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Electromagnetic couplings in a collective model of the nucleon

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Abstract

We study the electromagnetic properties of the nucleon and its excitations in a collective model. In the ensuing algebraic treatment all results for helicity amplitudes and form factors can be derived in closed form in the limit of a large model space. We discuss nucleon form factors and transverse electromagnetic couplings in photo- and electroproduction, including transition form factors that can be measured at new electron facilities.

1 Introduction

The availability and construction of facilities with intense polarized photons and CW electron accelerators in the GeV and multi-GeV region with high luminosity beams and large acceptance detectors signals a new era of hadron spectroscopy exploiting the electromagnetic probe. Ongoing and planned experiments for exclusive processes and polarization measurements in light quark baryons are in need of accurate theoretical interpretation and guidance. This, combined with the lack of exact solutions of QCD in the nonperturbative regime and the present inability of lattice calculations to address the entire excitation spectrum, motivate and necessitate the development of QCD-inspired models for baryon spectroscopy.

Extensive calculations of baryon observables were carried out within the quark potential model in its nonrelativistic [1] or relativized [2] form, which set the standard in the field. Such models emphasize the single-particle aspects of quark dynamics for which only a few low-lying configurations in the confining potential contribute significantly. On the other hand, flux-tube string quark models [3], as well as some regularities in the observed spectra (*e.g.* linear Regge trajectories [4], parity doubling [5]) hint that an alternative, more collective type of dynamics may play a role in the structure of baryons. Algebraic methods are particularly suitable for analyzing collective forms of dynamics, as demonstrated in nuclear and molecular physics [6]. In this contribution we use such methods to study the electromagnetic couplings of nonstrange baryons. Form factors and helicity amplitudes are far more sensitive to details in wave functions than the mass spectrum, and hence they provide a good tool to test (and possibly distinguish) different models of baryon structure.

2 A collective model of baryons

In a flux-tube picture low-lying baryon resonances correspond approximately to three quarks moving in an adiabatic potential generated by a Y-shaped junction of three flux-tubes [3]. With that in mind, we have recently suggested an algebraic model of baryons [7], in which the nucleon has the string configuration of Figure 1. The string is idealized as a thin string with a distribution of mass, charge and magnetization $g(\beta)$ where β is the coordinate along the string. The relevant degrees of freedom of the string configuration of Figure 1 are the two relative Jacobi coordinates

$$\vec{\rho} \;=\; (\vec{r}_1 - \vec{r}_2)/\sqrt{2} \;, \qquad \vec{\lambda} \;=\; (\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3)/\sqrt{6} \;, \tag{1}$$

where $\vec{r}_1$, $\vec{r}_2$ and $\vec{r}_3$ denote the end points of the string configuration. In an algebraic treatment we use instead second quantization in which the two relative Jacobi coordinates and their conjugate momenta are replaced by two vector boson creation $(b^{\dagger}_{\rho}, b^{\dagger}_{\lambda})$ and annihilation operators (b_{ρ}, b_{λ}). The 36 bilinear products of creation and annihilation operators generate the Lie algebra of $U(6)$, the symmetry group of the harmonic oscillator quark model [8]. In this case, the model space consists of a given harmonic oscillator shell labeled by $n = n_{\rho} + n_{\lambda}$. In order to allow for coupling between various oscillator shells (needed to achieve collectivity), we enlarge the model space by adding a scalar boson $(s^{\dagger}, s)$, under the restriction that the total number of bosons $N = n_{\rho} + n_{\lambda} + n_s$ (vector plus scalar) is conserved. The 49 bilinear products of the seven (six vector and one scalar) creation and annihilation operators generate the Lie algebra of $U(7)$. In this case all states of the model space belong to the symmetric representation $[N]$ of $U(7)$, which contains the harmonic oscillator shells with $n = n_{\rho} + n_{\lambda} = 0, 1, \ldots, N$. The s-boson does not introduce a new degree of freedom because it satisfies the constraint $\hat{n}_s = \hat{N} - \hat{n}_{\rho} - \hat{n}_{\lambda}$. All operators of interest, such as mass operators and transition operators, are expressed in terms of the generators of $U(7)$. The full algebraic structure for baryons is obtained by combining the geometric part with the usual spin-flavor and color parts into the spectrum generating algebra of $\mathcal{G} = U(7) \otimes SU_{sf}(6) \otimes SU_c(3)$.

For nonstrange baryons the S_3 permutation symmetry between the identical constituent parts can be incorporated exactly in $U(7)$ [7]. The resulting (one- and two-body) S_3-invariant mass operator describe the dynamics of the configuration in Figure 1, in which the length of the strings and the relative angles between them are equal. In this case the planar shape is an oblate top with D_{3h} point group symmetry. The classification of states under D_{3h} is equivalent to the classification under permutations and parity [9]. The eigenstates of the oblate top are characterized by $(v_1,v_2);K,L_t^P$, where (v_1,v_2) denote the vibrations (stretching, bending); K is the projection of the rotational angular momentum L on the body-fixed symmetry axis, P the parity and t the transformation property of the states under D_3 (a subgroup of D_{3h}), or equivalently the symmetry type under S_3. The antisymmetry of the total baryon wave function implies that the permutation symmetry of the geometric part is the same as that of the spin flavor part. Therefore, there are three equivalent ways to label the permutation symmetry of the baryon wave function, either by the dimension of the $SU_{sf}(6)$ representation or by the representation of the permutation group S_3 or by that of the point group D_3: $56 \leftrightarrow S \leftrightarrow A_1$, $20 \leftrightarrow A \leftrightarrow A_2$, $70 \leftrightarrow M \leftrightarrow E$.

In the present approach nonstrange resonances are identified with rotations and vibrations of a string with the symmetry of an oblate top. The resulting mass spectrum exhibits rotational states (K,L_t^P) arranged in bands built on top of each vibration (v_1,v_2). The corresponding wave functions are spread over many oscillator shells and hence are truly collective. Although the underlying dynamics is quite different, the fit for the mass spectrum is comparable [7] to that obtained in quark potential models.

3 Electromagnetic couplings

The distinguishing feature between different models of hadrons is their form factors. This holds especially in the case of baryons whose size ($\sim$ 1 fm) is comparable to the scale of their excitation energies ($\sim$ 300 MeV). For electromagnetic couplings the calculation of elastic and transition form factors proceeds through a nonrelativistic reduction of the coupling of point-like constituents to the photon field. The resulting transition operator contains a nonrelativistic part, a spin-orbit part, a nonadditive part and higher order corrections [10]. In this contribution we limit ourselves to the nonrelativistic part, which for transverse couplings consists of a magnetic and an electric contribution

$$\mathcal{H} \;=\; 6\sqrt{\pi/k_0}\,\mu e_3 \left[k s_{3,+}\hat{U} - \hat{T}_+/g \right] \;. \tag{2}$$

The $\hat{z}$-axis is taken along the direction of the photon momentum $\vec{k} = k\hat{z}$, k_0 is the photon energy, and e_j, $\vec{s_j}$ and $\mu = \mu_j = ge/2m_q$ denote the charge, spin and magnetic moment of the j-th constituent.

Helicity amplitudes and form factors of interest in photo- and electroproduction are proportional to the matrix elements of $\mathcal{H}$ between initial and final states. To evaluate these matrix elements in the algebraic model, the operators $\hat{U}$ and $\hat{T}_+$ are expressed in terms of generators of the $U(7)$ algebra [7]

$$\begin{aligned} \hat{U} &= \mathrm{e}^{-ik\beta \hat{D}_{\lambda,z}/X_D} \;, \\ \hat{T}_+ &= \frac{i m_q k_0 \beta}{2X_D} \left(\hat{D}_{\lambda,+}\, \mathrm{e}^{-ik\beta \hat{D}_{\lambda,z}/X_D} + \mathrm{e}^{-ik\beta \hat{D}_{\lambda,z}/X_D}\, \hat{D}_{\lambda,+} \right) \;. \end{aligned} \tag{3}$$

The dipole operator $\hat{D}_{\lambda,m} = (b^{\dagger}_{\lambda} \times s - s^{\dagger} \times \tilde{b}_{\lambda})^{(1)}_{m}$, is the generator of $U(7)$ with the same transformation properties as the Jacobi coordinate λ_m. The coefficient X_D is a normalization factor given by the reduced matrix element of the dipole operator $X_D = |\langle 1^-_E || \hat{D}_{\lambda} || 0^+_{A_1} \rangle|$ and β represents a scale of the coordinate.

The matrix elements of the operators in Eq. (3) can be obtained by noting that the operator appearing in the exponent is a generator of $U(7)$. Therefore, the matrix elements of $\hat{U}$ and $\hat{T}_+$ can be expressed in terms of group elements of $U(7)$, which can be evaluated exactly without having to make any further approximations. There exist limiting situations in which these group elements can be derived in closed form. In the limit of a large model space ($N \to \infty$) we recover for the harmonic oscillator the familiar expressions in terms of exponentials, and for the oblate top we find expressions in terms of spherical Bessel functions [7]. In keeping with a collective description of baryons, we assume that the charge and magnetic moment are not concentrated at the end points of the string configuration of Figure 1, but instead are distributed along the strings. Hereto we fold the matrix elements of the operators in Eq. (3) with a probability distribution for the charge and magnetization of the form $g(\beta) = (\beta^2/2a^3)\exp(-\beta/a)$ to obtain the distributed-string or collective form factors

$$\begin{aligned} F(k) &= \int \mathrm{d}\beta\, g(\beta)\, \langle \psi_f | \hat{U} | \psi_i \rangle \;, \\ G_+(k) &= \int \mathrm{d}\beta\, g(\beta)\, \langle \psi_f | \hat{T}_+ | \psi_i \rangle \;. \end{aligned} \tag{4}$$

Here a is a scale parameter.

In Table 1 we show the $N \to \infty$ results for some resonances which in the present approach are interpreted as rotational excitations of an oblate top. They are all characterized by $(v_1, v_2) = (0, 0)$. These closed expressions allow one to study the dependence on momentum transfer. For small values of k (long wavelength limit) the transition form factors $F(k)$ show the threshold behavior $\sim k^L$. For large values of k all form factors drop as powers of k. This property is well-known experimentally and is in contrast with harmonic oscillator quark models in which all form factors fall off exponentially (see Figure 4). The elastic form factor $F(k)$ drops as k^{-4}, whereas the transition form factors for all rotational excitations drop as k^{-3}. The form factors $G_+(k)$ drop as the derivatives of $F(k)$.

The analysis of electromagnetic form factors presented in this contribution is based on the collective form factors of Table 1 which are obtained in the limit of a large model space ($N \to \infty$). The choice for the probability distribution $g(\beta)$ is such that we recover the familiar dipole form for the elastic form factor $F(k)$. The value of the scale parameter a in $g(\beta)$ is determined by the proton charge radius $a = \langle r^2 \rangle_p^{1/2} = 0.249$ fm.

3.1 Transition form factors

In photo- and electroproduction of baryon resonances the transverse couplings are expressed in terms of helicity amplitudes, A_ν ($\nu = 1/2, 3/2$). Their measurement is an essential part of the research program at current and new electron facilities. Helicity amplitudes correspond to specific matrix elements of $\mathcal{H}$,

$$A_\nu = 6\sqrt{\pi/k_0}\,\mu \left[\beta_\nu\, kF(k) + \alpha_\nu\, G_+(k)/g\right] \;, \tag{5}$$

and are all expressed in terms of the radial integrals of Eq. (4) and Table 1. The spin-flavor dependence of the helicity amplitudes is contained in the α_ν and β_ν factors [7] and are common to all models sharing the same spin-flavor structure.

In Figures 2 and 3 we show the transition form factors for proton couplings to the N(1520)D_{13} and the N(1680)F_{15} resonances. All calculations are done in the Breit frame and include the sign of the subsequent strong decay of the resonance $N^* \rightarrow N+\pi$, which is calculated by assuming that the meson is emitted from a single constituent. The collective form factors (solid lines) give a fair agreement with the data, whereas the harmonic oscillator form factors (dotted lines) fall off too quickly. For both resonances, the rapid change in the corresponding asymmetry parameter $(A_{1/2}^2 - A_{3/2}^2)/(A_{1/2}^2 + A_{3/2}^2)$ from -1 to $+1$ is reproduced. Since the behavior of the asymmetry parameter is determined by the spin-flavor part and does not depend on the spatial part, the results obtained with the collective and the harmonic oscillator form factors are identical.

3.2 Nucleon form factors

The elastic electromagnetic form factors of the nucleon can be evaluated in a similar way. For the electric form factor of the proton we find (by construction) a dipole form

$$G_E^p \;=\; \frac{1}{(1+k^2a^2)^2} \;. \tag{6}$$

In Figure 4 we show the dependence of G_E^p on Q^2 ($= k^2$ for elastic transitions). On the other hand, the neutron electric form factor vanishes identically in this scheme, $G_E^n = 0$. This is a consequence of the spin-flavor content of the nucleon wave function.

A non-vanishing neutron electric form factor can be obtained by breaking the $SU_{sf}(6)$ symmetry *e.g.* via the hyperfine interaction [12], or by breaking the D_3 spatial symmetry *e.g.* by allowing for a quark-diquark structure [13] and flavor-dependent mass terms. If we assume a flavor dependent probability distribution, $g_u(\beta) = (\beta^2/2a_u^3)\exp(-\beta/a_u)$ and $g_d(\beta) = (\beta^2/2a_d^3)\exp(-\beta/a_d)$, we obtain

$$G_E^p \;=\; \frac{4}{3(1+k^2a_u^2)^2} - \frac{1}{3(1+k^2a_d^2)^2} \;,$$

$$G_E^n = \frac{2}{3(1+k^2a_u^2)^2} - \frac{2}{3(1+k^2a_d^2)^2} \; . \tag{7}$$

The two scale parameter $a_u = 0.258$ fm and $a_d = 0.285$ fm are determined by fitting simultaneously the proton and neutron charge radii. Figure 5 shows a comparison with the neutron electric form factor. The proton electric form factor is hardly changed when Eq. (6) is replaced by Eq. (7). We note that a relatively small breaking of the flavor symmetry is sufficient to give a good fit of both the neutron and the proton electric form factors. Similarly, for the magnetic form factors we find

$$\begin{aligned} G_M^p &= \frac{8\mu}{9(1+k^2a_u^2)^2} + \frac{\mu}{9(1+k^2a_d^2)^2} \; , \\ G_M^n &= -\frac{4\mu}{9(1+k^2a_d^2)^2} - \frac{2\mu}{9(1+k^2a_u^2)^2} \; . \end{aligned} \tag{8}$$

In the photoproduction limit the ratio of neutron and proton magnetic form factors is that of the corresponding magnetic moments $\mu_n/\mu_p = -0.685$ which agrees very well with the value $-2/3$ calculated from Eq. (8). On the basis of perturbative QCD one expects that for large values of Q^2 the above ratio approaches $-1/2 + \mathcal{O}(\ln Q^2)$ [16]. With the same values of the scale parameters as for the electric form factors we calculate this ratio to be -0.54, in agreement with the p-QCD value. Without the flavor dependence in the probability distribution ($a_u = a_d$) the ratio is $-2/3$, independent of Q^2.

4 Summary, conclusions and outlook

We have presented a study of electromagnetic couplings of baryon resonances in a collective model of the nucleon. All helicity amplitudes and form factors of interest can be expressed in closed form in the limit of a large model space ($N \rightarrow \infty$). We found a consistent description of photo- and electroproduction of the experimentally well-known N(1520)D_{13} and N(1680)F_{15} resonances. An application to the nucleon elastic form factors showed that by introducing a flavor dependent probability distribution we could simultaneously describe the proton and neutron electric form factors as well as the change in the ratio of their magnetic form factors.

Work on the calculation of photo- and electroproduction of all low lying resonances including the effects of higher order terms in the transition operator and the extension to longitudinal couplings is currently in progress.

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Table 1: Collective form factors of Eq. (4) for large N. $H(x) = \arctan x - x/(1+x^2)$. The final states are labeled by $[f, L^P]_{(v_1,v_2);K}$, where f denotes the dimension of the $SU_{sf}(6)$ representation. The initial state is $[56, 0^+]_{(0,0);0}$.

Final state	$F(k)$	$G_+(k)/m_q k_0 a$
$[56, 0^+]_{(0,0);0}$	$\frac{1}{(1+k^2a^2)^2}$	0
$[70, 1^-]_{(0,0);1}$	$-i\,\sqrt{3}\,\frac{ka}{(1+k^2a^2)^2}$	$\sqrt{2}F(k)/ka$
$[56, 2^+]_{(0,0);0}$	$\frac{1}{2}\sqrt{5}\left[\frac{-1}{(1+k^2a^2)^2} + \frac{3}{2k^3a^3}H(ka)\right]$	$\sqrt{6}F(k)/ka$
$[70, 2^+]_{(0,0);2}$	$-\frac{1}{2}\sqrt{15}\left[\frac{-1}{(1+k^2a^2)^2} + \frac{3}{2k^3a^3}H(ka)\right]$	$\sqrt{6}F(k)/ka$

Figure 1: Collective model of baryons and its idealized string configuration (the charge distribution of the proton is shown as an example).

Figure 2: Helicity amplitudes A_{ν}^{p} in 10^{-3} (GeV)$^{-1/2}$ for the N(1520)D_{13} resonance. The curves correspond to the collective form factor (solid lines) and the harmonic oscillator form factor (dotted lines). The experimental data are taken from the compilation in [11].

Figure 3: Same as Figure 2, but for the N(1680)F_{15} resonance.

Figure 4: Comparison between the experimental proton electric form factor G_E^p, the collective form factor (solid lines) and the harmonic oscillator form factor (dotted lines). The experimental data, taken from a compilation in [14], and the calculations are divided by the dipole form factor, $F_D = 1/(1+Q^2/0.71)^2$. The curves are obtained a) from Eq. (6) and the harmonic oscillator form factor $\exp(-k^2\beta^2/6)$ with $\beta = a\sqrt{12}$, and b) from Eq. (7) and the corresponding expression for the harmonic oscillator form factor with $\beta_u = a_u\sqrt{12}$ and $\beta_d = a_d\sqrt{12}$.

Figure 5: Comparison between the experimental neutron electric form factor G_E^n, the collective form factor (solid line) and the harmonic oscillator form factor (dotted line). The experimental data are taken from a compilation in [15].