron energy spectrum with the theoretical prediction of ACCMM model, Eq.

(25), using p_F as a free parameter. We fixed $m_{sp} = 0.15$ GeV and $m_q = m_c = 1.5$ GeV, which are the values commonly used by experimental analyses. We derive the value of p_F using χ^2 analysis, and we obtain

$$p_F = 0.27 \pm {}^{0.22}_{0.27} \text{ GeV}. \quad (26)$$

The minimum χ^2 equals to 0.59 with $p_F = 0.27$ GeV. However, the result, Eq. (26), is found to be strongly dependent on the input value of m_c : if we instead use smaller m_c , both the best fit value of p_F and the minimum χ^2 increase, and *vice versa*. In Fig. 1, we also show the theoretical ACCMM model spectrums with $p_F = 0, 0.27, 0.49$ GeV, corresponding to dashed-, full-, dotted-line, respectively. The experimental data and the theoretical predictions are all normalized to the semileptonic branching ratio, $BR(B \rightarrow e\nu X) = 9.6\%$, following the result of ARGUS [14].

In Section 2, we theoretically calculated the Fermi momentum parameter p_F , and obtained $p_F = 0.50 \sim 0.54$ GeV. We note that the theoretically calculated values are slightly outside off one σ standard deviation comparing with the best fit value of the experimental data. However, since the experimental spectrum has still large uncertainties, we cannot exclude the validity of relativistic quark model for calculation of p_F yet, nor ACCMM model itself to apply to experimental analyses to find CKM parameters, $|V_{cb}|$ and/or $|V_{ub}/V_{cb}|$. In near future, once we get much more data from asymmetric B factories, it would be very interesting to extract the precise value of p_F once again.

In our previous work [5], we investigated the dependence of $|V_{ub}/V_{cb}|$ on p_F in ACCMM model, and found rather strong dependence as a function of a parameter p_F ,

$$\begin{aligned} 10^2 \times |V_{ub}/V_{cb}|^2 &= 0.57 \pm 0.11 \quad (\text{ACCMM with } p_F = 0.3 \text{ [15]}), \\ &= 1.03 \pm 0.11 \quad (\text{ACCMM with } p_F = 0.5), \\ &= 1.02 \pm 0.20 \quad (\text{Isgur } et \text{ al. [16]}). \end{aligned} \quad (27)$$

As can be seen easily, those values between ACCMM model with $p_F = 0.3$ GeV and Isgur *et al.* model are in large disagreement. However, if we use $p_F = 0.5$ GeV, the result of the ACCMM model becomes 1.03, and these two models are in a good agreement for the value of $|V_{ub}/V_{cb}|$.

4. Conclusions

We conclude that the value $p_F \sim 0.3$ GeV, which has been commonly used in experimental analyses, has no theoretical or experimental clear justification, even though there has been recently an assertion that the prediction of heavy quark effective theory approach [17], far from the end-point region, gives approximately equal shape to the ACCMM model with $p_F \sim 0.3$ GeV. Therefore, it is strongly recommended to determine the value of p_F more reliably and independently, when we think of the importance of its role in experimental analyses. It is particularly important in the determination of the value of $|V_{ub}/V_{cb}|$. A better determination of p_F is also interesting theoretically since it has its own physical correspondence related to the Fermi motion inside B meson. In this context we calculated theoretically the value of p_F in the relativistic quark model using quantum mechanical variational method. It turns out that $p_F = 0.50 \sim 0.54$ GeV, which is not far from the value of p_F determined by comparing ACCMM model prediction and the model independent lepton energy spectrum of ARGUS measurement, $p_F = 0.27 \pm_{0.27}^{0.22}$ GeV. The theoretically determined value of p_F is almost independent of input parameters, α_s , m_b , m_c , m_{sp} and *etc.* On the other hand, the experimentally determined value is strongly affected by the value of the input parameter m_c .

By using the same framework, we then calculated the ratio f_B/f_D , and obtained the result which is larger, by the factor of about 1.3, than $\sqrt{M_D/M_B}$ given by the naive nonrelativistic analogy. This result is in a pretty good agreement with the recent Lattice calculations. We also calculated the ratio $(M_{B^*} - M_B)/(M_{D^*} - M_D)$,

whose results suggest the subtlety of the mechanism which gives rise to the value of this ratio.

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Figure Captions

Fig. 1 The normalized lepton energy spectrum of $B \rightarrow X_c l \nu$ for the whole region of electron energy from the recent ARGUS [14] measurement. Also shown are the theoretical ACCMM model predictions, Eq. (25), using $p_F = 0, 0.27, 0.49$ GeV, corresponding to dashed-, full-, dotted-line, respectively. The minimum χ^2 equals to 0.59 with $p_F = 0.27$ GeV. We fixed $m_{sp} = 0.15$ GeV and $m_q = m_c = 1.5$ GeV.

