hep-ph/9506471
June 1995

Spontaneous CP violation in supersymmetric models with four Higgs doublets

Manuel Masip[a,b] and Andrija Rašin[a]

[a] *Department of Physics*
University of Maryland
College Park, MD 20742, U.S.A.

[b] *Departamento de Física Teórica y del Cosmos*
Universidad de Granada
18071 Granada, Spain

Abstract

We consider supersymmetric extensions of the standard model with two pairs of Higgs doublets. We study the possibility that CP violation is generated spontaneously in the scalar sector via vacuum expectation values (VEVs) of the Higgs fields. Using a simple geometrical interpretation of the minimum conditions we prove that the minimum of the tree-level scalar potential for these models is allways real. We show that complex VEVs can appear once radiative corrections and/or explicit *soft* CP violating terms are added to the effective potential.

PACS numbers: 11.30.Er, 11.30.Qc, 12.60.Jv

1 Introduction

In order to introduce CP violation into gauge models one can consider two different approaches [1]. CP violating phases could be intrinsic to the parameters of the original Lagrangian or, alternatively, they could be spontaneous in the sense that all parameters of the theory are real, but the vacuum expectation values (VEVs) of the scalar Higgs fields are complex. Experimental evidence shows that CP violation is *small* but nonzero (it has only been measured in kaon physics). In general, this fact provides a *naturalness* criterium that can be used to decide which approach to generate CP violation seems favoured for a definite model. The first approach will be more natural (*i.e.*, require less fine tuned idependent parameters) in models where after phase redefinitions of the fields one is left with few independent phases, whereas spontaneous CP violation (SCPV) seems preferred for models with a large number of parameters and a rich enough scalar sector.

In the standard model, for example, assuming the parameters in the Lagrangian complex, all phases can be absorbed by field redefinitions except for two: the QCD phase θ and the phase ϕ in the Cabibbo-Kobayashi-Maskawa (CKM) matrix. The most popular mechanism proposed to solve the strong CP problem (that we shall not treat here) requires the introduction of a global symmetry [2], whereas the CKM phase appears allways multiplied by the small mixing V_{13} and do not require a fine tuned value to explain the kaon system. In this case SCPV, possible only by extending the Higgs and/or fermion sectors, is less economical. In contrast, in supersymmetric (SUSY) extensions of the standard model the origin of CP violation is more involving, due essentially to the large number of soft SUSY breaking parameters in the Lagrangian. For arbitrary complex parameters there are several new sources of CP violation, and the prediction of a neutron electric dipole moment within the experimental limits requires cancellations of two or three orders of magnitude. In this framework, the idea of SCPV seems appealing.

Unfortunately, it is known that in the minimal SUSY extension of the standard model (MSSM) there is no spontaneous generation of CP violation[1].

The first SUSY extension where SCPV has been found is in models with gauge singlets [5, 6], although in the simplest case the complex VEVs appear only for one-loop effective potentials with very strong top corrections. The addition of singlets does not spoil the gauge unification of the MSSM, relaxes bounds on the mass of the lightest neutral scalar, and offers a possible explanation to the μ problem (the Higgs mass term in the superpotential). Moreover, as shown by Pomarol [6], the phases generated spontaneously in these models could be enough to explain CP violation in kaon physics (*i.e.*, no complex Yukawas would be necessary). The singlet models, however, seem to imply either the presence of light Higgs scalars and heavy squarks or small complex phases, which in turn invoke some degree of fine tuning.

In this article we study the possibility of SCPV in SUSY models with two pairs of Higgs doublets. These models are an obvious generalization of the minimal case (they do not introduce new species, just *double* the Higgs sector). They occur naturally in left-right symmetric scenarios, where at least two bidoublets are required in order to get realistic fermion masses and mixings. It was also shown [7] that (unlike the minimal or singlet SUSY models) four Higgs doublet models can have a large $\tan\beta$ without fine tuning or too light charginos. Like in the singlet case, the lightest scalar in the four Higgs doublet models has *not* necessarily a tree-level mass smaller than the Z mass. The fact that four doublet models require an intermediate scale to be consistent with gauge unification could also be an advantage [8], since it might be more in line with recent data on $\alpha_s(M_Z)$ than minimal unification scenarios.

Since more than one Higgs doublet couples to quarks of a given charge, these models

[1] In fact, SCPV in MSSM is in principle possible once the radiative corrections are included [3]. However, this model contains a too light boson, and is thus experimentally excluded [4].

have in principle too large flavor changing neutral currents (FCNC) and require a mechanism to suppress some Yukawa couplings (usually, an approximate flavor symmetry [9]). If the couplings are complex with phases of order one these models also tend to have too large CP violation in the kaon system and too large neutron electric dipole moment [10], requiring an additional mechanism to suppress phases or couplings. These questions, briefly addressed in this paper, will be studied in detail elsewhere in the framework of models with approximate global symmetries [11, 12].

2 The Higgs sector and the minimum conditions

We start defining the model and establishing the conditions for the minimum of the tree-level potential. For previous work on SUSY models with four Higgs doublets see for example [13].

The Higgs sector of the model contains two pairs of $SU(2)$ doublet superfields, (H_1, H_3) and (H_2, H_4), with hypercharges -1 and $+1$, respectively. We denote these doublets by

$$H_{1(3)} = \begin{pmatrix} \phi^0_{1(3)} \\ \phi^-_{1(3)} \end{pmatrix}, \quad H_{2(4)} = \begin{pmatrix} \phi^+_{2(4)} \\ \phi^0_{2(4)} \end{pmatrix}. \tag{1}$$

The most general superpotential with four higgs doublets is then given by

$$\begin{aligned} W &= Q(\mathbf{h}_1 H_1 + \mathbf{h}_3 H_3) D^c + Q(\mathbf{h}_2 H_2 + \mathbf{h}_4 H_4) U^c + L(\mathbf{h}^e_1 H_1 + \mathbf{h}^e_3 H_3) E^c \\ &+ \mu_{12} H_1 H_2 + \mu_{32} H_3 H_2 + \mu_{14} H_1 H_4 + \mu_{34} H_3 H_4, \end{aligned} \tag{2}$$

where Q stand for quark doublets, D^c for down quark singlets, U^c for up quark singlets, L for lepton doublets, E^c for charged lepton singlets, and $\mathbf{h}_i$ are the Yukawa matrices (family indices are omitted).

Including soft SUSY breaking terms, the most general tree level scalar potential involving only Higgs fields is given by

$$V = m_1^2 H_1^\dagger H_1 + m_2^2 H_2^\dagger H_2 + m_3^2 H_3^\dagger H_3 + m_4^2 H_4^\dagger H_4 +$$

$$
\begin{aligned}
&+ \quad (m_{12}^2 H_1 H_2 + h.c.) + (m_{32}^2 H_3 H_2 + h.c.) + \\
&+ \quad (m_{14}^2 H_1 H_4 + h.c.) + (m_{34}^2 H_3 H_4 + h.c.) + \\
&+ \quad (m_{13}^2 H_1^\dagger H_3 + h.c.) + (m_{24}^2 H_2^\dagger H_4 + h.c.) + V_D^{4HD},
\end{aligned} \tag{3}
$$

where V_D^{4HD} is the D-term part of the potential. For the neutral components of the doublets one has

$$
V_D^{4HD} = \frac{1}{8}(g^2 + g'^2)[\phi_1^{0\,\dagger}\phi_1^0 + \phi_3^{0\,\dagger}\phi_3^0 - \phi_2^{0\,\dagger}\phi_2^0 - \phi_4^{0\,\dagger}\phi_4^0]^2. \tag{4}
$$

We will assume that the theory is CP invariant, *i.e.*, all the couplings and mass parameters are real. We do not assume any higher energy scales or accidental cancellations, but we suppose that there is a part of the parameter space giving minima of the potential which do not break the electric charge.

After spontaneous symmetry breaking, the Higgs fields will acquire VEVs which are possibly complex (from now on we drop the 0 superscript to specify neutral fields):

$$
<\phi_1> = \frac{1}{\sqrt{2}} v_1 \; ; \; <\phi_3> = \frac{1}{\sqrt{2}} v_3 e^{i\delta_3} \,, \tag{5}
$$

and

$$
<\phi_2> = \frac{1}{\sqrt{2}} v_2 e^{i\delta_2} \; ; \; <\phi_4> = \frac{1}{\sqrt{2}} v_4 e^{i\delta_4} \,, \tag{6}
$$

where we have used a global hypercharge transformation to rotate away the phase of $<\phi_1>$. The vacuum expectation value of the scalar potential is then

$$
\begin{aligned}
<V> &= \frac{1}{2}m_1^2 v_1^2 + \frac{1}{2}m_2^2 v_2^2 + \frac{1}{2}m_3^2 v_3^2 + \frac{1}{2}m_4^2 v_4^2 + m_{12}^2 v_1 v_2 \cos\delta_2 + m_{13}^2 v_1 v_3 \cos\delta_3 + \\
&+ \quad m_{14}^2 v_1 v_4 \cos\delta_4 + m_{32}^2 v_3 v_2 \cos(\delta_3 + \delta_2) + m_{34}^2 v_3 v_4 \cos(\delta_3 + \delta_4) + \\
&+ \quad m_{24}^2 v_2 v_4 \cos(\delta_2 - \delta_4) + \frac{1}{32}(g^2 + g'^2)[v_1^2 + v_3^2 - v_2^2 - v_4^2]^2.
\end{aligned} \tag{7}
$$

The conditions at the minimum are

$$
\frac{\partial V}{\partial v_1} = m_1^2 v_1 + m_{12}^2 v_2 \cos\delta_2 + m_{13}^2 v_3 \cos\delta_3 + m_{14}^2 v_4 \cos\delta_4 + v_1 g(\mathbf{v}) = 0 \,,
$$

$$\begin{aligned}
\frac{\partial V}{\partial v_2} &= m_2^2 v_2 + m_{12}^2 v_1 \cos\delta_2 + m_{32}^2 v_3 \cos(\delta_3+\delta_2) + m_{24}^2 v_4 \cos(\delta_2-\delta_4) - v_2 g(\mathbf{v}) = 0\,, \\
\frac{\partial V}{\partial v_3} &= m_3^2 v_3 + m_{32}^2 v_2 \cos(\delta_3+\delta_2) + m_{13}^2 v_1 \cos\delta_3 + m_{34}^2 v_4 \cos(\delta_3+\delta_4) + v_3 g(\mathbf{v}) = 0\,, \\
\frac{\partial V}{\partial v_4} &= m_4^2 v_4 + m_{24}^2 v_2 \cos(\delta_2-\delta_4) + m_{34}^2 v_3 \cos(\delta_3+\delta_4) + m_{14}^2 v_1 \cos\delta_4 - v_4 g(\mathbf{v}) = 0 \quad (8)
\end{aligned}$$

where $g(\mathbf{v}) = \frac{1}{8}(g^2+g'^2)[v_1^2+v_3^2-v_2^2-v_4^2]$, and

$$\begin{aligned}
\frac{\partial V}{\partial \delta_2} &= m_{12}^2 v_1 v_2 \sin\delta_2 + m_{32}^2 v_3 v_2 \sin(\delta_3+\delta_2) + m_{24}^2 v_2 v_4 \sin(\delta_2-\delta_4) = 0\,, \\
\frac{\partial V}{\partial \delta_3} &= m_{32}^2 v_3 v_2 \sin(\delta_3+\delta_2) + m_{13}^2 v_1 v_3 \sin\delta_3 + m_{34}^2 v_3 v_4 \sin(\delta_3+\delta_4) = 0\,, \\
\frac{\partial V}{\partial \delta_4} &= -m_{24}^2 v_2 v_4 \sin(\delta_2-\delta_4) + m_{34}^2 v_3 v_4 \sin(\delta_3+\delta_4) + m_{14}^2 v_1 v_4 \sin\delta_4 = 0. \qquad (9)
\end{aligned}$$

The seven equations in (8) and (9) contain seven unknows: the four VEVs v_i ($i = 1, ..., 4$) and the three phases δ_i ($i = 2, 3, 4$), and thus can be in principle solved. The last three equations have a trivial solution where all the sines vanish. However, a necessary condition for the theory to have spontaneous CP violation is that at least one of the phases δ_i is different from 0 or π. Therefore we are looking for nontrivial solutions to (9). The easiest way to solve this problem is to realize that there is a geometrical object that satisfies these equations[2]. It is defined by 3 triangles, each of which has two of the angles δ_i as shown in Figure 1. Appropriately, we call this object "tri-triangle". Six of the nine sides of the tri-triangle are independent. Addition of the sine laws of the three triangles gives

$$\begin{aligned}
(a-x)\sin\delta_2 \;&+\; c\sin(\delta_3+\delta_2) \;+\; d\sin(\delta_2-\delta_4) = 0\,, \\
c\sin(\delta_3+\delta_2) \;&+\; (b-z)\sin\delta_3 \;+\; f\sin(\delta_3+\delta_4) = 0\,, \\
-d\sin(\delta_2-\delta_4) \;&+\; f\sin(\delta_3+\delta_4) \;+\; (e-y)\sin\delta_4 = 0\,. \qquad (10)
\end{aligned}$$

[2]This is similar to most searches for SCPV in models with two phases. In that case, the nontrivial solution is found when the two phases can be fit as angles in a triangle with sides related to the VEVs and mass parameters in the potential [14].

Comparing (9) and (10) we find the correspondence between the six independent distances in the tri-triangle and the six independent quantities $m_{ij}^2 v_i v_j$:

$$\begin{aligned} a - x = m_{12}^2 v_1 v_2 \quad &, \quad c = m_{32}^2 v_2 v_3 \,, \\ b - z = m_{13}^2 v_1 v_3 \quad &, \quad d = m_{24}^2 v_2 v_4 \,, \\ e - y = m_{14}^2 v_1 v_4 \quad &, \quad f = m_{34}^2 v_3 v_4 \,. \end{aligned} \tag{11}$$

Using the tri-triangle we can eliminate the cosines of the phases δ_i in Eqs. (8) in terms of the sides:

$$\begin{aligned} v_1 \frac{\partial V}{\partial v_1} &= m_1^2 v_1^2 + (-\frac{ab}{c} + \frac{xe}{d} - \frac{yz}{f}) + v_1^2 g(\mathbf{v}) = 0 \\ v_2 \frac{\partial V}{\partial v_2} &= m_2^2 v_2^2 + (-\frac{ac}{b} + \frac{dx}{e}) - v_2^2 g(\mathbf{v}) = 0 \\ v_3 \frac{\partial V}{\partial v_3} &= m_3^2 v_3^2 + (-\frac{bc}{a} - \frac{zf}{y}) + v_3^2 g(\mathbf{v}) = 0 \\ v_4 \frac{\partial V}{\partial v_4} &= m_4^2 v_4^2 + (\frac{ed}{x} - \frac{yf}{z}) - v_4^2 g(\mathbf{v}) = 0 \,. \end{aligned} \tag{12}$$

To solve the four equations above and find the VEVs v_i we need first to express the combinations of sides in (12) in terms of the masses and VEVs in Eq. (11). The VEVs would give us the sides of the tri-triangle and from them we would construct the tri-triangle and read the angles δ_i. This would complete the search for the CP violating minimum of the scalar potential.

As we will prove in the next section, no such solution for the VEVs v_i can be found without fine tuning the mass parameters. This is because supersymmetry and the gauge symmetries dictate a too restricted form for the scalar potential (a simple D-term contribution and no trilinear terms). In the rest of the paper we show explicitly why this is so and point out minimal modifications that would induce nontrivial phases.

3 The minimum

To solve (12) we need to express the combinations involving the nine sides of the tri-triangle in terms of masses and VEVs (*i.e.*, in terms of the six distances in Eq. (11)). This requires solving a 4th order equation; we simplify this procedure by making a redefinition of the Higgs fields. We rewrite the scalar potential for the neutral fields in matrix notation:

$$\begin{aligned} V &= \begin{pmatrix} \phi_1^\dagger & \phi_3^\dagger \end{pmatrix} \begin{pmatrix} m_1^2 & m_{13}^2 \\ m_{13}^2 & m_3^2 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_3 \end{pmatrix} + \begin{pmatrix} \phi_2^\dagger & \phi_4^\dagger \end{pmatrix} \begin{pmatrix} m_2^2 & m_{24}^2 \\ m_{24}^2 & m_4^2 \end{pmatrix} \begin{pmatrix} \phi_2 \\ \phi_4 \end{pmatrix} \\ &+ [\begin{pmatrix} \phi_1 & \phi_3 \end{pmatrix} \begin{pmatrix} m_{12}^2 & m_{14}^2 \\ m_{32}^2 & m_{34}^2 \end{pmatrix} \begin{pmatrix} \phi_2 \\ \phi_4 \end{pmatrix} + h.c.] \\ &+ \frac{1}{8}(g^2+g'^2)[\begin{pmatrix} \phi_1^\dagger & \phi_3^\dagger \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_3 \end{pmatrix} - \begin{pmatrix} \phi_2^\dagger & \phi_4^\dagger \end{pmatrix} \begin{pmatrix} \phi_2 \\ \phi_4 \end{pmatrix}]^2 . \end{aligned} \tag{13}$$

The first two matrices above can be diagonalized through two unitary transformations (in our case two rotations, since the mass matrices are real and symmetric) of the neutral scalar fields: $\begin{pmatrix} \phi_1' \\ \phi_3' \end{pmatrix} = \mathbf{U}_1 \begin{pmatrix} \phi_1 \\ \phi_3 \end{pmatrix}$; $\begin{pmatrix} \phi_2' \\ \phi_4' \end{pmatrix} = \mathbf{U}_2 \begin{pmatrix} \phi_2 \\ \phi_4 \end{pmatrix}$. Note that the quartic term in the potential will not change its form and can be obtained just by replacing unprimed by primed fields. Then, without loss of generality we can go to a basis where $m_{13}'^2 = m_{24}'^2 = 0$ (from now on we drop the prime to specify rotated quantities). In Fig. 1 this corresponds to a tri-triangle where the sides $1/d$, $1/e$, and $1/x$ disappear (that triangle becomes infinite) and the quantities b and z become equal. Such an object depends on four independent distances and contains three angles that vary (for different choices of the distances) between 0 and π. The minimum conditions can be immediately read from Eq.(11) for this case:

$$\begin{aligned} a = m_{12}^2 v_1 v_2 \quad &, \quad c = m_{32}^2 v_2 v_3 \, , \\ y = -m_{14}^2 v_1 v_4 \quad &, \quad f = m_{34}^2 v_3 v_4 \, . \end{aligned} \tag{14}$$

It is easy to see that if all the masses m_{ij}^2 in Eq. (2) are positive the absolute minimum will be real. A necessary condition to have an absolute minimum with complex phases is that

$$m_{12}^2 m_{14}^2 m_{32}^2 m_{34}^2 < 0\,. \tag{15}$$

The signs of the masses m_{ij}^2 in Eq. (14) satisfy that constraint. As long as the choice of signs of the mass terms satisfies (15), the minimum can be obtained from the above tri-triangle just by redefining the angles δ_i.

Now we try to solve the minimum conditions (12) which correspond to this particular tri-triangle. The quantity b is not an independent distance, the solution can be determined in terms of a, c, f, and y (given in Eq. (14)) in Figure 1. We find

$$\frac{1}{b} = \frac{1}{v_1 v_3} h(\mathbf{v})\,, \tag{16}$$

where

$$h(\mathbf{v}) = \sqrt{m_{12}^2 m_{34}^2 - m_{14}^2 m_{32}^2}\sqrt{\frac{\frac{1}{m_{32}^2 m_{34}^2} v_1^2 - \frac{1}{m_{12}^2 m_{14}^2} v_3^2}{m_{12}^2 m_{32}^2 v_2^2 - m_{14}^2 m_{34}^2 v_4^2}}\,. \tag{17}$$

Then the conditions (12) read

$$\begin{aligned}
v_1 \frac{\partial V}{\partial v_1} &= v_1^2\,[\,m_1^2 - \frac{m_{12}^2 m_{34}^2 - m_{14}^2 m_{32}^2}{m_{32}^2 m_{34}^2}\frac{1}{h(\mathbf{v})} + g(\mathbf{v})\,] = 0 \\
v_2 \frac{\partial V}{\partial v_2} &= v_2^2\,[\,m_2^2 - m_{12}^2 m_{32}^2 h(\mathbf{v}) - g(\mathbf{v})\,] = 0 \\
v_3 \frac{\partial V}{\partial v_3} &= v_3^2\,[\,m_3^2 + \frac{m_{12}^2 m_{34}^2 - m_{14}^2 m_{32}^2}{m_{12}^2 m_{14}^2}\frac{1}{h(\mathbf{v})} + g(\mathbf{v})\,] = 0 \\
v_4 \frac{\partial V}{\partial v_4} &= v_4^2\,[\,m_4^2 + m_{14}^2 m_{34}^2 h(\mathbf{v}) - g(\mathbf{v})\,] = 0\,.
\end{aligned} \tag{18}$$

Since these four equations depend only on two combinations of VEVs, namely $g(\mathbf{v})$ and $h(\mathbf{v})$, they are uncompatible (unless a fine tuned value of the masses is chosen). We conclude that no tri-triangle like solution exists for the tree level potential in Eq. (7), and the three phases δ_i of the minima are necessarily zero or π. This situation is somewhat similar to the no-go theorem for simple singlet models [5]. In the tree level four

Higgs model, SCPV is not possible because of the specific form of the supersymmetric potentials.

4 Modifications of the model

We find two types of modifications or additions that would change this result, allowing for CP violating minima. The first possibilty would require to add to the effective potential new terms in order to avoid the cancellations producing that only two combinations of the four VEVs appear in conditions (18). Since we already considered the most generic Lagrangian consistent with (softly broken) SUSY and gauge invariance, one is only left (of course, unless new fields are introduced) with effective radiative corrections from order one Yukawa couplings. In order to change at least two of the four conditions (18) one needs two strong couplings. These could be the two couplings of the top quark to H_2 and H_4 or a top coupling combined with a bottom coupling to H_1 or H_3[3]. Notice that in the second case the minimum conditions (9) will keep the same form as in the tree level case, and therefore the tri-triangle solution works also here. In order to suppress FCNC in these cases the simplest idea is to invoke some additional symmetry that will suppress the Yukawa couplings of the additional Higsses (H_3 and H_4). Thus the second case, with one top and one bottom Yukawa coupling large, seems preferred.

A second possibility to obtain complex minima which does not rely on radiative effects would imply a substantial (but, in our opinion, well motivated) change on the definition of the model. We have assumed that all couplings in the potential are real and all phases are generated spontaneously. It seems plausible, however, to relax this requirement and introduce *soft* CP violation in the μ terms (see Eq. (3)). These terms could have their origin in some higher scale (singlet VEVs) with no effects on the rest

[3]Note that since now the Z mass has contributions from four VEVs, it is possible to have order one bottom Yukawa couplings even for order one $\tan\beta \equiv \sqrt{\frac{v_2^2+v_4^2}{v_1^2+v_3^2}}$.

of parameters. It turns out that one can absorb three of the four μ phases by Higgs field redefinitions, resulting into a new Lagrangian with only one complex phase δ_5[4]. The origin of CP violation would not be entirely spontaneous, but it would not appear neither in an *uncontrolled* way. This scenario seems more flexible for a treatment of FCNC in terms of an additional symmetry[11, 12] since no *a priori* conditions on the sizes of the Yukawa couplings have been set. It also avoids the typical domain wall problems of theories with spontaneous breaking of discrete symmetries.

5 Conclusions

The models with four Higgs doublets are another well motivated minimal extension of the MSSM. We have studied the possibility of SCPV in these models. We found a simple geometrical interpretation of the minimum equations that allowed us to understand the conditions for SCPV. Although no complex minima of the tree-level potential are possible, we singled out two interesting possibilities (radiative effects and explicit μ phases) that modify the potential so that CP violating phases appear. A more detailed analysis of these possibilities will be discussed elsewhere [12].

We thank R. Mohapatra for helpful suggestions and comments. The work of M. M. has been partially supported by a grant from the Junta de Andalucía (Spain). The work of A. R. was supported by the NSF grant No. PHY9421385. A. R. would like to thank G. Anderson, J. Kim, A. Melfo, M. Primc and G. Senjanović for discussions on solutions to the minimum equations.

[4]This would *break* one of the equations of (9) and destroy the tri-triangle solution. However, since the trivial CP conserving solution is also destroyed, the *only* solution is automatically CP violating.

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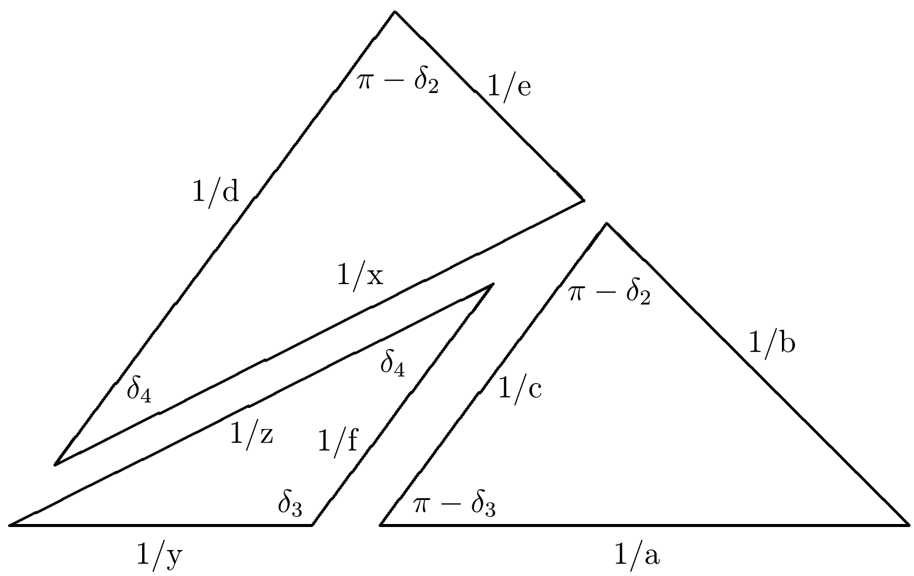

Figure 1: The geometrical object which represents the CP nontrivial solution of equations (9) consists of three triangles. Each triangle contains two of the three angles δ_2, δ_3 and δ_4. The sides of the triangles are denoted by 1/a, 1/b, 1/c, 1/d, 1/e, 1/f, 1/x, 1/y, 1/z.