Refutation of C. W. Misner's claims in his article "Yılmaz Cancels Newton"

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Abstract

It is shown that an article by C. W. Misner [1] contains serious errors. In particular, the claim that the Yılmaz theory of gravitation cancels the Newtonian gravitational interaction is based on a false premise. With the correct premise the conclusion of the article regarding the absence of gravitational interactions applies to general relativity and not to the Yılmaz theory.

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In an article by C. W. Misner [1] the expression referring to the stress-energy of an ideal fluid,

$$\chi_\mu{}^\nu = (\rho + P)u_\mu u^\nu - P\delta_\mu{}^\nu \;, \tag{1}$$

is called "relativistic matter", implying that it would be equated to the matter part $\tau_\mu{}^\nu$ in the Yılmaz field equations [2]

$$\frac{1}{2}G_\mu{}^\nu = \tau_\mu{}^\nu + t_\mu{}^\nu \tag{2}$$

$$D_\nu G_\mu{}^\nu \equiv 0 \tag{3}$$

$$\partial_\nu(\sqrt{-g}\,\tau_\mu{}^\nu) \equiv 0 \;, \tag{4}$$

which are written in Cartesian coordinates as we shall later present the solution for $\chi_\mu{}^\nu$ in such coordinates. [See Appendix A for conventions, definitions and coordinate-independent forms of $\tau_\mu{}^\nu$, $t_\mu{}^\nu$ and ∂_ν.]

But, of course, calling $\chi_\mu{}^\nu$ "relativistic matter" does not make it so, hence one is first obligated to find out what it is. We can find this out from the first and second approximation to the Newtonian theory in the equilibrium limit.

$$\chi_\mu{}^\nu = (\rho + P)u_\mu u^\nu - P\delta_\mu{}^\nu \;\Rightarrow\; \begin{pmatrix} \rho & & & \\ & -P & & \\ & & -P & \\ & & & -P \end{pmatrix} . \tag{5}$$

We need only three pieces of information for this, namely,

$$g_{00} \simeq 1 + 2\Phi \;, \quad g_{ij} \simeq -\delta_{ij}(1 - 2\Phi) \tag{6}$$

$$\Phi = \frac{r^2}{4R^2} \tag{7}$$

$$P = -\frac{3}{8}\frac{r^2 - r_0{}^2}{R^4} \;, \tag{8}$$

where $1/R^2 = 2\rho/3$. (Note that the pressure is a second-order quantity in terms of the energy density ρ.) In the Newtonian approximation these lead to the following relations:

$$\partial_i P \simeq \frac{1}{2}\partial_i g_{00}\chi^{00} \tag{9}$$

for the pressure gradient and

$$\rho\,\partial_i\Phi = -\partial_i P \;, \tag{10}$$

because of the equilibrium. Also, from (6) we get to first order $\tau_0{}^0 \simeq \rho$.

Units are chosen $c = 1$ and $4\pi G = 1$ to avoid having c and $4\pi G$ appear in most of the equations. We use $\chi_\mu{}^\nu$ to denote the above expression instead of $T_\mu{}^\nu$ because the use of $T_\mu{}^\nu$ can be confusing, sometimes denoting $T_\mu{}^\nu = \tau_\mu{}^\nu$ and sometimes $\frac{1}{2}G_\mu{}^\nu = T_\mu{}^\nu$. Also we let (5) represent the equilibrium case, $\rho\,dv^i/dt = 0$, of a perfect fluid sphere since no solution for the more general non-equilibrium case is presented by [1] or by us.

We can now test whether $\chi_\mu{}^\nu$ could be $\tau_\mu{}^\nu$ as [1] assumes. In the Newtonian limit we have $\sqrt{-g}P \simeq P$ to second order, so that $\partial_\nu(\sqrt{-g}\,\chi_0{}^\nu) = 0$ and

$$\partial_\nu(\sqrt{-g}\,\chi_i{}^\nu) = -\partial_i(\sqrt{-g}P) \simeq -\partial_i P \neq 0 \;, \tag{11}$$

hence the Freud identity (4) is not satisfied. This shows that $\chi_\mu{}^\nu$ can not possibly be equated to $\tau_\mu{}^\nu$ even in the Newtonian approximation.

Let us next test whether $\chi_\mu{}^\nu$ could be identified with $\tau_\mu{}^\nu + t_\mu{}^\nu$ in which case it would satisfy the Bianchi identity (2). Let us write

$$\chi_\mu{}^\nu = \tau_\mu{}^\nu + t_\mu{}^\nu \tag{12}$$

and take the covariant divergence. Using the Freud identity (4) this gives

$$D_\nu\chi_i{}^\nu = \frac{1}{\sqrt{-g}}\partial_\nu(\sqrt{-g}\,\tau_i{}^\nu) - \frac{1}{2}\partial_i g_{\alpha\beta}\tau^{\alpha\beta} + D_\nu t_i{}^\nu \tag{13}$$

$$= -\frac{1}{2}\partial_i g_{\alpha\beta}\tau^{\alpha\beta} + D_\nu t_i{}^\nu \simeq -\frac{1}{2}\partial_i g_{00}\tau^{00} + \partial_\nu t_i{}^\nu \;. \tag{14}$$

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In this approximation we may use Eqs. (9) and (A.3) of Appendix A to get

$$D_\nu \chi_i{}^\nu = -\partial_i P - \rho\, \partial_i \Phi \ , \tag{15}$$

which by (10) verifies the Bianchi identity

$$D_\nu \left(\frac{1}{2} G_i{}^\nu\right) = D_\nu \chi_i{}^\nu = \rho\, \partial_i \Phi - \rho\, \partial_i \Phi \equiv 0 \ . \tag{16}$$

Although the identity has been verified to second order for simplicity, it is valid to all orders.

What we have just proved is that $\chi_\mu{}^\nu$, which appears at first sight to be solely matter stress-energy, is in reality $\tau_\mu{}^\nu + t_\mu{}^\nu$, that is, $\chi_\mu{}^\nu$ is composed of both matter and field stress-energies. [This is clearly visible in the actual solution of the equilibrium problem presented in Appendix B.] In other words, Misner's premise that $\chi_\mu{}^\nu$ is matter alone and that it would be identified with the matter part $\tau_\mu{}^\nu$ of Yılmaz' theory, is false. Misner's claims in [1] can not be valid because the basic premise on which they depend is incorrect.

In principle we can stop here and go no further with [1]. However, [1] makes other incorrect statements, clouding the basic understanding of the subject. For this reason we add a number of notes. These notes provide additional information and, where needed, explicit calculations to indicate that [1] simply fails to convey the true situation.

Notes

1. Since Misner considers $\chi_\mu{}^\nu$ to be purely matter, as is clear from the discussion after his Eq. (1.1), he makes the identification

$$\chi_\mu{}^\nu = \tau_\mu{}^\nu \tag{17}$$

in his Eq. (1.2), which we have earlier shown to be his false premise. Therefore, according to Misner's Eq. (1.1), Einstein's field equations are written

$$\frac{1}{2} G_\mu{}^\nu = \tau_\mu{}^\nu \ . \tag{18}$$

Now, to second order in equilibrium we get from the left-hand side of equation (17)

$$\partial_\nu(\sqrt{-g}\,\chi_i{}^\nu) \simeq -\partial_i P \ , \tag{19}$$

whereas from the right-hand side we get by the Freud identity (4)

$$\partial_\nu(\sqrt{-g}\,\tau_\mu{}^\nu) = 0 \ . \tag{20}$$

Combining, we find

$$-\partial_i P = 0 \ . \tag{21}$$

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But with his Eq. (4.4) Misner attributes this result to Yılmaz' theory.

On the other hand, as we have earlier shown, starting with the correct premise $\chi_\mu{}^\nu = \tau_\mu{}^\nu + t_\mu{}^\nu$ one derives the result

$$-\partial_i P - \rho\,\partial_i \Phi = 0 \;, \tag{22}$$

which Misner attributes to Einstein's theory in Eq. (4.4). Thus the conclusions that Misner draws from that equation are actually true in reverse. Ironically, therefore, Misner's conclusions regarding the absence of gravitational interaction applies to general relativity, and not to the Yılmaz theory.

2. In Appendix A of [1] the total stress-energy in the Newtonian limit is stated as

$$T_N^{ik} = \rho u^i u^k + P^{ik} + t_N^{ik} \;, \tag{23}$$

that is, the Newtonian field stress-energy t_N^{ik} is a necessary part of the total stress-energy. It is then clear that, in order for a theory to reduce to the Newtonian theory in the limit, this field stress-energy must be recovered. This means that the Newtonian theory must be recovered to first and second order in the limit (a first-order correspondence is not sufficient) since t_N^{ik} is a second order quantity. This contradicts the statement in the abstract of [1] that the Newtonian limit would not be affected by the field stress-energy.

3. In the past two of us [3] have challenged the relativity community to find an acceptable explanation of the simple and basic Cavendish experiment using general relativity. So far, we have not received an adequate response to this challenge. Instead, as in [1], the issue is carried into areas not related to the force between the two spatially separated bodies in the Cavendish experiment. As can be seen [1] does not provide an explanation of the Cavendish experiment by general relativity.

4. The new theory is in principle a fundamental microscopic theory, and, as such, deals with particles and waves. In such a theory thermodynamic properties are to arise from the motions and collisions of the particles and not from the continuum equations of a classical perfect fluid. The continuum limit is to be arrived at through statistical averages. It is, however, gratifying that even without this averaging a continuum solution to the theory is possible as exhibited in Appendix B.

5. It is well known that general relativity has severe problems with quantization. Recently it has been found that this is due to the absence of $t_\mu{}^\nu$ in the field equations of general relativity. When $t_\mu{}^\nu$ is present as in the Yılmaz theory, the gravitational field can be quantized via Feynman's method [4]. The reason for this is that in order to quantize a field theory by Feynman's method one has to have a field lagrangian and one can not have have field lagrangian without having a field stress energy. Further, it has been found that the quantized theory is finite. It appears that with the new theory the dream of generations of physicists is being realized.

6. The article [1] complains about the time-independent exact N-particle interactive solution

$$g_{00} = e^{2\Phi} \;, \quad g_{ij} = -\delta_{ij}\, e^{-2\Phi} \tag{24}$$

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$$\Phi = -\sum_A \frac{m_A}{|\mathbf{x} - \mathbf{x}_A|} \tag{25}$$

of the new theory, saying that if it is exact and independent of time, then nothing can move. This statement can not be correct because in the Newtonian theory the Poisson equation has exactly such a solution. We can therefore do whatever is done in the Newtonian theory to introduce equations of motion and apply it to the Newtonian limit of the new theory. If a theory did not have such time-independent N-body solutions then one would have to worry since the Newtonian theory has them. The terminology of calling such solutions "static" is a misnomer. They should be called instantaneous-action solutions, corresponding to the $c \to \infty$ limit, since the explicit time dependence of Φ drops out in this limit.

7. A most important feature of the new theory is that the Bianchi identity is satisfied both by the left-hand side and the right-hand side of the field equations *as an identity*, whereas in general relativity the left-hand side satisfies the Bianchi identity identically but the right-hand side does not. We are told that Einstein himself was aware of this and that is why he many times said "My equation is like a house with two wings; the left-hand side is made of fine marble, but the right-hand side is perishable wood". It is said that it was his "dream" to find a right-hand side that also satisfies the Bianchi identity, but this was judged to be too difficult or impossible, and it was given up. Instead, the divergence of the right-hand side is forced to zero. But then this becomes a *constraint* on matter or on the field (or both), making the theory mathematically overdetermined.

8. Finally, let it be understood that we wish to implement Einstein's concept of gravitation as curved spacetime, in the most satisfactory way. With the advent of modern symbolic calculation software (Mathematica, Maple, etc.), available to everyone, we calculate in the spirit of Leibniz' maxim for the resolution of scientific disputes, "Calculemus", and go with what we see. All such calculations (which we invite interested persons to carry out for themselves) lead us to one single overall conclusion: In order to be physically and mathematically correct, the conventional dictum, "the right-hand side of the field equations is all stress-energy, *except* the gravitational field stress-energy", must be changed into "the right-hand side of the field equations is all stress-energy, *including* the gravitational field stress-energy. Without the inclusion of the gravitational field stress-energy $t_\mu{}^\nu$ there is no interaction between bodies of finite mass because, as in the Newtonian case, the gravitational force density is the divergence of $t_\mu{}^\nu$.

We shall not dwell on other minor misconceptions in [1]. Enough has been said of its major misconceptions to indicate that its conclusions must be taken in the reverse direction.

Appendix A. Conventions, terminology, and the Yılmaz theory

We define the metric $g_{\mu\nu}$ and the Newtonian potential Φ so that to first order $g_{00} = 1 + 2\Phi$,

$g_{ik} = -\delta_{ik}(1 - 2\Phi)$ and $\chi_0{}^0 = \rho \simeq \sigma$. This causes the Poisson equation to take the form

$$\nabla^2 \Phi = \sigma \, , \tag{A.1}$$

where ∇^2 is the ordinary Laplacian and $\sigma = \sigma_{\text{ord}}$ is the ordinary mass density as in the Newtonian theory. We define the gravitational field stress-energy in the Newtonian limit as (Misner's $t_\mu{}^\nu$ is the negative of ours)

$$t_\mu{}^\nu = -\partial_\mu \Phi \, \partial^\nu \Phi + \frac{1}{2} \delta_\mu{}^\nu \partial_\lambda \Phi \, \partial^\lambda \Phi \, , \tag{A.2}$$

from which follows

$$\partial_\nu t_\mu{}^\nu = \nabla^2 \Phi \, \partial_\mu \Phi = \sigma \, \partial_\mu \Phi \, . \tag{A.3}$$

These equations refer to the time-independent case, as they will be applied only to the equilibrium state of a sphere of perfect fluid. In first order $\sigma \simeq \rho$. The non-equilibrium case can be treated by letting $v^i \neq 0$, $dv^i/dt \neq 0$ but no solution for this more general case is presented, neither by [1] nor by us, so we must refrain from generalities for which we have no solutions. For example, in [1] the expression $\rho \, dv^i/dt$ frequently appears. To be honest, we should really set this term to zero. As we will see, there is plenty to discuss and understand already for the case of equilibrium.

In [1] the quantity
$$T^{\mu\nu} = (\rho + P)u^\mu u^\nu - Pg^{\mu\nu} \tag{A.4}$$

is introduced, stating that it represents the stress-energy tensor of matter. We write this expression with mixed indices and denote it as

$$\chi_\mu{}^\nu = (\rho + P)u_\mu u^\nu - P\delta_\mu{}^\nu \, . \tag{A.5}$$

There are two reasons for this. One is that the use of $T^{\mu\nu}$ can be ambiguous, sometimes denoting the right-hand side of $\frac{1}{2}G_\mu{}^\nu = T_\mu{}^\nu$ and sometimes denoting the matter part $\tau_\mu{}^\nu$ of the Yılmaz theory. The second reason is that we really do not know what $\chi_\mu{}^\nu$ is. We would like to call it $\chi_\mu{}^\nu$ as if it is an unknown and find out what it is, as is done in the text.

The equations in the Yılmaz theory are

$$\frac{1}{2}G_\mu{}^\nu = \tau_\mu{}^\nu + t_\mu{}^\nu \tag{A.6}$$

$$D_\nu G_\mu{}^\nu \equiv 0 \tag{A.7}$$

$$\bar{\partial}_\nu(\sqrt{-\kappa}\,\tau_\mu{}^\nu) \equiv 0 \, . \tag{A.8}$$

The definitions of $\tau_\mu{}^\nu$ and $t_\mu{}^\nu$ are explicitly given by

$$\tau_\mu{}^\nu = \frac{1}{4\sqrt{-\kappa}} \bar{\partial}_\alpha \left[\mathbf{g}^{\alpha\lambda}\mathbf{g}^{\nu\rho}(\bar{\partial}_\rho \mathbf{g}_{\mu\lambda} - \bar{\partial}_\lambda \mathbf{g}_{\mu\rho}) + \delta_\mu{}^\nu \bar{\partial}_\beta \mathbf{g}^{\beta\alpha} - \delta_\mu{}^\alpha \bar{\partial}_\beta \mathbf{g}^{\beta\nu} \right] \tag{A.9}$$

$$t_\mu{}^\nu = W_\mu{}^\nu - \frac{1}{2}\delta_\mu{}^\nu W_\lambda{}^\lambda \tag{A.10}$$

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where

$$W_\mu{}^\nu = \frac{1}{8\sqrt{-\kappa}} \mathbf{g}^{\nu\lambda} \left[\bar{\partial}_\lambda \mathbf{g}_{\alpha\beta} \bar{\partial}_\mu \mathbf{g}^{\alpha\beta} - 2\,\bar{\partial}_\lambda(\sqrt{-\kappa})\,\bar{\partial}_\mu(\frac{1}{\sqrt{-\kappa}}) - 2\,\bar{\partial}_\alpha \mathbf{g}_{\lambda\beta} \bar{\partial}_\mu \mathbf{g}^{\alpha\beta} \right] \qquad (A.11)$$

$$\mathbf{g}^{\mu\nu} = \sqrt{-\kappa}\,g^{\mu\nu}\,, \qquad \mathbf{g}_{\mu\nu} = \frac{1}{\sqrt{-\kappa}}\,g_{\mu\nu}\,, \qquad\qquad\qquad (A.12)$$

where $\bar{\partial}_\nu$ denotes the covariant derivative with respect to the metric $\eta_\mu{}^\nu$ of the local flat space [5] and

$$\sqrt{-\kappa} = \frac{\sqrt{-g}}{\sqrt{-\eta}}\,. \qquad\qquad\qquad (A.13)$$

If the coordinates are locally Lorentzian then $\bar{\partial}_\nu$ is just the ordinary derivative ∂_ν as in the text. To make the exposition simpler we chose Lorentzian coordinates, although in general ∂_ν should be replaced everywhere by $\bar{\partial}_\nu$. The virtue of this procedure is that these definitions and the Freud decomposition $\frac{1}{2}G_\mu{}^\nu = \tau_\mu{}^\nu + t_\mu{}^\nu$ are valid using any local flat-space coordinate system.

The concept of the covariant derivative with respect to the coordinates of a local flat space was introduced by N. Rosen [6] in 1940. Rosen later used this concept to formulate his bimetric theory, which is different from the Yılmaz theory. Yılmaz uses it to define the matter and field stress-energy tensors $\tau_\mu{}^\nu$ and $t_\mu{}^\nu$ in a coordinate-independent way. Physically $\tau_\mu{}^\nu$ and $t_\mu{}^\nu$ are identified through correspondence to the Newtonian theory and to special relativity.

It can be seen that the Yılmaz theory is far easier to work with than general relativity, especially when the four-index Riemann tensor $R^\rho_{\mu\nu\sigma}$ is not needed. For then the only things to calculate are $\tau_\mu{}^\nu$ and $t_\mu{}^\nu$. In the usual practice of general relativity one does not pay much attention to the Freud identity, but it is clear that it plays a fundamental role in providing an unambigous definition of the matter and field components of the total stress-energy.

Appendix B. Interior solution for the perfect fluid sphere

From what is said above it is clear that $\chi_\mu{}^\nu = (\rho + P)u_\mu u^\nu - P\delta_\mu{}^\nu$ is of the form $\tau_\mu{}^\nu + t_\mu{}^\nu$, hence it does not belong to general relativity. If it did, the $t_\mu{}^\nu$ would be missing. For as stated in the introduction of [1] the formulation of general relativity is that the right-hand side of the field equations is "everything *except* the gravitational field stress-energy". The corresponding statement in the new theory is that the right-hand side is "everything *including* the gravitational field stress-energy". One may ask whether it is true that the usual Schwarzschild interior solution in reality belongs to the new theory? The answer is, almost yes but not quite.

We present below an iterative solution valid to second order for the ideal fluid sphere, which is compared with the Schwarzschild interior solution evaluated to the same order. The

Schwarzschild metric [7] and our metric are both given in Cartesian coordinates to make the comparison intuitively simple. The solution in the new theory is determined by the following physical information. We require that

$$\frac{1}{2}G_\mu{}^\nu = \chi_\mu{}^\nu ,$$ (B.1)

where $\chi_\mu{}^\nu$ has the form (5), and

$$\chi_0{}^0 = \rho = \tau_0{}^0 + t_0{}^0 , \quad \chi_i{}^j = -\delta_i{}^j P ,$$ (B.2)

$$\tau_0{}^0 = \sigma_{\text{cov}} = \text{the covariant Laplacian of } \Phi ,$$ (B.3)

with the pressure P being given by (8).

Schwarzschild: $\quad g_{00} = \left[\frac{3}{2} \left(\frac{1-\Phi_0}{1+\Phi_0} \right) - \frac{1}{2} \left(\frac{1-\Phi}{1+\Phi} \right) \right]^2 , \quad g_{ij} = -\delta_{ij} \frac{1}{(1+\Phi)^2}$ (B.4)

Yılmaz: $\quad g_{00} = e^{2(\Phi-3\Phi_0)(1-\Phi)} , \quad g_{ij} = -\delta_{ij} e^{-2\Phi} ,$ (B.5)

where

$$\Phi = \frac{1}{6}\sigma_{\text{ord}}\, r^2$$ (B.6)

(Note that g_{00} has the form $e^{2(\Phi+\Psi)}$, where Φ is a function of matter density and Ψ is a function of pressure.) These metrics give, respectively,

$$\frac{1}{2}G_\mu{}^\nu \qquad\qquad\qquad \tau_\mu{}^\nu \qquad\qquad\qquad t_\mu{}^\nu$$ (B.7)

Schwarzschild: $\quad \begin{pmatrix} \sigma_{\text{ord}} & 0 \\ \hline 0 & -\delta_i{}^j P \end{pmatrix} = \begin{pmatrix} \sigma_{\text{ord}} + S_0{}^0 & 0 \\ \hline 0 & -\delta_i{}^j P + S_i{}^j \end{pmatrix} + \begin{pmatrix} t_0{}^0 & 0 \\ \hline 0 & t_i{}^j \end{pmatrix}$ (B.8)

Yılmaz: $\quad \begin{pmatrix} \sigma_{\text{cov}} + t_0{}^0 & 0 \\ \hline 0 & -\delta_i{}^j P \end{pmatrix} = \begin{pmatrix} \sigma_{\text{cov}} & 0 \\ \hline 0 & -\delta_i{}^j P + S_i{}^j \end{pmatrix} + \begin{pmatrix} t_0{}^0 & 0 \\ \hline 0 & t_i{}^j \end{pmatrix}$ (B.9)

$$\rho \simeq \sigma_{\text{cov}} + t_0{}^0$$
$$\sigma_{\text{ord}} \simeq \sqrt{-g}\,\sigma_{\text{cov}}$$
$$\frac{dP}{dr} \simeq -\rho\,\frac{d\Phi}{dr} ,$$

which are valid to first and the second order. These solutions differ because in the new theory the $\frac{1}{2}G_\mu{}^\nu = \rho$ includes not only the matter density σ_{cov} but also the energy density of the field, $t_0{}^0$, thus satisfying the mass-energy correspondence of special relativity.

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Note that the material stresses $S_\mu{}^\nu$ do not have the same form as $t_\mu{}^\nu$, yet at equilibrium where $\Phi = r^2/4R^2$ they cancel to form the pressure. This can be seen from

$$S_0{}^0 = -(\nabla\Phi)^2 + \Phi\,\nabla^2\Phi \qquad\qquad t_0{}^0 = -\frac{1}{2}(\nabla\Phi)^2$$

$$S_1{}^1 = -\Phi_{,2}^2 - \Phi_{,3}^2 + \frac{1}{2}\Phi(\Phi_{,22} + \Phi_{,33}) \qquad t_1{}^1 = -\frac{1}{2}(\Phi_{,2}^2 + \Phi_{,3}^2 - \Phi_{,1}^2) \ . \qquad\text{(B.10)}$$

$$S_1{}^2 = -\Phi_{,1}\Phi_{,2} + \frac{1}{2}\Phi\Phi_{,12} \qquad\qquad t_1{}^2 = \Phi_{,1}\Phi_{,2}$$

All other components are obtainable by symmetry. The essential point is that $t_\mu{}^\nu$ is present and forms a fundamental part of the total stress-energy as in the new theory.

The second-order solution above can be iterated to higher orders in Φ. For example, to third order the solution is

$$g_{00} = \exp[2(\Phi - 3\Phi_0)(1 - \Phi) + \frac{4}{3}\Phi^3 - 6\Phi^2\Phi_0 + 4\Phi\Phi_0^2] \qquad\text{(B.11)}$$

$$g_{ik} = -\delta_{ik}\,e^{-2\Phi} \ . \qquad\text{(B.12)}$$

For the Yılmaz solution $\tau_\mu{}^\nu$ and $t_\mu{}^\nu$ have the following expressions:

$$\tau_t{}^t = \sigma_{\text{cov}} \qquad\qquad\qquad t_t{}^t = -\frac{\sigma_{\text{cov}}^2}{18}r^2$$

$$\tau_x{}^x = -\frac{\sigma_{\text{cov}}^2}{18}(3r_0^2 - 4r^2 + 2x^2) \qquad t_x{}^x = \frac{\sigma_{\text{cov}}^2}{18}(2x^2 - r^2)$$

$$\tau_x{}^y = -\frac{\sigma_{\text{cov}}^2}{9}xy \qquad\qquad t_x{}^y = \frac{\sigma_{\text{cov}}^2}{9}xy$$

$$\tau_x{}^z = -\frac{\sigma_{\text{cov}}^2}{9}xz \qquad\qquad t_x{}^z = \frac{\sigma_{\text{cov}}^2}{9}xz$$

$$\tau_y{}^x = -\frac{\sigma_{\text{cov}}^2}{9}yx \qquad\qquad t_y{}^x = \frac{\sigma_{\text{cov}}^2}{9}yx$$

$$\tau_y{}^y = -\frac{\sigma_{\text{cov}}^2}{18}(3r_0^2 - 4r^2 + 2y^2) \qquad t_y{}^y = \frac{\sigma_{\text{cov}}^2}{18}(2y^2 - r^2) \qquad\text{(B.13)}$$

$$\tau_y{}^z = -\frac{\sigma_{\text{cov}}^2}{9}yz \qquad\qquad t_y{}^z = \frac{\sigma_{\text{cov}}^2}{9}yz$$

$$\tau_z{}^x = -\frac{\sigma_{\text{cov}}^2}{9}zx \qquad\qquad t_z{}^x = \frac{\sigma_{\text{cov}}^2}{9}zx$$

$$\tau_z{}^y = -\frac{\sigma_{\text{cov}}^2}{9}zy \qquad\qquad t_z{}^y = \frac{\sigma_{\text{cov}}^2}{9}zy$$

$$\tau_z{}^z = -\frac{\sigma_{\text{cov}}^2}{18}(3r_0^2 - 4r^2 + 2z^2) \qquad t_z{}^z = \frac{\sigma_{\text{cov}}^2}{18}(2z^2 - r^2) \ ,$$

where

$$\sigma_{\text{ord}} = \nabla^2\Phi \qquad\text{(B.14)}$$

$$\sigma_{\text{cov}} = -\frac{1}{\sqrt{-g}}\, \partial_\nu(\sqrt{-g}\, \partial^\nu \Phi) \tag{B.15}$$

$$\simeq \frac{1}{\sqrt{-g}}\nabla^2\Phi \tag{B.16}$$

$$\simeq \sigma_{\text{ord}}\,(1 + 2\Phi)\,. \tag{B.17}$$

For completeness the corresponding exterior solutions are shown below:

$$\text{Schwarzschild:} \qquad g_{00} = \left(\frac{1 + \dfrac{\Phi}{2}}{1 - \dfrac{\Phi}{2}}\right)^2, \qquad g_{ij} = -\delta_{ij}\left(1 - \frac{\Phi}{2}\right)^4 \tag{B.18}$$

$$\text{Yılmaz:} \qquad g_{00} = e^{2\Phi}\,, \qquad g_{ij} = -\delta_{ij}\, e^{-2\Phi}\,, \tag{B.19}$$

where

$$\Phi = -\frac{M}{r} \tag{B.20}$$

These metrics give, respectively,

$$\frac{1}{2}G_\mu{}^\nu \qquad\qquad \tau_\mu{}^\nu \qquad\qquad t_\mu{}^\nu \tag{B.21}$$

$$\text{Schwarzschild:} \qquad \left(\begin{array}{c|c} 0 & 0 \\ \hline 0 & 0 \end{array}\right) = \left(\begin{array}{c|c} 0 & 0 \\ \hline 0 & 0 \end{array}\right) + \left(\begin{array}{c|c} 0 & 0 \\ \hline 0 & 0 \end{array}\right) \tag{B.22}$$

$$\text{Yılmaz:} \qquad \left(\begin{array}{c|c} t_0{}^0 & 0 \\ \hline 0 & t_i{}^j \end{array}\right) = \left(\begin{array}{c|c} 0 & 0 \\ \hline 0 & 0 \end{array}\right) + \left(\begin{array}{c|c} t_0{}^0 & 0 \\ \hline 0 & t_i{}^j \end{array}\right) \tag{B.23}$$

where

$$t_0{}^0 = -\frac{1}{2}(\nabla\Phi)^2$$

$$t_1{}^1 = -\frac{1}{2}(\Phi_{,2}^2 + \Phi_{,3}^2 - \Phi_{,1}^2) \tag{B.24}$$

$$t_1{}^2 = \Phi_{,1}\Phi_{,2}$$

All other components are obtainable by symmetry. Note that in the Schwarzschild solution $t_\mu{}^\nu$ is missing.

For the Yılmaz solution, $\tau_\mu{}^\nu$ and $t_\mu{}^\nu$ have the following expressions:

$$\tau_t{}^t = 0 \qquad\qquad t_t{}^t = -\frac{M^2}{2r^4}$$

$$\tau_x{}^x = 0 \qquad\qquad t_x{}^x = \frac{M^2}{2r^6}(2x^2 - r^2)$$

$$\tau_x{}^y = 0 \qquad\qquad t_x{}^y = \frac{M^2}{r^6}xy$$

$$\tau_x{}^z = 0 \qquad\qquad t_x{}^z = \frac{M^2}{r^6}xz$$

$$\tau_y{}^x = 0 \qquad\qquad t_y{}^x = \frac{M^2}{r^6}yx$$

$$\tau_y{}^y = 0 \qquad\qquad t_y{}^y = \frac{M^2}{2r^6}(2y^2 - r^2) \qquad\qquad \text{(B.25)}$$

$$\tau_y{}^z = 0 \qquad\qquad t_y{}^z = \frac{M^2}{r^6}yz$$

$$\tau_z{}^x = 0 \qquad\qquad t_z{}^x = \frac{M^2}{r^6}zx$$

$$\tau_z{}^y = 0 \qquad\qquad t_z{}^y = \frac{M^2}{r^6}zy$$

$$\tau_z{}^z = 0 \qquad\qquad t_z{}^z = \frac{M^2}{2r^6}(2z^2 - r^2) \,,$$

Incidentally, in the Yılmaz theory there are no black holes in the sense of event horizons, but there can be stellar collapse as observed. Radially directed light can always escape, although red-shifted. However, there are no point singularities since the invariant curvature quantities for the exterior such as the Ricci invariant $R = R_\mu{}^\mu = 2M^2/(r^4 e^{2M/r})$ and the Kretschmann invariant $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} = 4M^2(7M^2 - 16Mr + 12r^2)/(r^8 e^{4Mr})$ do not diverge. In fact, they go to zero as the radius goes to zero. This seems to mean that even in stellar collapse and in the early universe a quantum theory of gravitation based on Yılmaz theory will not lead to inconsistency.

Note that if we hold the statement of general relativity as "the right-hand side is everything *except* field stress-energy", neither of the interior solutions discussed above belongs to general relativity because, contrary to the statement above, the field stress-energy $t_\mu{}^\nu$ is fully present in both. The main difference between these solutions is that in the Schwarzschild solution $\frac{1}{2}G_0{}^0 = \sigma_{\text{ord}}$, whereas in the Yılmaz theory $\frac{1}{2}G_0{}^0 = \sigma_{\text{cov}} + t_0{}^0$. This shows that only the latter solution has the correct special-relativistic correspondence.

In the Yılmaz theory the exterior solution has also a field-energy density $t_0{}^0$. Since $\Phi_{\text{int}} = r^2/4R^2$, $\Phi_{\text{ext}} = -M/r$ and $t_0{}^0 = -(\nabla\Phi)^2/(8\pi)$, we find that

$$\mathcal{E} = \int_0^{r_0} t_0{}^0(\text{int.})\, dV + \int_{r_0}^\infty t_0{}^0(\text{ext.})\, dV \qquad\qquad \text{(B.26)}$$

$$= -\frac{M^2}{10 r_0} - \frac{M^2}{2 r_0} = -\frac{3}{5}\frac{M^2}{r_0} \ , \tag{B.27}$$

which is exactly the Newtonian field energy of the fluid sphere. The interior Schwarzschild metric does not belong to general relativity since the right-hand side of the field equations includes field stress-energy, contrary to the statement of general relativity.

Thus in the Yılmaz theory the interior and the exterior field energies are as in the Newtonian theory, whereas in general relativity there is no field energy in the exterior. Clearly general relativity does not have a unique correspondence with the Newtonian theory. For in the exterior case $t_\mu{}^\nu$ is assumed to be zero (in fact, set to zero by $\frac{1}{2}G_\mu{}^\nu = 0$), whereas in the interior no such condition exists, hence $t_\mu{}^\nu$ inadvertently sneaks in. Thus there is an inconsistency between the exterior and interior solutions in general relativity, whereas in the Yılmaz theory the expressions of $\tau_\mu{}^\nu$ and $t_\mu{}^\nu$ are unambiguously defined both in the exterior and the interior (see Appendix A).

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