Rescaling of Nuclear Structure Functions

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Abstract

It is shown that nucleonic structure functions are $x-$ and Q^2-rescaled in nuclei. The $x-$rescaling accounts for nuclear effects in the case of exact scaling, while the Q^2-rescaling is responsible for a corresponding modification of quantum corrections. This result is obtained in the leading order for all flavour combinations and connects the two known models for the EMC-effect. Electroproduction and gluonic nuclear structure functions are calculated.

1. There is still no consensus on the origin of the nuclear EMC-effect[1]. In addition to the theoretical importance of this phenomenon, it is now becoming practically important to know nuclear quark and gluon structure functions (SF) in order to search for novel effects in hadron-nucleus and in nucleus-nucleus collisions. Among several models for the EMC effect, we focus on the Q^2-rescaling model [2] and the x-rescaling model [3]. In some papers [4, 5] it was concluded that the $x-$rescaling alone cannot explain the EMC-effect. An ansatz that both types of rescaling can be combined has been exploited in Refs.[6, 7]. A possible duality of these two models was discussed in Ref.[8], where an interesting correspondence between the conventional nuclear theory and the QCD properties of nucleons was conjectured. We show that there is indeed a deep correspondence between these two types of rescaling in nuclei but our conclusions differ from the conclusions of Ref.[8]. We claim that the $x-$rescaling accounts for the nuclear modification of nucleonic SF in the tree approximation for intrinsic partons, while the Q^2-rescaling represents a corresponding modification of quantum corrections. Therefore these two types of rescaling are self-complementary and must be taken both into account, but are not equivalent to each other. We prove the ansatz of Ref.[7] and reveal the QCD content of the $(x, Q^2)-$rescaling. We also extend this model to flavour singlet combinations and calculate the gluonic EMC-effect.

2. We assume that in the region of intermediate and large x the distances involved are short enough and photons interact with separated nucleons in nuclei. We consider only nucleons as nuclear constituents and discuss below the possibility of other constituents. The photon-nucleus amplitude $T_A(q, P_A)$ can be represented in the convolution form

$$T_A(q, P_A) = \int d^4p \, c(P_A, p) \, f(p) \, T_N(q, p), \tag{1}$$

where $T_N(q,p) = \int d^4x \, e^{-iqx} \; < p|T(J(\frac{x}{2})J(-\frac{x}{2}))|p >$ (here x is the coordinate variable, but it is the Bjorken variable elsewhere), $c(P_A, p)$ are normalization factors absorbing the flux factors[4, 9], $f(p)$ is the four-momentum distribution of nucleons in nuclei. The Lorentz and flavour indices are omitted for simplicity. In the covariant Feynman formalism we can write

$$p \equiv P_A - P_{A-1} = (m_N - E/(A-1), -\vec{P}_{A-1}) = (m_N - \epsilon, \, \vec{p}), \tag{2}$$

where E and $\vec{P}_{A-1}$ are an excitation energy and a three-momentum of the spectator (A-1)-nucleus, which in contrast to the target nucleus may not be in its ground state, ϵ has the meaning of a nucleon separation energy. From $(e, e'p)-$experiments it follows[10] that the nucleon four-momentum distribution can be approximately described for medium nuclei as

$$|\vec{p}| \leq p_F = 260MeV, \ \ \epsilon \approx 35MeV. \tag{3}$$

In the large $(-q^2)$ limit and with fixed $x = -q^2/2pq$, the operator product expansion (OPE) can be used to obtain

$$T_N(q,p,g(\mu),\mu) = \sum_{i,j} (\frac{2pq}{-q^2})^j \, C_i^{(j)}(q,g(\mu),\mu) \, O_i^{(j)}(p,g(\mu),\mu), \tag{4}$$

where $C_i^{(j)}$ and $O_i^{(j)}$ are Wilson coefficients and reduced matrix elements, respectively, j is the spin of an operator of type i, μ is the scale parameter and $g(\mu)$ is the renormalized coupling constant. We now claim that p from (2) can be substituted in (4). The OPE is used to extract two factors with two scales: the target-independent Wilson coefficients and the reduced matrix elements containing unperturbative target-dependent effects at the scale $\sim m_N$. The condition for OPE is $(-q^2) \gg k^2$, where k characterises interactions in a target. Parton interactions, which fulfill this condition, are accounted for by the regular part of the amplitudes (the reduced matrix elements). With respect to this condition, there is no difference between intra- and internucleon interactions of partons. Therefore we find reasonable to include in the regular part of amplitudes the interactions of partons from different nucleons as well. The result of these interactions is approximately represented by off-mass-shell momenta p.

The $p-$dependence of T_N appears not only in the factors $(2pq/-q^2)^j$ in (4) (this is accounted for by the $x-$rescaling) but also in the reduced matrix elements $O_i^{(j)}$. Let us prove that $O_i^{(j)}$ are p-dependent. In the region of asymptotic q and p, the solution of the renormgroup (RG) equation for T_N is

$$T_N(\sigma q, \sigma p, g(\mu), \mu) = T_N(q, p, \overline{g}(t,\mu), \mu), \tag{5}$$

where σ is a rescaling factor, $t = ln(\sigma)$ and $\overline{g}$ is the running coupling constant. The solutions of the RG equation for $O_i^{(j)}$ and $C_i^{(j)}$ can be written as[11]

$$\begin{aligned} C_i^{(j)}(\sigma p, g(\mu),\mu) &= C_i^{(j)}(p,\overline{g}(t,\mu),\mu)\, exp(-\int_0^t dx\,\gamma_i^{(j)}(x)), \\ O_i^{(j)}(\sigma q, g(\mu),\mu) &= O_i^{(j)}(q,\overline{g}(t,\mu),\mu)\, exp(+\int_0^t dx\,\gamma_i^{(j)}(x)), \end{aligned} \tag{6}$$

where $\gamma_i^{(j)}$ is a corresponding anomalous dimension. The last equation takes place only if the matrix element is $p-$dependent. Otherwise the exponent from the former of the equations (6) will not be canceled in the expansion (4) for the l.h.s. of (5). This means that $O_i^{(j)}$ must be $p-$dependent in order to secure (5). Since this is true for asymptotic momenta, this is also true in a general case. Note that this proof can be done for arbitrary momenta in the MS-scheme. From the definition of $O_i^{(j)}$ and $C_i^{(j)}$, these values depend on momenta squared. By defining

$$\overline{C}_i^{(j)}(q^2, g(\mu),\mu) \equiv C_i^{(j)}(q, g(\mu),\mu),\;\; \overline{O}_i^{(j)}(p^2, g(\mu),\mu) \equiv O_i^{(j)}(q, g(\mu),\mu), \tag{7}$$

the photon - bound nucleon amplitude can be rewritten as

$$T_N(q,p) = \sum_i (\frac{x_0}{y})^{-i}\; \overline{C}_i(q^2, g(\mu),\mu)\, \overline{O}_i(p^2, g(\mu),\mu), \quad y = \frac{pq}{m_N q_0}, \tag{8}$$

where $x_0 = -q^2/2m_N q_0$. The $x-$rescaling due to y takes place always when $(pq) \neq (m_N q_0)$, i.e. even for free moving nucleons with $p^2 = m_N^2$ and in the tree approximation when $\overline{C}_i^{(j)}$ and $\overline{O}_i^{(j)}$ are constant. The p^2-dependence of $\overline{O}_i^{(j)}$ has nontrivial effects only when $p^2 \neq m_N^2$ *and* quantum corrections are included, as we now show.

The p^2-dependence of $\overline{O}_i^{(j)}$ cannot be calculated within the perturbative QCD. But we can transform this dependence into the calculable q^2-dependence of $\overline{C}_i^{(j)}$. For a Lorentz-invariant function of p^2, the transformation $p^2 \to \sigma^2 p^2$ of external momenta squared is equivalent to the linear rescaling $p \to \sigma p$ of those momenta. Therefore $\overline{O}_i^{(j)}$ for bound nucleons can be found by linearly rescaling nucleonic momenta. For bound nucleons, the rescaling parameter σ is given by

$$\sigma = (\frac{p^2}{m_N^2})^{1/2} = (\frac{(m_N-\epsilon)^2 - \vec{p}^{\,2}}{m_N^2})^{1/2} < 1. \tag{9}$$

In the tree approximation the scaling is exact, the anomalous dimensions vanish and both $\overline{C}_i^{(j)}$ and $\overline{O}_i^{(j)}$ become constant[11]. This means that the p^2-dependence of reduced matrix elements takes place due to quark-gluon loops. If the external nucleonic momenta are linearly rescaled then loop momenta would also be linearly rescaled by the same factor σ. Thus, the p^2-dependence of reduced matrix elements can be represented by the linear rescaling of quark and gluon momenta in bound nucleons. This conclusion is the starting point in the derivation of the Q^2-rescaling in the model of Ref.[2]. The difference is that in Ref.[2] the rescaling parameter was not given by (9), but was assumed to be

$$\sigma' = \lambda_N/\lambda_A, \tag{10}$$

where λ is a confinement size for a corresponding target. In contrast to σ, which can be related to empirical nuclear values, the parameters $\lambda_{N,A}$ are poorly known and allow to fit the EMC-effect by the Q^2-rescaling alone.

Following the conjecture of Ref.[2], we assume that the linear rescaling of parton momenta may be compensated by the same rescaling of the normalization scale μ,

$$\overline{O}_i^{(j)}(p^2, g(\mu_N), \mu_N) \approx \overline{O}_i^{(j)}(m_N^2, g(\mu_N/\sigma), \mu_N/\sigma). \tag{11}$$

Here μ_N is an arbitrary parameter, interpreted in Ref.[2] as the scale at which the valence-quark approximation works well for leading twist operators. Our derivation is not limited by the valence quark approximation. Using the fact that final results are not sensitive to μ_N, if it is of the order of hadronic mass, we substitute $\mu_N = m_N$ in numerical calculations.

The derivation of the final equation is now straightforward. The products $(\overline{C}_i^{(j)}\overline{O}_i^{(j)})$ are normalization-independent provided the normalization scale is the same for both factors. The $\mu-$dependence of $\overline{C}_i^{(j)}$ is known within the perturbation theory and, in the leading order, is given by

$$\overline{C}_i^{(j)}(q^2, g(\mu), \mu) \sim (\frac{\alpha_s(q^2)}{\alpha_s(\mu^2)})^{\gamma_i^{(j)}/2\beta}, \tag{12}$$

where $\gamma_i^{(j)}$ and β are the lowest order gamma and beta functions. From this equation it follows that $\overline{C}_i^{(j)}(q^2, g(m_N/\sigma), m_N/\sigma) = \overline{C}_i^{(j)}(\xi q^2, g(m_N), m_N)$ and the rescaling parameter ξ is given by

$$\xi = \sigma^{-2\alpha_s(m_N^2)/\alpha_s(Q^2)}. \tag{13}$$

In the leading order, ξ is independent of the operator spin and flavour. This means that in this order all SF moments are rescaled by the same ξ and we can write the final result in terms of SF,

$$F^A(x_0, Q^2) = \int d^4p\, f(p)\, (pq/m_N q_0)\, F^N(x_0/y, \xi Q^2), \tag{14}$$

where F can be F_2, F_3 or gluon distribution functions (the factor $(pq/m_N q_0)$ is the reminiscence of the normalization factor in (1)).

We self-consistently rescaled both the tree-approximation part and the Q^2-evolution of SF, whereas only the latter is rescaled in Ref.[2]. Consequently, the changes of scale in these two approaches are different: the average σ from (9) is about 0.94 for the iron nucleus but σ' from Ref.[2] is about 0.87 for the same nucleus, i.e. $(1-\sigma') \approx 2(1-\sigma)$. Therefore in the approach of Ref.[2] the EMC-effect was fitted by the Q^2-rescaling alone, whereas in the present model the combined $(x, Q^2)-$rescaling represents the full result. Note that in Ref.[8] it was implied that *all* parton momenta (not just valence quark momenta) would be rescaled by σ'. In this case bound nucleon momenta, which are combinations of parton momenta, should also be rescaled by σ'. As it follows from (4) and (8), this leads to an $x-$rescaling as in (14), but with $y = \sigma'$. However, the numerical results of Ref.[2] leave no room for such additional $x-$rescaling.

3. We have calculated nuclear SF using the Q^2-dependent parametrization of free nucleon SF from Ref.[12] (Set 1) and the nucleon distribution given by (3). The parameter Λ_{QCD} was fixed as in Ref.[12]. The nucleus to nucleon ratios of electroproduction SF are shown in Fig.1. The Q^2-rescaling further suppresses nuclear SF at medium and large x. The description of the data is reasonable, except for the large-x region. In that region the contribution of nucleon-nucleon short-range correlations, neglected in this paper, may improve the agreement with the data[6]. The R increasing as $x \to 1$ is automatically accounted for by the $x-$rescaling in our model, but not in the model of Ref.[2]. Note that the Fermi motion contribution can disturb the agreement with the data, obtained in Ref.[2], even at $x \leq 0.6$ (see Ref.[9] for the Fermi motion effect). We conclude that the $(x, Q^2)-$rescaling can explain the bulk of the EMC-effect. In some papers[13, 14], the $x-$rescaling has been

combined with a modification of nucleonic SF for off-mass-shell nucleons. That modification gives an additional suppression of nuclear SF and allows to describe the EMC-effect without the Q^2-rescaling we discussed. There is no contradiction between the present paper and Refs.[13, 14]: we suggested an alternative p^2-dependent modification of SF for bound nucleons, based on the RG technique. In this approach, the p^2-dependence of SF takes place only when the Q^2-dependence of SF is non-trivial.

The gluonic SF ratios, calculated with only the $x-$rescaling and with the combined $(x, Q^2)-$rescaling, are also shown in Fig.1. The Q^2-rescaling is very important in this case. As an illustration, we applied the calculated quark and gluon nuclear distributions to estimate the nuclear effectiveness α in the bottomonium production in proton-nucleus collisions at 800 GeV. We used the model for this reaction from Ref.[15]. In this reaction, the dominant contribution is due to gluon-gluon fusion subprocess. In Fig.2, α is compared for free nucleon (dashed line) and nuclear (solid line) SF calculated according to (14). The description of the data is satisfactory except for the region $x \geq 0.25$ ($x_F \leq 0$). In this region the comover interaction[17], not taken into account in the model of Ref.[15], is expected to be significant. The EMC-effect explains only a small fraction of the observed bottomonium suppression but is not negligible.

It is known that the $x-$rescaling leads to a violation of the momentum sum rule for off-mass-shell nucleons. This violation can be compensated either by excess pions[18, 3] or by excess gluons[19, 15] in nuclei. The lack of the DY production enhancement for nuclear targets, as well as some results for the quarkonium production on nuclei[15], indicates in favour of excess gluons in nuclei. In Fig.2, we also show our result for α taking into account the excess gluon contribution, assumed in Ref.[15]. We can say that this contribution does not contradict the data. A possible substructure of nuclear binding forces has been discussed in several papers[8, 19, 15]. The nuclear rescaling we studied here is not based on an assumption about the origin of binding forces.

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References

[1] J.J.Aubert et al, Phys.Lett. 123B (1983) 275.

[2] F.E.Close, R.L.Jaffe, R.G.Roberts and G.G.Ross, Phys.Rev. D 13 (1985) 1004.

[3] S.V.Akulinichev, S.A.Kulagin and G.M.Vagradov, Phys.Lett. 158B (1985) 485.

[4] L.L.Frankfurt and M.I.Strikman, Phys.Lett. 183B (1987) 254.

[5] G.L.Li, K.F.Liu and G.E.Brown, Phys.Lett. 213B (1988) 531.

[6] C.Ciofi degli Atti and S.Liuti, Phys.Lett. 225B (1989) 215.

[7] G.V.Dunne and A.W.Thomas, Nucl.Phys. A455 (1986) 701.

[8] F.E.Close, R.G.Roberts and GG.Ross, Phys.Lett. 168B (1986) 400.

[9] S.V.Akulinichev and S.Shlomo, Phys.Lett. 234B (1990) 170.

[10] S.Frullani and J.Mougey, Adv.Nucl.Phys. 14 (1984) 1.

[11] T.-P.Cheng and L.-F.Li, *Gauge Theory of Elementary Particle Physics*, Clarendon Press, Oxford (1984).

[12] D.W.Duke and J.F.Owens, Phys.Rev. D30 (1984) 49.

[13] F.Gross and S.Liuti, Phys.Rev. C45 (1992) 1374.

[14] S.A.Kulagin, G.Piller and W.Weise, Phys.Rev. C50 (1994) 1154.

[15] S.V.Akulinichev, INR preprint 0887-95(1995)(nucl-th/9504032).

[16] D.M.Alde et al, Phys.Rev.Lett. 66 (1991) 133.

[17] S.J.Brodsky and P.Hoyer, Phys.Rev.Lett. 63 1989) 1566.

[18] M.Ericson and A.W.Thomas, Phys.Lett. 128B (1983) 112.

[19] L.L.Frankfurt, M.I.Strikman and S.Liuti, Phys.Rev.Lett. 65 (1990) 1725.

Figure captions.

Fig.1. The electroproduction cross section ratio for A=56: the short-dashed line describes the $x-$rescaling alone, the solid and long-dashed lines describe the $(x, Q^2)-$rescaling calculated at Q^2=10 GeV^2 and Q^2=100 Gev^2, respectively. The data for electroproduction are from Ref.[1]. The ratio for the gluon distribution functions for A=56: the dashed-dotted line describes the $x-$rescaling alone and the dotted line describes the $(x, Q^2)-$rescaling calculated at Q^2=100 GeV^2.

Fig.2 The nuclear effectiveness for bottomonium production in proton-nucleus collisions at 800 GeV calculated with the nuclear structure function from Eq.(14) (solid line) and with the free nucleon structure functions (long-dashed line). The former result with the excess gluon contribution included is shown by the short-dashed line. The data (diamonds for $(1S)-$states and crosses for $(2S+3S)-$states) are from Ref.[16].