Hartle-Hawking state in supersymmetric minisuperspace

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Abstract

The Hartle-Hawking 'no-boundary' state is constructed explicitely for the recently developed supersymmetric minisuperspace model with non-vanishing fermion number.

Spatially homogeneous models both in gravity and in supergravity have enjoyed some popularity in recent years as a testing ground for new ideas in quantum cosmology. One such idea, which has been discussed extensively in the literature, is the proposal by Hartle and Hawking for the construction of the 'wave-function of the universe', including gravity [1]. According to this proposal the quantum state of the universe is formally given by the Euclidean path-integral of exp[-action] over all compact 4-geometries, containing a given compact 3-geometry (the argument of the wave-function) as its only boundary. This is why it is also called the 'no-boundary' state. While this idea of striking (but also deceptive) simplicity could be partially implemented, e.g. in spatially homogeneous minisuperspace models, like a closed Friedmann universe with a scalar field [1] or an anisotropic Bianchi type IX universe with a cosmological constant [2] its use in *supersymmetric* minisuperspace models has caused some difficulty.

The supersymmetric Friedmann model without matter was treated successfully [3] but

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*Permanent address: Research Institute for Solid State Physics, P.O. Box 49, H1525 Budapest, Hungary lacks sufficient degrees of freedom to permit a physically meaningful discussion of this issue. The inclusion of a spatially homogeneous supersymmetric scalar matter-field has, so far, led only to explicit solutions of the wormhole-type [3,4]. The first treatments of the spatially homogenous supersymmetric anisotropic Bianchi type IX model without matter field concluded that a Hartle-Hawking state would not exist in such a model [5–8], the only permitted state being that of a 'worm-hole' in the completely empty or filled fermion sectors which had previously been found in [9]. Subsequently it was shown [10] that the particular $SO(3)$ symmetry of Bianchi type IX permits an alternative homogeneity ansatz for the Rarita-Schwinger field, and that its application replaces the permitted 'worm-hole' state in the empty or filled fermion sector by a 'no-boundary' state in the same sector. In a recent paper [11] we reexamined the supersymmetric minisuperspace models of Bianchi type in class A [12] without matter fields and showed that, contrary to previous expectations, they posses *infinitely many* physical states. Hence, the question of the existence and form of a 'no-boundary' state in such models must be reconsidered. In the present paper we (i) apply the theory of [11] to the supersymmetric Bianchi type IX model without matter, and with the conventional homogeneity condition for the Rarita-Schwinger field, and (ii) construct the Hartle-Hawking 'no-boundary'-state for that model explicitely. The dependence of that state on the 3-metric turns out to be the same as in [10] (see also [13]), where the alternative homogeneity condition was applied. However, the dependence on the spatially homogeneous Rarita-Schwinger field is completely different from [10]. It turns out to be a state near the middle of the fermion number spectrum, between the completely empty and the completely filled fermion sectors. This state has a much better chance to permit an extension to full supergravity, because it was proven that the physical states in full supergravity cannot ly in the empty and filled fermion sectors [14]. A brief account of our results has already been given in a recent conference report [15].

Let us begin recapitulating some notation and results of [11] which are necessary here. The starting point is the Langrangean of $N = 1$ supergravity in the notations defined in [16]. Space-time is assumed to be foliated by space-like 3-surfaces which are homogeneous under the action of a 3-dimensional homogeneity group which is here assumed to be $SO(3)$. A symmetric basis of 1-forms ω^p then exists ($p = 1, 2, 3$), satisfying $h^{1/2} d\omega^p = \frac{1}{2}\delta^{pq}\epsilon_{qrs}\omega^r \wedge \omega^s$, where h_{pq} with $h = \det h_{pq}$ are the purely time-dependent components of the spatial 3-metric, and ϵ_{qrs} are the components of the 3-dimensional Levi-Civita tensor. The volume of the underlying 3-sphere is $V = \int \omega^1 \wedge \omega^2 \wedge \omega^3 = 16\pi^2$. In the metric representation the independent variables are given by the spatial components of the tetrad ${e_p}^a (a = 0, 1, 2, 3)$ satisfying ${e_p}^a e_{qa} = h_{pq}$, and the spatial components of the Grassmannian Rarita-Schwinger field ${\psi_p}^\alpha$, ${\bar{\psi}_p}^{\dot{\alpha}}$. We shall here adopt the homogeneity conditions ${e_p}^a = {e_p}^a(t)$, ${\psi_p}^\alpha = {\psi_p}^\alpha(t)$ and shall *not* make use of the alternative homogeneity condition for ${\psi_p}^\alpha$ consistent with $SO(3)$ which was proposed in [10]. Introducing canonical momenta, Poisson brackets and finally Dirac brackets in order to eliminate the appearing second class constraints one finds canonical expressions for the supersymmetry generators S_α, $\bar{S}_{\dot{\alpha}}$ and the Lorentz generators $J_{\alpha\beta}$, $\bar{J}_{\dot{\alpha}\dot{\beta}}$ of the following form

$$\begin{aligned} S_\alpha &= -\mathcal{C}^{\dot{\alpha}\beta}_{pr} \left(\frac{1}{2} V \delta^{pq} {e_q}^a + \tfrac{i}{2} {p_+}^{pa} \right) \sigma_{a\alpha\dot{\alpha}} {\pi^r}_\beta \\ \bar{S}_{\dot{\alpha}} &= \left(\frac{1}{2} V \delta^{pq} {e_q}^a - \tfrac{i}{2} {p_+}^{pa} \right) \sigma_{a\alpha\dot{\alpha}} {\psi_p}^\alpha \end{aligned} \tag{1}$$

and

$$\begin{aligned} J_{\alpha\beta} &= +\tfrac{1}{2} (\sigma^{ac}\epsilon)_{\alpha\beta} \left(e_{pa} {p_+}^p{}_c - e_{pc} {p_+}^p{}_a \right) \\ &\quad - \tfrac{1}{2} \left(\psi_{p\alpha} {\pi^p}_\beta + \psi_{p\beta} {\pi^p}_\alpha \right) \\ \bar{J}_{\dot{\alpha}\dot{\beta}} &= -\tfrac{1}{2} (\epsilon\bar{\sigma}^{ac})_{\dot{\alpha}\dot{\beta}} \left(e_{pa} {p_+}^p{}_c - e_{pc} {p_+}^p{}_a \right) . \end{aligned} \tag{2}$$

For all conventions regarding the σ-matrices and $\epsilon_{\alpha\beta}$ we refer to [16]. The kernel $\mathcal{C}^{\dot{\alpha}\alpha}_{pq}$ is defined as

$$\mathcal{C}^{\dot{\alpha}\alpha}_{pq} = -\tfrac{1}{2Vh^{1/2}} \left(i h_{pq} n^a - \epsilon_{pqr} e^{ra} \right) {\bar{\sigma}_a}^{\dot{\alpha}\alpha} \tag{3}$$

n^a is the future oriented unit vector normal on the space-like 3-surfaces and its components are functions of the ${e_p}^a$. The variables p^p_{+a} and the Grassmannian ${\pi^p}_\alpha$ are the 'Dirac-conjugates' of ${e_p}^a$ and ${\psi_p}^\alpha$ in the sense that the only non-vanishing Dirac-brackets are

$$\{e_p{}^a, p_+{}^q{}_b\}^* = \delta_p{}^q \delta_b{}^a$$

$$\{\psi_p{}^\alpha, \pi^q{}_\beta\}^* = -\delta_p{}^q \delta_\beta{}^\alpha\,. \tag{4}$$

Canonical quantization is performed in the metric $(e_p{}^a, \psi_p{}^\alpha)$-representation by putting

$$p_+{}^p{}_a = -i\hbar(\partial/\partial e_p{}^a) \qquad \pi^p{}_\alpha = -i\hbar(\partial/\partial \psi_p{}^\alpha)\,. \tag{5}$$

There is an ordering ambiguity in the expression for S_α because the kernel (3) does not commute with p^p_{+a}. Here we shall deviate from ref. [11] and adopt the choice of the ordering as displayed explicitely in eq. (1), while in [11] we ordered the kernel $\mathcal{C}^{\dot\alpha\beta}_{pr}$ to the right of $p_+{}^{pa}$ before quantizing. While, at least so far, no reason of principle is visible to prefer one choice of ordering over the other (or over any mixed ordering in between), the ordering chosen here will actually simplify in an essential way the form of eq. (22) below.

With the adopted choice of operator ordering we find the explicit graded generator algebra

$$\Big[S_\alpha, S_\beta\Big]_+ = 0 = \Big[\bar S_{\dot\alpha}, \bar S_{\dot\beta}\Big]_+ \tag{6}$$

$$\Big[S_\alpha, \bar S_{\dot\alpha}\Big]_+ = -\frac{\hbar}{2} H_{\alpha\dot\alpha} \tag{7}$$

$$\Big[H_{\alpha\dot\alpha}, S_\beta\Big]_- = -i\hbar \varepsilon_{\alpha\beta} \bar D_{\dot\alpha}{}^{\dot\beta\dot\gamma} \bar J_{\dot\beta\dot\gamma} \tag{8}$$

$$\begin{aligned}\Big[H_{\alpha\dot\alpha}, \bar S_{\dot\beta}\Big]_- &= i\hbar \varepsilon_{\dot\alpha\dot\beta} J_{\beta\gamma} D_\alpha{}^{\beta\gamma} \\ &= i\hbar \varepsilon_{\dot\alpha\dot\beta}\Big[D_\alpha{}^{\beta\gamma} J_{\beta\gamma} + i\hbar \bar E_\alpha{}^{\dot\gamma\dot\delta} \bar J_{\dot\gamma\dot\delta} - \frac{i\hbar n^a}{V h^{1/2}} \sigma_{a\alpha\dot\gamma} \bar S^{\dot\gamma}\Big]\end{aligned} \tag{9}$$

and the well-known commutators with $J_{\alpha\beta}$, $\bar J_{\dot\alpha\dot\beta}$ reflecting Lorentz transformations. The operator $H_{\alpha\dot\alpha}$ is here *defined* by the anti-commutator (7), but we have checked that it classically differs only by terms proportional to Lorentz generators from $\tilde H_{\alpha\dot\alpha}$ defined by the diffeomorphism and Hamiltonian generators H^p and H via $\tilde H_{\alpha\dot\alpha} = \sigma_{a\alpha\dot\alpha}(e_p{}^a H^p + n^a H)$. The structure functions $D_\alpha{}^{\beta\gamma}$, $\bar D^{\dot\beta\dot\gamma}_{\dot\alpha}$, $\bar E^{\dot\gamma\dot\delta}_\alpha$ are Grassmannian odd functions of $e_p{}^a$, $\psi_p{}^\alpha$. While their explicit form is not essential, for the following, we shall here list them for completeness and future reference

$$D_\alpha{}^{\beta\gamma} = n^b e_p{}^c \varepsilon^{\beta\delta} (\sigma_b \bar\sigma_c)_\delta{}^\gamma \Big[h^{-1/2} \delta^{pq} \varepsilon_{\alpha\rho} \psi_q{}^\rho + \epsilon^{pqr} \sigma_{a\alpha\dot\alpha} C^{\dot\alpha\sigma}_{sq} \varepsilon_{\sigma\rho} \psi_r{}^\rho \Big(\frac{V}{2} \delta^{st} e_t{}^a + \frac{i}{2} p_+{}^{sa}\Big)\Big] \tag{10}$$

$$\bar{E}_{\alpha}{}^{\dot{\gamma}\dot{\delta}} = \left(\frac{i}{2Vh^{1/2}}\right)\left(n^a e^{pb} + n^b e^{pa}\right)\epsilon_{\alpha\gamma}\bar{\sigma}_a{}^{\dot{\gamma}\gamma}\bar{\sigma}_b{}^{\dot{\delta}\beta}\psi_{p\beta} \tag{11}$$

$\bar{D}^{\dot{\beta}\dot{\gamma}}_{\dot{\alpha}}$ and $E_{\dot{\alpha}}{}^{\gamma\delta}$ are given by the matrix-adjoints of these expressions. Due to the different ordering chosen, the algebra (6)-(9) differs slightly from a corresponding result given in [11], but both forms are, of course, fully consistent.

As all generators in (6)-(9) appear on the right-hand side the 'graded' algebra closes not only classically, but also quantum mechanically. Due to the Jacobi-identity for commutators this result is even sufficient to prove that the only remaining commutator $[H_{\alpha\dot{\alpha}}, H_{\beta\dot{\beta}}]_-$ evaluates to structure functions multiplied with generators S_γ, $\bar{S}_{\dot{\gamma}}$, $J_{\gamma\delta}$, $\bar{J}_{\dot{\gamma}\dot{\delta}}$, $H_{\gamma\dot{\gamma}}$ on the right, i.e. we find that this spatially homogeneous model has a closed generator algebra and is free from anomalies.

Let us now turn to the physical states of the system in the sense of Dirac [17], i.e. the states which are annihilated by all the generators S_α, $\bar{S}_{\dot{\alpha}}$, $J_{\alpha\beta}$, $\bar{J}_{\dot{\alpha}\dot{\beta}}$, $H_{\alpha\dot{\alpha}}$. These states Ψ_F can be parametrized by the conserved fermion number $\psi_p{}^\alpha \partial/\partial\psi_p{}^\alpha = F$ and have the form

$$\Psi_0 = \exp\left[\frac{V}{2\hbar}\delta^{pq}h_{pq}\right] \tag{12}$$

$$\Psi_2 = \bar{S}_{\dot{\alpha}}\bar{S}^{\dot{\alpha}} f(h_{pq}) \tag{13}$$

$$\Psi_4 = S^\alpha S_\alpha g(h_{pq}) \prod_{r=1}^{3}(\psi_r)^2 \tag{14}$$

$$\Psi_6 = \exp\left[-\frac{V}{2\hbar}\delta^{pq}h_{pq}\right] \prod_{r=1}^{3}(\psi_r)^2 \,. \tag{15}$$

Here the amplitudes f and g appearing in the 2- and 4-fermion sector, respectively, are functions of the metric h_{pq} only, which makes all states (12)-(15) Lorentz-invariant and serves to satisfy the Lorentz-constraints. The functions f, g satisfy Wheeler-DeWitt equations, which are obtained by applying S_α to Ψ_2 and $\bar{S}^{\dot{\alpha}}$ to Ψ_4, respectively, and using the algebra (6)-(9) [9]. In the first case we obtain

$$\left(H^{(0)}_{\alpha\dot{\alpha}} - \frac{\hbar^2}{Vh^{1/2}} n^a \sigma_{a\alpha\dot{\alpha}}\right) f(h_{pq}) = 0 \tag{16}$$

where we have used the identity $[\bar{S}^{\dot{\alpha}}, \sigma_{a\alpha\dot{\alpha}} n^a/h^{1/2}] = 0$ to factor out $\bar{S}^{\dot{\alpha}}$ to the left. We discarded the possibility that the right hand side of eq. (16) could be non-zero and proportional to a bosonic function annihilated by $\bar{S}^{\dot\alpha}$. The reason is that all such functions are known to vanish in *full* supergravity [14]. Here $H^{(0)}_{\alpha\dot\alpha}$ consists only of the bosonic terms of $H_{\alpha\dot\alpha}$, i.e. of those terms which remain if $\pi^p{}_\alpha$ is first brought to the right using its anti-commutation relation with $\psi_p{}^\alpha$, and is then equated to zero. In the 4-fermion sector we find in an analogous manner

$$H^{(1)}_{\alpha\dot\alpha} g(h_{pq}) = 0\,, \tag{17}$$

however, $H^{(1)}_{\alpha\dot\alpha}$ is now obtained from $H_{\alpha\dot\alpha}$ by bringing $\psi_p{}^\alpha$ to the right, using its anti-commutation relation with $\pi^p{}_\alpha$, and then equating it to zero.

To get explicit expressions it is useful to parametrize the spatial metric by $h_{pq} = \Omega_{pi}(e^{2\boldsymbol{\beta}})_{ij}\Omega_{qj}$ where Ω_{pi} is a rotation matrix, depending on three Euler angles, and

$$\left(e^{2\boldsymbol{\beta}}\right)_{ij} = e^{2\alpha}\,\mathrm{diag}\,\left(e^{2\beta_+ + 2\sqrt{3}\beta_-}, e^{2\beta_+ - 2\sqrt{3}\beta_-}, e^{-4\beta_+}\right)\,. \tag{18}$$

It is important to note that the rotation matrix Ω_{pi} and the parameters α, β_+, β_- are unique functions of the tetrad $e_p{}^a$. The diffeomorphism constraint

$$e_p{}^a \bar\sigma_a^{\dot\alpha\alpha} H^{(0)}{}_{\alpha\dot\alpha} f(h_{pq}) = 0 = e_p{}^a \bar\sigma_a^{\dot\alpha\alpha} H^{(1)}{}_{\alpha\dot\alpha} g(h_{pq}) \tag{19}$$

is then satisfied by taking $f(h_{pq})$ and $g(h_{pq})$ as *independent* of the Euler angles of the rotation matrices, thus $f = f(\alpha, \beta_+, \beta_-)$, $g = g(\alpha, \beta_+, \beta_-)$. There only remains the Hamiltonian constraint

$$n^a \bar\sigma_a^{\dot\alpha\alpha}\left(H_{\alpha\dot\alpha}{}^{(0)} - \frac{\hbar^2}{V h^{1/2}} n^b \sigma_{b\alpha\dot\alpha}\right) f(\alpha, \beta_+, \beta_-) = 0 \tag{20}$$

$$n^a \bar\sigma_a^{\dot\alpha\alpha} H_{\alpha\dot\alpha}{}^{(1)} g(\alpha, \beta_+, \beta_-) = 0\,. \tag{21}$$

The latter reads explicitely,

$$\Bigg[-\frac{\hbar^2}{V^2}\left(\frac{\partial}{\partial\alpha}\right)^2 + \frac{\hbar^2}{V^2}\left(\frac{\partial}{\partial\beta_+}\right)^2 + \frac{\hbar^2}{V^2}\left(\frac{\partial}{\partial\beta_-}\right)^2 + \left(\frac{\partial\phi}{\partial\alpha}\right)^2 - \left(\frac{\partial\phi}{\partial\beta_+}\right)^2 - \left(\frac{\partial\phi}{\partial\beta_-}\right)^2 + \frac{\hbar}{V}\left(-\frac{\partial^2\phi}{\partial\alpha^2} + \frac{\partial^2\phi}{\partial\beta_+^2} + \frac{\partial^2\phi}{\partial\beta_-^2}\right)\Bigg] g(\alpha, \beta_+, \beta_-) = 0 \tag{22}$$

with the abbreviation

$$\phi = \frac{1}{2}\delta^{pq}h_{pq} = \frac{1}{2}e^{2\alpha}\left(2e^{2\beta_+}\cosh 2\sqrt{3}\beta_- + e^{-4\beta_+}\right) . \tag{23}$$

Due to our judicious choice of ordering in eq. (1) a term proportional to $\frac{\hbar^2}{V}e^{-3\alpha}g(\alpha,\beta_+,\beta_-)$ is avoided in eq. (21), while the corresponding term is present in eq. (20), which we will not need in the following, however. In fact we shall here only be interested in some special explicit solutions of eq. (22), as it will turn out that the Hartle-Hawking state we are looking for is among them. First we note that a very simple solution of eq. (22) is given by $g(h_{pq}) \sim \exp(-V\phi(h_{pq})/\hbar)$, but this solution, inserted in eq. (14), gives $\Psi_4 = 0$, i.e. it only gives the trivial solution. Remarkably, however, there are four equally simple linearly independent further solutions of eq. (22) which give nontrivial results for Ψ_4. The first of these is the desired Hartle-Hawking state, namely

$$\begin{aligned} g(h_{pq}) = \exp\Big[-\frac{V}{2\hbar}e^{2\alpha}(2e^{2\beta_+}(\cosh 2\sqrt{3}\beta_- - 1) \\ + e^{-4\beta_+} - 4e^{-\beta_+}\cosh\sqrt{3}\beta_-)\Big] . \end{aligned} \tag{24}$$

The other three states are

$$\begin{aligned} g(h_{pq}) = \exp\Big[-\frac{V}{2\hbar}e^{2\alpha}(4e^{2\beta_+}(\sinh(\sqrt{3}\beta_-)^2 + e^{-4\beta_+} \\ + 4e^{-4\beta_+}\cosh\sqrt{3}\beta_-)\Big] . \end{aligned} \tag{25}$$

and the two further expressions obtained by rotating the (β_+,β_-)-axis around $\beta_+ = 0 = \beta_-$ twice by 120°-degrees, respectively. The final form of the Hartle-Hawking state in the 4-fermion sector, i.e. Ψ_4, is obtained as a function of ${\psi_p}^a$ and ${e_p}^a$ by acting with the operator $(S^\alpha S_\alpha)$ on $g(h_{pq})\Pi(\psi_r)^2$. To perform this step one should express the invariants α, β_+, β_- of the spatial metric in terms of the matrix elements h_{pq} which are functions of the tetrad via the relation $h_{pq} = {e_p}^a e_{qa}$.

Let us now discuss the result further. The result (24) coincides in form with the amplitude of the Hartle-Hawking state in the filled-fermion sector found in [10] by assuming a different homogeneity condition for the Rarita-Schwinger field. By contrast, here we have assumed the usual homogeneity condition ${\psi_p}^\alpha = {\psi_p}^\alpha(t)$ and the amplitude (24), via eq. (17), corresponds to a state in the middle of the fermion-number spectrum, namely in the 4-fermion sector. This change is highly wellcome, because quantum states in the empty and filled fermion sector are known [14] not to exist in full supergravity, where the existing states are grouped around the middle of the fermion-number spectrum, corresponding to the Dirac-vacuum of the gravitino. Therefore the state (14), (24) may now well have a counterpart in full supergravity.

That eq. (24) indeed gives the Hartle-Hawking state can be seen as follows: First of all the real exponential form of $g(h_{pq})$ shows that no classically allowed domain of the spatial metric is described by this wave-function. (This is an agreement with the known fact that no empty closed Friedmann universe can exist classically, but may exist as a quantum fluctuation. However, it is in contrast to the classical possibility of an empty anisotropic Bianchi-type IX mixmaster universe [18]. Such classically allowed mixmaster solutions must therefore correspond to *other* solutions of eqs. (16) or (17)). The spatial metric therefore exists in this wave-function only due to classically forbidden tunnelling processes. To exhibit these in a semi-classical way let us write $g(h_{pq})$ in the form $g(h_{pq}) \sim \exp[-\frac{V}{\hbar}\mathrm{I}(h_{pq})]$ thereby defining the Euclidean action $\mathrm{I} = \mathrm{I}(\alpha, \beta_+, \beta_-)$. Then the semi-classical(i.e. most probable) tunnelling path parametrized by a suitable affine parameter λ satisfies the first-order differential equations

$$\begin{aligned} p_\alpha &= \frac{\partial \mathrm{I}}{\partial \alpha} = -\frac{d\alpha}{d\lambda} \\ p_{\beta_\pm} &= \frac{\partial \mathrm{I}}{\partial \beta_\pm} = \frac{d\beta_\pm}{d\lambda} \, . \end{aligned} \tag{26}$$

With solutions $\alpha(\lambda)$, $\beta_+(\lambda)$, $\beta_-(\lambda)$ the corresponding 4-metric has the form

$$ds^2 = \left(3\sqrt{V} e^{3\alpha} d\lambda^2 + (e^{2\boldsymbol{\beta}})_{pq}\omega^p\omega^q\right) \, . \tag{27}$$

Eqs. (27) with $\mathrm{I} = \frac{1}{2}e^{2\alpha}(2e^{2\beta_+}(\cosh 2\sqrt{3}\beta_- - 1) + e^{-4\beta_+} - 4e^{-\beta_+}\cosh\sqrt{3}\beta_-)$ are, in fact, well known [19]. They have been solved [19] to give the 4-metric of a compact Riemannian 4-space filling in, without singularity, any given 3-geometry of Bianchi type IX whose metric tensor is parametrized by α, β_-, β_+. For our spatially homogeneous model this is the property which defines the 'no-boundary' state, at least semi-classically.But since eq. (24) also solves the fully quantum mechanical Wheeler DeWitt equation (21) is an exact quantum amplitude with the required semiclassical property and hence, indeed the exact Hartle-Hawking state of the supersymmetric Bianchi-type IX model. The states (25) can be discussed in a similar manner. However, in these cases, the semiclassical tunnelling paths extending the given 3-geometry turn out to describe *non-compact* 4-geometries, as one of the scale-parameters grows without bound in the limit $\alpha \to -\infty$, even though the other two scale-parameters and the metric 3-volume shrink to zero. Hence, these states (and similarly the states Ψ_0, Ψ_6 of (12), (15)) do not qualify as 'no-boundary states'.

In summary, giving an explicit solution of all constraints of a quantized supersymmetric spatially homogenous cosmological model without matter or cosmological constant we have found a state in one of the sectors in the middle of the spectrum of fermion numbers which qualifies as the 'no-boundary' state of this system. The explicit form (24) shows that this state, for values of the overall scale-parameter e^{α} large compared to the Planck-length, strongly favors isotropic metrics ($\beta_+, \beta_- \to 0$). It will, of course, be interesting to extend this analysis e.g. by allowing for a cosmological constant [20], or a matter field, or treating the case of full supergravity [21]. While such extensions are technically more demanding the present analysis gives clear indications how one may proceed.

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