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On Non-standard Couplings among the Electroweak Vector Bosons[†]

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Abstract

Application of a Stueckelberg transformation allows one to connect various Lagrangians which have been independently proposed for non-standard couplings. We discuss the reduction of the number of independent parameters in the Lagrangian and compare symmetry arguments with dimensional arguments.

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1 Introduction

As previously shown [1, 2], reasonable ($SU(2)$ symmetry) constraints, weaker than the ones embodied in the standard $SU(2)_L \times U(1)_Y$ electroweak theory, allow one to systematically reduce the number of free parameters compatible with relativistic invariance of the trilinear and quadrilinear couplings among the electroweak vector bosons. The results are of theoretical as well as of practical interest. From the theoretical side they clarify the connection between non-standard couplings and symmetries. From the practical side, a systematic reduction by additional symmetry principles of the number of free parameters allowed by relativistic invariance is mandatory if reasonable bounds on such parameters are to be obtained by future measurements, e.g. in $e^+e^- \to W^+W^-$.

In the present note we demonstrate how to systematically extend any $U(1)_{em}$-gauge-invariant Lagrangian describing couplings among vector bosons to become invariant under local $SU(2)_L \times U(1)_Y$ transformations. The procedure is simple. By applying a Stueckelberg transformation to the given Lagrangian, local $SU(2)_L \times U(1)_Y$ invariance becomes manifest via the introduction of three (unphysical) scalar degrees of freedom [3, 4, 5], which are non-linearly realized. In a second substitution, one linearizes the theory with respect to the scalar degrees of freedom by introducing a physical scalar (Higgs boson).

Both the non-linear [6, 7] and the linear [8, 9, 10, 11] realization of $SU(2)_L \times U(1)_Y$ symmetry for non-standard couplings among the vector bosons have been given before. This is not the point of the present paper. The aim of the present note is twofold. First of all, we show how the different Lagrangians independently advocated for and discussed in the literature are *connected by a simple transformation* of the vector-boson interactions via introducing scalar degrees of freedom. Even though the different Lagrangians are well-known, this simple interrelation via a Stueckelberg transformation has to the best of our knowledge never been explicitly presented. Secondly, and as a consequence of the aforementioned interrelation of the different Lagrangians, it will become obvious that *no additional arguments are gained* concerning the strategy for reducing the number and nature of the non-standard interactions of the vector bosons among each other if these interactions are supplemented by interactions with scalar degrees of freedom.

The auxiliary scalar fields in the non-linear realization allow for a transition to arbitrary gauges. This is of relevance for loop calculations. The introduction of a physical Higgs particle formally improves the degree of divergence of loops calculated within the theory [10, 12]. However, due to the well-known ambiguities inherently connected with loop calculations in non-renormalizable theories, the interpretation of the results of such calculations is quite controversial[1]. On the other hand, it may be worthwhile to explore what kind of Higgs interactions are generated, once one allows for non-standard couplings.

In section 2, we briefly consider standard $SU(2)_L \times U(1)_Y$ interactions among vector bosons. This is necessary for the motivation and a transparent presentation of the $SU(2)$ symmetry assumptions (local $SU(2)$ as well as the so-called custodial $SU(2)$, known as $SU(2)_C$) to be employed in section 3 with respect to the general relativistically invariant

[1] Assuming that non-standard couplings are generated by one-loop effects of unknown heavy particles, one should keep in mind that these one-loop-generated terms must not be inserted in loops again within a consistent one-loop calculation [13].

Ansatz for non-standard couplings among the vector bosons. Final conclusions will be drawn in section 4.

2 Standard Interactions, non-linearly and linearly realized $SU(2)_L \times U(1)_Y$ symmetry

We start from the gauge group $SU(2)_L \times U(1)_Y$ [14, 15]. Introducing the weak-isospin triplet $W^i_\mu(x)$, $(i = 1, 2, 3)$, and a weak hypercharge singlet $B^0_\mu(x)$, we have

$$\mathcal{L}_{\rm YM} = -\frac{1}{2}\,\mathrm{tr}\,(W^{\mu\nu}W_{\mu\nu}) - \frac{1}{2}\,\mathrm{tr}\,(B^{\mu\nu}B_{\mu\nu}), \tag{2.1}$$

where

$$\begin{aligned} W_\mu &= W^i_\mu \frac{\tau_i}{2}, \qquad & W_{\mu\nu} &= W^i_{\mu\nu}\frac{\tau_i}{2} &= \partial_\mu W_\nu - \partial_\nu W_\mu + ig[W_\mu, W_\nu], \\ B_\mu &= B^0_\mu \frac{\tau_3}{2}, \qquad & B_{\mu\nu} &= B^0_{\mu\nu}\frac{\tau_3}{2} &= \partial_\mu B_\nu - \partial_\nu B_\mu. \end{aligned} \tag{2.2}$$

In the absence of a mass term (and even without auxiliary scalar fields), the Lagrangian (2.1) is invariant under local $SU(2)_L \times U(1)_Y$ transformations,

$$\begin{aligned} &W_\mu \to S W_\mu S^\dagger - \frac{i}{g} S\partial_\mu S^\dagger, \qquad S = \exp\left(\frac{i}{2} g\alpha_i\tau_i\right), \qquad && \alpha_i = \alpha_i(x),\ i = 1, 2, 3, \\ &B_\mu \to B_\mu - \partial_\mu\beta\frac{\tau_3}{2}, && \beta = \beta(x), \end{aligned} \tag{2.3}$$

which imply the corresponding transformations of the field strength tensors,

$$\begin{aligned} W_{\mu\nu} &\to S\,W_{\mu\nu}\,S^\dagger, \\ B_{\mu\nu} &\to B_{\mu\nu}. \end{aligned} \tag{2.4}$$

Under local electromagnetic gauge transformations, $U(1)_{em}$, specified by

$$\begin{aligned} S &= \exp\left(\frac{i}{2}\,e\,\chi\tau_3\right), \\ B_\mu &\to B_\mu - \partial_\mu\chi\frac{\tau_3}{2}\frac{e}{g'}, \end{aligned} \tag{2.5}$$

the fields transform as[2]

$$B^0_\mu \to B^0_\mu - \frac{e}{g'}\partial_\mu\chi, \qquad B^0_{\mu\nu} \to B^0_{\mu\nu}, \tag{2.6}$$

$$W^3_\mu \to W^3_\mu - \frac{e}{g}\partial_\mu\chi, \qquad W^3_{\mu\nu} \to W^3_{\mu\nu}, \tag{2.7}$$

[2]For completeness we note that $W^\pm_\mu \equiv \frac{1}{\sqrt{2}}(W^1_\mu \mp i\,W^2_\mu)$ and $W^\pm_{\mu\nu} \equiv \frac{1}{\sqrt{2}}(W^1_{\mu\nu} \mp i\,W^2_{\mu\nu})$ with $W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - g\,\epsilon_{ijk}W^j_\mu W^k_\nu$.

and

$$W_\mu^\pm \to \exp(\pm ie\chi)\, W_\mu^\pm, \qquad W_{\mu\nu}^\pm \to \exp(\pm ie\chi)\, W_{\mu\nu}^\pm. \tag{2.8}$$

A mass term which allows for mixing among neutral bosons and preserves $U(1)_{em}$ is given by

$$\mathcal{L}_{\rm mass} = M_W^2 \,{\rm tr}\left(W_\mu - \frac{g'}{g} B_\mu\right)^2. \tag{2.9}$$

Upon diagonalization, (2.9) together with (2.1) yields the vector-boson part of the Lagrangian of the electroweak standard model [14, 15] in its unitary gauge.

The mass term (2.9) by assumption has the important property of being invariant under global $SU(2)_L$ transformations in the limit of a decoupling B_μ field, $g' \to 0$, i.e., "intrinsic $SU(2)$ violation" [1] is excluded for the charged and neutral (unmixed) masses, all given by $M_{W^\pm} \equiv M_{W^0} \equiv M_W$ in (2.9). This symmetry of the Lagrangian (2.1), (2.9) is also known as "custodial" $SU(2)$ and coincides with the global $SU(2)$ broken by current mixing employed by Bjorken and Hung and Sakurai [16]. It guarantees the validity of Weinberg's mass relation [15] between the $W^\pm$ and the Z^0 masses.

Empirically, by combining the measurements of the $W^\pm$ mass with the precision data on $e^+e^- \to Z_0 \to$ (leptons, quarks), one finds a deviation from unity in the ratio $M_{W^0}/M_{W^\pm}$ which is given by [17]

$$\Delta x^{\rm exp} \equiv 1 - \frac{M_{W^0}^2}{M_{W^\pm}^2} = (9.6 \pm 4.7 \pm 0.2) \cdot 10^{-3}. \tag{2.10}$$

The first error in this result is statistical, while the second one corresponds to the error in the input value of $\alpha(M_Z^2)^{-1} = 128.87 \pm 0.12$. Even though $\Delta x^{\rm exp}$ in (2.10) deviates from zero, (2.10) provides strong empirical support for $SU(2)_C$ symmetry of the Lagrangian. Indeed, the apparent violation of $SU(2)_C$ in (2.10) can be fully explained as a consequence of the fermion loop correction dominated by the top-quark loop and given by [17]

$$\Delta x_{\rm ferm} \simeq 12 \cdot 10^{-3} \text{ for } m_t = 180\ GeV. \tag{2.11}$$

Accordingly, there is strong motivation to extend the validity of $SU(2)_C$ to the (non-standard) couplings among the electroweak vector bosons to be discussed in section 3.

We return to our discussion of the symmetry properties of the Lagrangian given by (2.1) and (2.9). The mass term as it stands appears to break local $SU(2)_L \times U(1)_Y$ symmetry. Local $SU(2)_L \times U(1)_Y$ symmetry becomes manifest, however, upon introducing auxiliary scalar fields (Goldstone fields) $\varphi_i(x)$ via the (non-Abelian) Stueckelberg transformation [4, 5]

$$W_\mu \to U^\dagger W_\mu U - \frac{i}{g} U^\dagger \partial_\mu U, \qquad B_\mu \to B_\mu, \tag{2.12}$$

where

$$U \equiv \exp\left(\frac{i}{2}\frac{g}{M_W}\varphi_i \tau_i\right). \tag{2.13}$$

This implies for the corresponding transformation of the field strength tensors

$$W_{\mu\nu} \to U^\dagger W_{\mu\nu} U, \qquad B_{\mu\nu} \to B_{\mu\nu}. \tag{2.14}$$

The substitution (2.12) leaves the Yang-Mills term (2.1) invariant. Introducing the covariant derivative,

$$D_\mu U \equiv \partial_\mu U + igW_\mu U - ig'UB_\mu, \tag{2.15}$$

the substitution (2.12) results in

$$W^\mu - \frac{g'}{g}B^\mu \to -\frac{i}{g}U^\dagger D^\mu U = \frac{i}{g}(D^\mu U)^\dagger U, \tag{2.16}$$

and the mass term in (2.9) takes the form

$$\mathcal{L}_{\text{mass}} = \frac{M_W^2}{g^2}\,\text{tr}\,\left[(D_\mu U)^\dagger (D^\mu U)\right]. \tag{2.17}$$

The local $SU(2)_L \times U(1)_Y$ transformations (2.3) and

$$U \to SU \exp\left(\frac{-i}{2}g'\beta\tau_3\right) \tag{2.18}$$

assure gauge invariance of the full electroweak Lagrangian including the vector-boson mass term. A suitable gauge fixing condition yields $U = 1$ [5] and takes us back to the original Lagrangian which is thus identified as the unitary gauge (U-gauge) of an $SU(2)_L \times U(1)_Y$ gauge invariant massive vector boson theory with mixing in the neutral sector. We note in passing that electromagnetic $U(1)_{em}$ gauge invariance of the Lagrangian in the original form (2.1), (2.9) is a prerequisite for local $SU(2)_L \times U(1)_Y$ invariance. An arbitrary $SU(2)_L \times U(1)_Y$ gauge transformation (2.3), (2.18) acts as an electromagnetic gauge transformation on the (U-gauge) Lagrangian (2.1), (2.9) [5].

In a final step, we linearize in the scalar fields $\varphi_i(x)$ and add an additional scalar field, the physical Higgs scalar $H(x)$, via the replacement [5, 6]

$$\begin{aligned} U \to \frac{\sqrt{2}}{v}\phi &\equiv \frac{1}{\sqrt{2}}\frac{g}{M_W}\phi \\ &= 1 + \frac{g}{2M_W}\left(H(x) + i\varphi_i(x)\tau_i\right), \end{aligned} \tag{2.19}$$

and the addition of the Higgs potential terms to the Lagrangian.

We have thus reconstructed [5, 18] the bosonic sector of the standard electroweak theory in four distinct steps

i) local $SU(2)_L \times U(1)_Y$ symmetry purely in the vector-boson sector (without scalar fields),

ii) custodial $SU(2)$ when introducing vector-boson masses,

iii) manifest local $SU(2)_L \times U(1)_Y$ symmetry via three scalar degrees of freedom within a non-linear framework, and

iv) linearisation and the introduction of a physical scalar (Higgs) particle.

The same sequence of steps will now be applied to the most general Ansatz allowed by relativistic invariance for couplings of vector bosons among each other.

3 Non-standard couplings among vector bosons

Upon diagonalization of the mass term (2.9) in the Lagrangian (2.1), (2.9) via

$$\begin{aligned} A_\mu &= \ c_W B^0_\mu + s_W W^3_\mu, \\ Z_\mu &= -s_W B^0_\mu + c_W W^3_\mu, \end{aligned} \tag{3.1}$$

one finds the standard trilinear interactions between the photon, the neutral Z and the charged $W^\pm$ bosons,

$$\begin{aligned} \mathcal{L}_{\text{int,SM}} &= -ie[A_\mu(W^{-\mu\nu}_{(A)} W^+_\nu - W^{+\mu\nu}_{(A)} W^-_\nu) + F^{(A)}_{\mu\nu} W^{+\mu} W^{-\nu}] \\ &\quad -ie\frac{c_W}{s_W}[Z_\mu(W^{-\mu\nu}_{(A)} W^+_\nu - W^{+\mu\nu}_{(A)} W^-_\nu) + Z^{(A)}_{\mu\nu} W^{+\mu} W^{-\nu}], \end{aligned} \tag{3.2}$$

where $W^{\pm\mu\nu}_{(A)} = \partial^\mu W^{\pm\nu} - \partial^\nu W^{\pm\mu}$, $F^{\mu\nu}_{(A)} = \partial^\mu A^\nu - \partial^\nu A^\mu$ and $Z^{\mu\nu}_{(A)} = \partial^\mu Z^\nu - \partial^\nu Z^\mu$ are the Abelian field strength tensors. Here, $e = g\, s_W = g'\, c_W$ denotes the electromagnetic coupling, where $s_W^2 \equiv 1 - c_W^2$. We generalize[3] (3.2) by allowing for deviations in the relative magnitude of the two terms making up the photon and the Z interaction, and by allowing for deviations in the absolute magnitude of the Z interaction. In addition, Lorentz- as well as C- and P-invariance allow for a dimension-six term, sometimes called quadrupole term. Accordingly, we supplement (3.2) by the non-standard couplings [19, 20]

$$\begin{aligned} \mathcal{L}_{\text{int,NS}} &= -iex_\gamma F_{\mu\nu} W^{+\mu} W^{-\nu} \\ &\quad -ie\delta_Z[Z_\mu(W^{-\mu\nu} W^+_\nu - W^{+\mu\nu} W^-_\nu) + Z_{\mu\nu} W^{+\mu} W^{-\nu}] \\ &\quad -iex_Z Z_{\mu\nu} W^{+\mu} W^{-\nu}, \\ &\quad +ie\frac{y_\gamma}{M^2_{W^\pm}} F_\mu{}^\nu W_\nu^{-\lambda} W_\lambda^{+\mu}, \\ &\quad +ie\frac{y_Z}{M^2_{W^\pm}} Z_\mu{}^\nu W_\nu^{-\lambda} W_\lambda^{+\mu}, \end{aligned} \tag{3.3}$$

characterized by the five free parameters,

$$\delta_Z, x_\gamma, x_Z, y_\gamma, y_Z, \tag{3.4}$$

which vanish for the standard case. We refer to the Lagrangian (3.3) as the "phenomenological non-standard Lagrangian" for interactions among vector-bosons. It has been widely used in the simulation, e.g. [20, 21], of the analysis of future data on, e.g., $e^+e^- \to W^+W^-$.

Electromagnetic gauge invariance of the Lagrangian (3.3) and invariance of the Z_μ field requires $W^{\pm\mu\nu}$ to transform according to (2.8) and to have the non-Abelian form

$$W^{\pm\mu\nu} = W^{\pm\mu\nu}_{(A)} \mp ig(W^{\pm\mu} W^\nu_3 - W^\mu_3 W^{\pm\nu}), \tag{3.5}$$

where W^μ_3 is to be replaced by the linear combination of the Z_μ and A_μ following from (3.1). The non-Abelian structure (3.5) induces quadrilinear interactions among two charged and two neutral vector bosons and fixes the strength of these interactions. Electromagnetic

[3] We restrict ourselves to C and P conserving extensions.

gauge invariance allows to add quadrilinear interactions among four charged bosons of arbitrary strength to (3.3). We fix the strength of these interactions by adopting the non-Abelian forms

$$
\begin{aligned}
F^{\mu\nu} &= F^{\mu\nu}_{(A)} + ie(W^{+\mu}W^{-\nu} - W^{-\mu}W^{+\nu}),\\
Z^{\mu\nu} &= Z^{\mu\nu}_{(A)} + ie\frac{c_W}{s_W}(W^{+\mu}W^{-\nu} - W^{-\mu}W^{+\nu})
\end{aligned}
\tag{3.6}
$$

for $F^{\mu\nu}$ and $Z^{\mu\nu}$ in (3.3) which with (3.1) supplement the tensors (3.5) by their neutral non-Abelian component $W_3^{\mu\nu}$ and by $B^{0\mu\nu}$.

In order to explicitly display and discuss the $SU(2)_L \times U(1)_Y$-symmetry properties of the non-standard interactions in (3.3), we transform to the W^3B^0 base [20] and the matrix notation already employed in (2.1), (2.9). Applying the transformation (3.1), we obtain

$$
\begin{aligned}
\mathcal{L}_{\text{int,NS}} = & \; 2\,c_1\,\mathrm{tr}\,[(W^\mu - \frac{g'}{g}B^\mu)W_{\mu\nu}(W^\nu - \frac{g'}{g}B^\nu)]\\
& +2\,c_2\,\mathrm{tr}\,[B_{\mu\nu}W^\mu W^\nu]\\
& +c_3\,\mathrm{tr}\,[\tau_3\,W_{\mu\nu}]\,\mathrm{tr}\,[\tau_3\,W^\mu W^\nu]\\
& +\frac{c_4}{M_W^2}\,\mathrm{tr}\,[{W_\mu}^\nu {W_\nu}^\lambda {W_\lambda}^\mu]\\
& +2\,\frac{c_5}{M_W^2}\,\mathrm{tr}\,[{B_\mu}^\nu {W_\nu}^\lambda {W_\lambda}^\mu],
\end{aligned}
\tag{3.7}
$$

where the five coefficients are linearly related to the five parameters in (3.3) via

$$
\begin{aligned}
c_1 &= iec_W\delta_Z,\\
c_2 &= -ie(c_W x_\gamma - s_W x_Z - s_W\delta_Z),\\
c_3 &= -ie(c_W x_Z + s_W x_\gamma),\\
c_4 &= -\frac{2}{3}\,i\,e\,(c_W y_Z + s_W y_\gamma),\\
c_5 &= ie(s_W\,y_Z - c_W\,y_\gamma).
\end{aligned}
\tag{3.8}
$$

The full vector-boson Lagrangian is obtained by adding (3.7) to the sum of (2.1) and (2.9). Having established electromagnetic gauge invariance of the Lagrangian (3.3), and consequently of (3.7), the various steps of the previous section may now be applied to this extended Lagrangian.

Requiring $SU(2)_L \times U(1)_Y$ symmetry to be realized by the vector-boson interactions themselves (step i)) immediately excludes all non-standard terms in (3.7) with the only exception of the term with coefficient $c_4 \neq 0$, i.e.,

$$
c_1 = c_2 = c_3 = c_5 = 0. \tag{3.9}
$$

We remain with a single non-vanishing independent parameter. From the explicit expressions of the coefficients in (3.8) one immediately finds $\delta_Z = x_\gamma = x_Z = 0$ and the relation

$$
y_Z = \frac{c_W}{s_W}y_\gamma \tag{3.10}
$$

first given in [2]. Note that (3.10) generalizes the relation $g_{ZWW} = e\,(c_W/s_W)$ in (3.2) to the quadrupole interaction.

We turn to exclusion of intrinsic $SU(2)$ violation [1], i.e., $SU(2)_C$ symmetry (step ii)). This requirement is much less restrictive, as it demands a vanishing value of only[4]

$$c_3 = 0, \tag{3.11}$$

in (3.7), while the interactions involving B_μ and $B_{\mu\nu}$ are allowed in analogy to the symmetry property of the standard vector-boson mass term (2.9). The requirement (3.11) implies

$$x_Z = -\frac{s_W}{c_W} x_\gamma, \tag{3.12}$$

a relation first given in [1]. Accordingly, upon imposing $SU(2)_C$ symmetry we remain with two independent parameters (δ_Z, x_γ) in the dimension-four part of the Lagrangian (3.3), (3.7), and with the additional two independent parameters (y_γ, y_Z) in the dimension-six part. Imposing the symmetry restriction (3.10) on the dimension-six terms, i.e., combining (3.10) with (3.12), leads to a Lagrangian with three independent parameters $(\delta_Z, x_\gamma, y_\gamma)$ which embodies $SU(2)_C$ in the dimension-four and local $SU(2)_L \times U(1)_Y$ in the dimension-six terms.

Finally, requiring $SU(2)_C$ and minimal coupling of the hypercharge field (no $B_{\mu\nu}$ term in (3.7)), implies $c_2 = c_3 = c_5 = 0$, i.e. (3.10), (3.12) as well as [22]

$$\delta_Z = \frac{x_\gamma}{c_W\, s_W}. \tag{3.13}$$

We remain with only two independent parameters, (x_γ, y_γ).

Even though $SU(2)_C$ is directly tested by the experimentally measured parameter $\Delta x^{\rm exp}$ in (2.10) which determines the mass ratio $M_{W^0}/M_{W^\pm}$, its extension to self couplings amounts to an assumption, after all. It is of interest, accordingly, that this underlying assumption cannot only be tested in multi-parameter cases but also in a single-free-parameter model. Simply imposing $c_1 = c_2 = c_4 = c_5 = 0$ in (3.7), (3.8) implies the constraint

$$x_Z = \frac{c_W}{s_W} x_\gamma \tag{3.14}$$

with $\delta_Z = y_\gamma = y_Z = 0$ which obviously yields $c_3 \neq 0$ and a single-free-parameter test of $SU(2)_C$ symmetry.

Local $SU(2)_L \times U(1)_Y$ symmetry of the $U(1)_{em}$ phenomenological non-standard Lagrangian (3.3), (3.7) becomes manifest by applying the Stueckelberg transformation (2.12), (2.14) on it (step iii)). Noting that the c_2 and c_3 terms in (3.7) are invariant under the substitution $W^\mu W^\nu \to (W^\mu - \frac{g'}{g} B^\mu)(W^\nu - \frac{g'}{g} B^\nu)$ and making use of (2.16), one finds

[4]Note that the decoupling limit $g' = e = s_W = 0$ of custodial $SU(2)$ symmetry in (3.7) has to be taken under the constraints $e/s_W = $ const as well as $c_3 = $ const and $c_4 = $ const (compare [20]).

$$\begin{aligned}
\mathcal{L}_{\mathrm{int,NS}} = \quad & 2\,\frac{c_1}{g^2}\,\mathrm{tr}\,[(D_\mu U)^\dagger W^{\mu\nu}(D_\nu U)] \\
& +2\,\frac{c_2}{g^2}\,\mathrm{tr}\,[B^{\mu\nu}(D_\mu U)^\dagger(D_\nu U)] \\
& +\frac{c_3}{g^2}\,\mathrm{tr}\,[U\,\tau_3\,U^\dagger\,W^{\mu\nu}]\,\mathrm{tr}\,[\tau_3\,(D_\mu U)^\dagger\,(D_\nu U)] \\
& +\frac{c_4}{M_W^2}\,\mathrm{tr}\,[W_\mu{}^\nu W_\nu{}^\lambda W_\lambda{}^\mu] \\
& +2\,\frac{c_5}{M_W^2}\,\mathrm{tr}\,[B_\mu{}^\nu U^\dagger W_\nu{}^\lambda W_\lambda{}^\mu U].
\end{aligned} \tag{3.15}$$

We stress that the Lagrangians (3.3), (3.7) and (3.15) are equivalent. Consequently, no new arguments become available concerning the reduction of the number of five free parameters by rewriting (3.3), (3.7) in the manifestly $SU(2)_L \times U(1)_Y$ gauge invariant form (3.15). Often, this point is not correctly presented in the literature. While all three dimension-four terms in (3.15) are given in [7, 12], the $SU(2)_C$-violating c_3-term is not present in e.g. [23], and the omission is not justified. The form (3.15) of the Lagrangian is of interest as it connects the subject of non-standard vector-boson interactions with the widely used investigations (e.g. [24]) on chiral Lagrangians. Also, (3.15) provides an intermediate step in introducing the Higgs scalar.

In the last step (step iv)), we now assume the existence of a Higgs scalar particle of mass m_H sufficiently below the unitarity limit of $m_H \simeq 1$ TeV. Carrying out the substitution (2.19) in (3.15), one obtains

$$\begin{aligned}
\mathcal{L}_{\mathrm{int,NS}} = \quad & \frac{c_1}{M_W^2}\,\mathrm{tr}\,[(D_\mu \phi)^\dagger W^{\mu\nu}(D_\nu \phi)] \\
& +\frac{c_2}{M_W^2}\,\mathrm{tr}\,[B^{\mu\nu}(D_\mu \phi)^\dagger(D_\nu \phi)] \\
& +\frac{c_3}{M_W^4}\,\frac{g^2}{4}\,\mathrm{tr}\,[\phi\,\tau_3\,\phi^\dagger\,W^{\mu\nu}\phi]\,\mathrm{tr}\,[\tau_3\,(D_\mu \phi)^\dagger\,(D_\nu \phi)] \\
& +\frac{c_4}{M_W^2}\,\mathrm{tr}\,[W_\mu{}^\nu W_\nu{}^\lambda W_\lambda{}^\mu] \\
& +\frac{c_5}{M_W^4}\,g^2\,\mathrm{tr}\,[B_\mu{}^\nu \phi^\dagger W_\nu{}^\lambda W_\lambda{}^\mu \phi].
\end{aligned} \tag{3.16}$$

As far as vector-boson interactions are concerned, (3.16) is equivalent to the phenomenological non-standard Lagrangian (3.3). The linearization of the scalar sector and the introduction of a physical Higgs boson does not change the vector boson self-interactions.

The introduction of the Higgs particle, when passing from (3.15) to (3.16), leads to a dimensional transmutation of the interaction terms. The Lagrangian (3.16) contains three dimension-six and two dimension-eight terms. Restricting oneself to dimension-six terms [9, 10, 12] in (3.16) implies

$$c_3 = c_5 = 0, \tag{3.17}$$

and thus the relations (3.10) and (3.12) [25] originally derived from symmetry arguments [1, 2].

The omission of the higher-dimension terms in (3.16) may be based on the expectation that such terms would lead to a less decent high-energy behavior of scattering cross sections than terms of lower dimension. This is not the case, however, for the c_3 and c_5 terms in (3.16). In fact, all terms in the Lagrangian (3.16) give rise to the same high-energy behavior of boson-boson scattering cross sections. To be specific, *as long as terms quadratic in the deviations from the standard model are neglected*, the cross sections for all boson-boson scattering processes become constant[5] (i.e. they are $\mathcal{O}(s^0)$, where $\sqrt{s}$ denotes the c.m. scattering energy) in the high-energy limit. This is true for all terms in (3.16)[6]. For the scattering of two vector-bosons into two vector-bosons the $\mathcal{O}(s^0)$ behavior for all terms has been shown in [26]. Using the formalism of [26], one easily generalizes this result of a constant, $\mathcal{O}(s^0)$, behavior to all two-boson into two-boson processes, i.e. for any combination of external Higgs bosons and vector-bosons in an arbitrary polarization state. Accordingly, the omission of the c_3 and c_5 terms is of no relevance for the high-energy behavior of scattering processes. The dimensional argument cannot be justified by a reference to differences in the high-energy behavior.

As no direct empirical evidence for the Higgs particle is available so far, it is gratifying that the three-parameter model of Lagrangian (3.3), (3.7) with the constraints (3.10) and (3.12) does not rely on the existence of a Higgs particle. Simple symmetry considerations alone are sufficient to exclude theoretically less favored scenarios.

Let us note, however, that the (linearly realized) $SU(2)_L \times U(1)_Y$ symmetry according to (3.16) specifies certain non-standard Higgs interactions, in case a light Higgs particle is realized in nature in conjunction with non-standard vector-boson self-couplings. The introduction of the Higgs particle improves the high-energy behavior of the standard electroweak theory from an $\mathcal{O}(s^0)$ to an $\mathcal{O}(s^{-1})$ behavior. As a consequence, also the high-energy behavior of the theory with non-standard couplings is improved from $\mathcal{O}(s)$ to $\mathcal{O}(s^0)$, as long as the non-standard parameters are small enough to be treated in linear approximation. The introduction of the non-standard Higgs interactions does not provide an additional argument, however, for omitting certain non-standard terms in (3.3), (3.7), (3.15), apart from those arguments already derived from symmetry considerations.

4 Conclusion

Various authors have independently derived the two aforementioned $SU(2)_L \times U(1)_Y$-gauge-invariant Lagrangians in a chiral Lagrangian approach and within an effective field theory with a light Higgs particle. Dimensional arguments were used to reduce the number of independent parameters. In the present note we have shown that the different Lagrangians emerge from a phenomenological Ansatz by applying a Stueckelberg transformation followed by a linearization of the scalar sector. The results of dimensional

[5]Remember that in the standard model all cross sections are $\mathcal{O}(s^{-1})$.

[6]If the Higgs boson is omitted from the theory, i.e. if one uses the sum of the Lagrangians (2.1), (2.17) and (3.15), the cross sections rise as $\mathcal{O}(s)$ independently of whether only c_1, c_2 and c_4 or also c_3 and c_5 are different from zero (if one neglects the terms quadratic in the non-standard couplings). The less decent high energy behavior as compared to the case with a Higgs boson is entirely due to the omission of the *standard* Higgs interactions [26, 27].

arguments concerning the reduction of the number of free parameters when applied to the Lagrangian with the Higgs scalar coincide with the result of the symmetry arguments suggested and employed a long time ago in the framework of the phenomenological Lagrangian. The dimensional arguments, however, cannot be justified from the high-energy behavior, as all five non-standard terms lead to the same behavior.

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