AREA-PRESERVING ALGEBRA STRUCTURE OF TWO-DIMENSIONAL SUPERGRAVITY

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Abstract

The effective action for 2d-gravity in Weyl-invariant regularization is extended to supersymmetric case. The super area-preserving invariance and cocyclic properties under general supergravitational transformations of the last action is shown.

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1 Introduction.

The Weyl symmetry in the string theory is a fundamental principle which governs the string dynamics [1].

The super Weyl transformations, which firstly had been considered by Howe [2] are useful for investigation of the superspace constraints in the supergravity theories [3].

Even being spoiled by quantum corrections the Weyl symmetry of classical theory implies the cocyclic property of the effective action. It is so restrictive that fixes action up to numerical constants [4]. This form of the effective action causes an independent interest as the theory of matter field in (super) Weyl invariant (super) gravitational background [5].

In previous work [6] we have studied transformation properties of the effective action of 2d-gravity under Weyl rescaling of metric. The well known non-local functional of effective action of 2d-gravity is

$$W[g] = \int R \frac{1}{\Box} R \tag{1.1}$$

The Weyl-coboundary of $\mathring{1}$

$$W[e^{\sigma} g] - W[g] \equiv S_L(\sigma, g) \tag{1.2}$$

coincides with the famous Liouville action

$$S_L(\sigma, g) = \int d^2 x \sqrt{g} (\frac{1}{2} g^{\alpha\beta} \partial_\alpha \sigma \partial_\beta \sigma + R\sigma) \tag{1.3}$$

The action $\mathring{3}$ satisfies to cocyclic condition

$$S_L(\sigma_1 + \sigma_2, g) - S_L(\sigma_1, e^{\sigma_2} g) - S_L(\sigma_2, g) = 0 \tag{1.4}$$

which follows from $\mathring{2}$.The relation $\mathring{4}$ allow us to consider $\mathring{3}$ as the anomaly arising from mesure's transformation in the bosonic string theory:

$$D_{e^{\sigma} g} X^{\mu} = e^{-D S_L(\sigma, g)} D_g X^{\mu} \tag{1.5}$$

where D is dimension of target space. The effective action of 2d-gravity, computed in the Weyl invariant regularization is simply functional $\mathring{1}$, depending on the Weyl invariant combination $g_{\alpha\beta}/\sqrt{g}$

$$\tilde{W}[g_{\alpha\beta}] = W\left[\frac{g_{\alpha\beta}}{\sqrt{g}}\right] = \int (R(g) - \Box \log \sqrt{g}) \sqrt{g} \frac{1}{\sqrt{g}\Box} (R(g) - \Box \log \sqrt{g}) \sqrt{g}. \tag{1.6}$$

and differ from $\mathring{1}$ by the local counterterm:

$$\tilde{W}[g] = W[g] + S_L(\sigma, g)|_{\sigma = -\log \sqrt{g}} \tag{1.7}$$

However the Weyl invariant action $\mathring{7}$ is not scalar under general coordinate transformation $Diff_2$ now, it is invariant only under area- preserving $SDiff_2$ subgroup of $Diff_2$. So under shifting $\mathring{7}$ we trade $Diff_2$ for $Weyl \times SDiff_2$ and corresponding functional mesure is not diffeomorphism invariant.

$$\tilde{D}_{f^*g} X^\mu = e^{-dS(f,g)} \tilde{D}_g X^\mu . \tag{1.8}$$

Here we used the notations

$$S(f,g) \equiv S_L(\sigma, g)|_{\sigma=\log \Delta_x^F}, \quad \Delta_x^F = \det \| \frac{\partial F^\beta}{\partial x^\alpha} \|$$

and F^α is inverse to f^α: $F^\pm(f^+, f^-) = x^\pm$

2 The super Liouville action

The extension of procedure, described above, to case of the 2d supergravity is transparent. The super Liouville action consist from two parts: kinetic and topological.

$$S_{S-L}(\sigma, \psi; e^a_\alpha, \chi_\alpha) = S_0 + S_1 \tag{2.1}$$

The kinetic part of $\mathring{9}$ is well known Neveu-Schwarz string action:

$$S_0 = \int d^2 x e (g^{\alpha\beta} \partial_\alpha \sigma \partial_\beta - i \bar{\psi} \gamma^\beta \gamma^\alpha \partial_\alpha \psi + 2 \bar{\chi}_\alpha \gamma^\beta \gamma^\alpha \psi \partial_\beta \sigma + \frac{1}{2} \bar{\psi} \psi \bar{\chi}_\alpha \gamma^\beta \gamma^\alpha \chi_\beta) \tag{2.2}$$

and topological part is

$$S_1 = \int d^2 x \epsilon^{\alpha\beta} (\omega_\alpha(e) + \frac{1}{2} \bar{\chi}_\alpha \gamma_3 \gamma^\mu \chi_\mu)(\partial_\beta \sigma + \bar{\chi}_\nu \gamma_\beta \gamma^\nu \psi) \tag{2.3}$$

The S_0 and S_1 are separately invariant under local supersymmetry transformations. Their sum satisfies to the cocyclic condition:

$$\begin{aligned} S_{S-L}(\sigma_1 + \sigma_2, \psi_1; e^a_\alpha, \chi_\alpha) &= S_{S-L}(\sigma_2, \psi_2; e^a_\alpha, \chi_\alpha) + \\ &+ S_{S-L}(\sigma_1, e^{-\frac{\sigma_2}{4}} \psi_1; e^{\frac{\sigma_2}{2}} e^a_\alpha, e^{\frac{\sigma_2}{4}} (\chi_\alpha + \gamma_\alpha \psi_2) \end{aligned} \tag{2.4}$$

In agreement with general analisis of the cocycles of the Weyl group [4] we note that expression $\mathring{9}$ is polynomial on parameters of the super Weyl group (σ and ψ) of degree no more that two (dimension of the space-time). The maximal order terms on σ and ψ i.e. S_0 are invariant under super Weyl transformations:

$$\begin{aligned} \psi &\rightarrow e^{-\frac{\sigma}{4}} \psi \\ e^a_\alpha &\rightarrow e^{\frac{\sigma}{2}} e^a_\alpha \\ \chi_\alpha &\rightarrow e^{\frac{\sigma}{2}} \chi + \gamma_\alpha \eta \end{aligned} \tag{2.5}$$

while the low-order part S_1 is not in complete agreement with general analisis of the cocycles of the Weyl group [4].

The superfield formulation is rather simple. In terms of superfield $\varphi = \sigma + \bar{\theta}\psi + \bar{\psi}\theta + B\bar{\theta}\theta$ expression $\mathring{9}$ takes the form

$$S_{S-L} = \int d^2x d^2\theta E(\frac{1}{2}D\varphi\bar{D}\varphi + \Re\varphi) \tag{2.6}$$

where $\Re$ is supercurvature.

The expression $\mathring{1}4$ is super Weyl coboundary of the non-local effective action

$$W[E] = \int \Re \frac{1}{D\bar{D}} \Re. \tag{2.7}$$

It has general form

$$(Anomaly)\Delta^{-1}(Anomaly) \tag{2.8}$$

where Δ is (super)Weyl invariant differential operator of order d (space-time dimension). To obtain super Weyl invariant expression one needs to replace zweibein and gravitino by super Weyl invariant combinations

$$e^a_\alpha \to \frac{e^a_\alpha}{\sqrt{g}}, \quad \chi_\alpha \to e^{-\frac{1}{4}\gamma^\beta}\gamma_\alpha\chi_\beta \tag{2.9}$$

To define, which part of two-dimensional superdiffeomorphysm group is compatible with super Weyl symmetry, it should to be find local supersymmetric and general coordinate transformations whose preserve gauge conditions:

$$\det \|e^a_\alpha\| = 1, \gamma^\alpha\chi_\alpha = 0 \tag{2.10}$$

The answer is following: it is diffeomorphysms with divergentless vector field and local supersymmetry with the parameter, which satisfies constraint

$$\gamma^\alpha(\partial_\alpha - \frac{1}{2}\omega_\alpha(e))\epsilon(x) = 0 \tag{2.11}$$

Let us notice that the commutator of such transformations gives rise to area-preserving diffeomorphysm. This is quite natural, because that transformations form maximal subalgebra of two-dimensional superdiffeomorphisms. This subgroup is defined by the symplectic form

$$\Omega = pdq - \theta^A d\theta^A \tag{2.12}$$

in phase superplane. The corresponding Poisson braket is

$$\{A, B\} = \epsilon^{\alpha\beta}\partial_\alpha A\partial_\beta B + \delta^{AB}\frac{\partial^L A}{\partial\theta^A}\frac{\partial^R B}{\partial\theta^B} \tag{2.13}$$

If we choose super Weyl invariant regularization for functional mesure, then under general superdiffeomorphysms it is transformed according to rule

$$\tilde{D}_{F*E}\Phi = e^{S(F,E)}\tilde{D}_E\Phi \tag{2.14}$$

Here F is superdiffeomorphysm, acting on supervielbein E, constrained by unit determinant and Φ is matter superfield.

3 Conclusion and outlook

As we seen, all regularities of ordinary 2d-gravity can be transformed to the case of supergravity in natural way.

It seems real to do this in four and higher dimensions.

This approach can be applied to w-gravity. Because the 2d supergravity in many aspects seems like to w-gravity [7] it is reasonable to try find nonchiral expression for the effective action of w_3-gravity in flat and general ordinary- and w_3- gravitational background.

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