Gravitons in one-loop quantum cosmology: correspondence between covariant and non-covariant formalisms

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Abstract. The discrepancy between the results of covariant and non-covariant one-loop calculations for higher-spin fields in quantum cosmology is analyzed. A detailed mode-by-mode study of perturbative quantum gravity about a flat Euclidean background bounded by two concentric 3-spheres, including non-physical degrees of freedom and ghost modes, leads to one-loop amplitudes in agreement with the covariant Schwinger-DeWitt

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method. This calculation provides the generalization of a previous analysis of fermionic fields and electromagnetic fields at one-loop about flat Euclidean backgrounds admitting a well-defined 3+1 decomposition.

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I.   INTRODUCTION

In recent papers devoted to one-loop calculations in quantum gravity and quantum cosmology [1-11], discrepancies were found between the covariant Schwinger-DeWitt method [12], where the scaling factor of one-loop amplitudes coincides with the A_2 coefficient in the heat-kernel expansion, and zeta-function regularization [13-14] with the corresponding mode-by-mode analysis of quantized fields.

In Ref. [1] it was shown that the $\zeta(0)$ value for gravitons calculated on the Riemannian de Sitter 4-sphere in terms of physical degrees of freedom [15] disagrees with that obtained in covariant formalism [16]. In Refs. [2-11] it was shown that the results of analogous calculations for gauge fields on manifolds with boundaries, as well as earlier results of Refs. [17-18], are in disagreement with the results of covariant calculations using the modified Schwinger-DeWitt formulae for manifolds with boundaries [19].

In Ref. [20] the attempt of investigating the reason of the discrepancy described above was made. Since for spin-$\frac{1}{2}$ fields there is no gauge freedom, different results cannot be due to inequivalent quantization techniques, but may have an entirely geometrical origin. The hypothesis was then put for-

ward that the reason of this discrepancy consists in the inappropriate use of 3+1 decomposition on the manifolds where such a split is ill-defined (apparently the 3+1 decomposition is ill-defined on the full 4-dimensional sphere or on the part of 4-sphere or 4-dimensional flat space bounded by a single 3-sphere, since the vector field matching the normal to the boundary is singular at the origin). The $\zeta(0)$ value for massless spin-1/2 fields on the manifold representing the part of flat Euclidean 4-dimensional space bounded by two concentric 3-spheres (such a manifold admits a well-defined 3+1 split) was calculated. It was shown that the result coincides with the covariant one obtained within the framework of the Schwinger-DeWitt method [3,19].

On evaluating $\zeta(0)$ for vacuum Maxwell theory subject to the Coulomb gauge *before* quantization, the one-loop result is in disagreement with the covariant one, even in the two-boundary case, where one has a well-defined 3+1 split [21]. The possible reason is that, on non-trivial backgrounds with boundaries, contributions of ghosts and non-physical degrees of freedom do not cancel each other. In Ref. [21], continuing the study of the quantization of the electromagnetic field on manifolds with boundaries appearing in Ref. [22], this suggestion was checked. It was shown that, if one deals with non-physical degrees of freedom and ghost modes within the Faddeev-Popov formalism, and if one studies a flat Euclidean background bounded by two concentric 3-spheres, the one-loop evaluation of quantum amplitudes agrees with the covariant result, confirming in such a way our hypotheses. A technique for the disentanglement of coupled gauge modes was also described and applied

for the first time.

Here, we calculate $\zeta(0)$ for gravitons on the part of flat 4-dimensional Euclidean space bounded by two concentric 3-spheres, taking into account the contributions of non-physical modes and ghosts. We show that in this case, which is more complicated from the technical point of view than the electromagnetic analysis of Refs. [18,21-22], one finds again agreement between the results of covariant and non-covariant calculations.

In our calculations we use the version of the generalized ζ-function technique elaborated in [8-10]. The main ideas are as follows. Zeta-functions are traces of complex powers of elliptic, self-adjoint, positive-definite differential operators. They have an analytic continuation to the complex plane as meromorphic functions with simple poles. Remarkably, they are regular at the origin together with their first derivative, and this enables one to define and compute the determinants of the corresponding operators and one-loop quantum amplitudes. With our notation, one writes $f_n(M^2)$ for the function occurring in the equation obeyed by the eigenvalues by virtue of boundary conditions, and $d(n)$ for the degeneracy of the eigenvalues. One then defines the function

$$I(M^2, s) \equiv \sum_{n=n_0}^{\infty} d(n) n^{-2s} \log f_n(M^2). \qquad (1.1)$$

Such a function has a unique analytic continuation to the whole complex-s plane as a meromorphic function, i.e.

$$I(M^2, s) = \frac{I_{pole}(M^2)}{s} + I^R(M^2) + O(s). \qquad (1.2)$$

The $\zeta(0)$ value is then obtained as [8-10]

$$\zeta(0) = I_{\log} + I_{pole}(\infty) - I_{pole}(0), \qquad (1.3)$$

where $I_{log} = I_{log}^R$ is the coefficient of $\log M$ from $I(M^2, s)$ as $M \to \infty$, and $I_{pole}(M^2)$ is the residue at $s = 0$. Remarkably, I_{log} and $I_{pole}(\infty)$ are obtained from the uniform asymptotic expansions of modified Bessel functions as their order tends to ∞ and $M \to \infty$, whereas $I_{pole}(0)$ is obtained from the limiting behaviour of such Bessel functions as $M \to 0$.

In Sec. II we write down the equations for basis functions for physical and non-physical degrees of freedom of the gravitational field with the corresponding ghost modes, and we find their solutions on the chosen background. In Sec. III we consider the boundary conditions and compute all contributions to the full $\zeta(0)$ value. Results and concluding remarks are presented in Sec. IV. The forms of the differential operators acting on metric perturbations are given in the Appendix.

II. EQUATIONS FOR BASIS FUNCTIONS AND THEIR SOLUTIONS

For the reasons described in the introduction, we study pure gravity at one-loop about a flat Euclidean background with two concentric 3-sphere boundaries. Our approach to quantization follows the Faddeev-Popov formalism (cf. [22]). Hence we deal with quantum amplitudes of the form

$$Z[boundary\ data] = \int_C \mu_1[g]\ \mu_2[\varphi]\ exp(-\widetilde{I}_E).$$

With our notation, C is the set of all Riemannian 4-geometries matching the boundary data, μ_1 is a suitable measure on the space of metrics, μ_2 is a suitable measure for ghosts, Φ_ν is an arbitrary gauge-averaging functional, and the total Euclidean action reads (in $c = 1$ units)

$$\widetilde{I}_E = I_{gh} + \frac{1}{16\pi G} \int_M {}^{(4)}R \sqrt{det\ g}\ d^4x + \frac{1}{8\pi G} \int_{\partial M} Tr\ K \sqrt{det\ q}\ d^3x$$
$$+ \frac{1}{16\pi G} \int_M \frac{1}{2\alpha} \Phi_\nu \Phi^\nu \sqrt{det\ g}\ d^4x. \tag{2.1}$$

Of course, K is the extrinsic-curvature tensor of the boundary, q is the induced 3-metric of ∂M, and α is a positive dimensionless parameter. The ghost action I_{gh} depends on the specific form of Φ_ν. Denoting by $h_{\mu\nu}$ the perturbation around the background 4-metric $g_{\mu\nu}$, one thus finds equations of motion of the kind

$$^\Phi h_{\mu\nu} = 0,$$

where $^\Phi$ is the 4-dimensional elliptic operator corresponding to the form of Φ_ν one is working with. Here we choose the de Donder gauge-averaging functional

$$\Phi_\nu^{DD} \equiv \nabla^\mu \left(h_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \hat{h} \right),$$

where ∇^μ is covariant differentiation with respect to $g_{\mu\nu}$, and $\hat{h} \equiv g^{\mu\nu}\ h_{\mu\nu}$. The corresponding $^{\Phi^{DD}}$ operator is the one obtained by analytic continuation of the standard D'Alembert operator, hereafter denoted by . The resulting eigenvalue equation is (see Appendix)

$$h_{\mu\nu} + \lambda h_{\mu\nu} = 0.$$

Now we can make the 3+1 decomposition of our background 4-geometry and expand h_{00}, h_{0i} and h_{ij} in hyperspherical harmonics as

$$h_{00}(x,\tau) = \sum_{n=1}^{\infty} a_n(\tau) Q^{(n)}(x), \tag{2.2}$$

$$h_{0i}(x,\tau) = \sum_{n=2}^{\infty} \left[b_n(\tau) \frac{\nabla_i Q^{(n)}(x)}{(n^2-1)} + c_n(\tau) S_i^{(n)}(x) \right], \tag{2.3}$$

$$h_{ij}(x,\tau) = \sum_{n=3}^{\infty} d_n(\tau) \left(\frac{\nabla_i \nabla_j Q^{(n)}(x)}{(n^2-1)} + \frac{c_{ij}}{3} Q^{(n)}(x) \right) + \sum_{n=1}^{\infty} \frac{e_n(\tau)}{3} c_{ij} Q^{(n)}(x)$$
$$+ \sum_{n=3}^{\infty} \left[f_n(\tau) \left(\nabla_i S_j^{(n)}(x) + \nabla_j S_i^{(n)}(x) \right) + k_n(\tau) G_{ij}^{(n)}(x) \right]. \tag{2.4}$$

Here $Q^{(n)}(x)$, $S_i^{(n)}(x)$ and $G_{ij}^{(n)}(x)$ are scalar, transverse vector and transverse-traceless tensor hyperspherical harmonics respectively, on a unit 3-sphere with metric c_{ij}. Their properties are described in Refs. [17-18,23].

The insertion of the expansions (2.2)-(2.4) into Eq. (2.1) leads to the following system of equations (decoupled modes will be treated separately):

$$\widehat{A}_n a_n(\tau) + \widehat{B}_n b_n(\tau) + \widehat{C}_n e_n(\tau) = 0, \tag{2.5}$$

$$\widehat{D}_n b_n(\tau) + \widehat{E}_n a_n(\tau) + \widehat{F}_n d_n(\tau) + \widehat{G}_n e_n(\tau) = 0, \tag{2.6}$$

$$\widehat{L}_n d_n(\tau) + \widehat{M}_n b_n(\tau) = 0, \tag{2.7}$$

$$\widehat{N}_n e_n(\tau) + \widehat{P}_n b_n(\tau) + \widehat{Q}_n a_n(\tau) = 0, \tag{2.8}$$

$$\widehat{H}_n c_n(\tau) + \widehat{K}_n f_n(\tau) = 0, \tag{2.9}$$

$$\widehat{R}_n f_n(\tau) + \widehat{S}_n c_n(\tau) = 0, \tag{2.10}$$

$$\widehat{T}_n k_n(\tau) = 0. \tag{2.11}$$

Since our background is flat, after setting $\alpha = 1$ in (2.1) the operators appearing in Eqs. (2.5)-(2.11) take the form (for all integer $n \geq 3$)

$$\widehat{A}_n \equiv \frac{d^2}{d\tau^2} + \frac{3}{\tau}\frac{d}{d\tau} - \frac{(n^2 + 5)}{\tau^2} + \lambda_n,$$

$$\widehat{B}_n \equiv \frac{4}{\tau^3},$$

$$\widehat{C}_n \equiv \frac{2}{\tau^4},$$

$$\widehat{D}_n \equiv \frac{d^2}{d\tau^2} + \frac{1}{\tau}\frac{d}{d\tau} - \frac{(n^2 + 4)}{\tau^2} + \lambda_n,$$

$$\widehat{E}_n \equiv \frac{2}{\tau}(n^2 - 1),$$

$$\widehat{F}_n \equiv \frac{4}{3}\frac{(n^2 - 4)}{\tau^3},$$

$$\widehat{G}_n \equiv -\frac{2}{3}\frac{(n^2 - 1)}{\tau^3},$$

$$\widehat{H}_n \equiv \frac{d^2}{d\tau^2} + \frac{1}{\tau}\frac{d}{d\tau} - \frac{(n^2 + 5)}{\tau^2} + \lambda_n,$$

$$\widehat{K}_n \equiv \frac{2}{\tau^3}(n^2 - 4),$$

$$\widehat{L}_n \equiv \frac{d^2}{d\tau^2} - \frac{1}{\tau}\frac{d}{d\tau} - \frac{(n^2 - 5)}{\tau^2} + \lambda_n,$$

$$\widehat{M}_n \equiv \frac{4}{\tau},$$

$$\widehat{N}_n \equiv \frac{d^2}{d\tau^2} - \frac{1}{\tau}\frac{d}{d\tau} - \frac{(n^2 + 1)}{\tau^2} + \lambda_n,$$

$$\widehat{P}_n \equiv -\frac{4}{\tau},$$

$$\widehat{Q}_n \equiv 6,$$

$$\widehat{R}_n \equiv \frac{d^2}{d\tau^2} - \frac{1}{\tau}\frac{d}{d\tau} - \frac{(n^2 - 4)}{\tau^2} + \lambda_n,$$

$$\widehat{S}_n \equiv \frac{2}{\tau},$$

$$\widehat{T}_n \equiv \frac{d^2}{d\tau^2} - \frac{1}{\tau}\frac{d}{d\tau} - \frac{(n^2 - 1)}{\tau^2} + \lambda_n. \tag{2.12}$$

Inserting the operator $\widehat{T}_n$ from Eq. (2.12) into Eq. (2.11) we can easily find the basis function describing the transverse-traceless symmetric tensor harmonics which usually are treated as physical degrees of freedom [1,15,17,23]

$$k_n(\tau) = \alpha_1 \tau I_n(M\tau) + \alpha_2 \tau K_n(M\tau), \quad n = 3,\ldots \tag{2.13}$$

where $M = \sqrt{-\lambda}$ and I and K are modified Bessel functions.

However, the equations (2.5)-(2.8) for scalar-type gravitational perturbations lead to a rather complicated entangled system as well as Eqs. (2.9)-(2.10), describing vector perturbations. In Ref. [21], where we have studied the analogous problem for the electromagnetic field, a method was used to decouple a similar entangled system for normal and longitudinal components of the 4-vector potential. The idea is that one can diagonalize a 2×2 operator matrix after multiplying it by two functional matrices. In some cases one can choose these functional matrices in such a way that the transformed operator matrix is diagonal and the corresponding differential equations for basis functions are decoupled. However, in the case of scalar-type gravitational perturbations we have a 4×4 operator matrix. To diagonalize such a matrix it is necessary to solve a system of 24 second-order algebraic equations with 24 variables. This problem seems a rather cumbersome one and we thus use another method. For this purpose, we assume that the solution of the system of equations (2.5)-(2.8) is some set of modified Bessel functions

with unknown index ν. Let us look for a solution of this system in the form

$$a_n(\tau) = \beta_1 \frac{W_\nu(M\tau)}{\tau}, \tag{2.14}$$

$$b_n(\tau) = \beta_2 W_\nu(M\tau), \tag{2.15}$$

$$d_n(\tau) = \beta_3 \tau W_\nu(M\tau), \tag{2.16}$$

$$e_n(\tau) = \beta_4 \tau W_\nu(M\tau). \tag{2.17}$$

Here, W_ν is a linear combination of modified Bessel functions I_ν and K_ν obeying the Bessel equation

$$\left(\frac{d^2}{d\tau^2} + \frac{1}{\tau}\frac{d}{d\tau} - \frac{\nu^2}{\tau^2} - M^2 \right) W_\nu(M\tau) = 0. \tag{2.18}$$

Now, inserting the functions (2.14)-(2.17) and the corresponding operators from Eq. (2.12) into the system of equations (2.5)-(2.8), and taking into account the Bessel equation (2.18), one finds the following system of equations for $\beta_1, \beta_2, \beta_3$ and β_4:

$$(\nu^2 - n^2 - 6)\beta_1 + 4\beta_2 + 2\beta_4 = 0,$$

$$6(n^2 - 1)\beta_1 + 3(\nu^2 - n^2 - 4)\beta_2 + 4(n^2 - 4)\beta_3 - 2(n^2 - 1)\beta_4 = 0,$$

$$4\beta_2 + (\nu^2 - n^2 + 4)\beta_3 = 0,$$

$$6\beta_1 - 4\beta_2 + (\nu^2 - n^2 - 2)\beta_4 = 0. \tag{2.19}$$

The condition for the existence of nontrivial solutions of the system (2.19) is the vanishing of its determinant, i.e.

$$(\nu^2 - n^2)^2 \left[(\nu^2 - n^2)^2 - 8(\nu^2 - n^2) - 16(n^2 - 1) \right] = 0. \tag{2.20}$$

The roots of Eq. (2.20) are

$$\nu^2 = n^2, \ \nu^2 = (n-2)^2, \ \nu^2 = (n+2)^2.$$

The positive values of ν provide the orders of modified Bessel functions. Now we can write down the β's corresponding to different values for ν's. For $\nu = n$ one has

$$\beta_4 = 3\beta_1, \ \beta_2 = \beta_3 = 0, \tag{2.21}$$

or

$$\beta_1 = 0, \ \beta_3 = -\beta_2, \beta_4 = -2\beta_2. \tag{2.22}$$

For $\nu = n - 2$ one has

$$\beta_2 = (n+1)\beta_1, \ \beta_3 = \frac{(n+1)}{(n-2)}\beta_1, \ \beta_4 = -\beta_1. \tag{2.23}$$

Last, for $\nu = n + 2$ one has

$$\beta_2 = -(n-1)\beta_1, \ \beta_3 = \frac{(n-1)}{(n+2)}\beta_1, \ \beta_4 = -\beta_1. \tag{2.24}$$

Having the Eqs. (2.21)-(2.24) we can get the basis functions for scalar-type gravitational perturbations (2.14)–(2.17)

$$\begin{aligned}
a_n(\tau) \ &= \frac{1}{\tau}\Big(\gamma_1 I_n(M\tau) + \gamma_3 I_{n-2}(M\tau) + \gamma_4 I_{n+2}(M\tau) \\
&\quad + \delta_1 K_n(M\tau) + \delta_3 K_{n-2}(M\tau) + \delta_4 K_{n+2}(M\tau)\Big),
\end{aligned} \tag{2.25}$$

$$\begin{aligned}
b_n(\tau) \ &= \gamma_2 I_n(M\tau) + (n+1)\gamma_3 I_{n-2}(M\tau) \\
&\quad -(n-1)\gamma_4 I_{n+2}(M\tau) + \delta_2 K_n(M\tau) \\
&\quad +(n+1)\delta_3 K_{n-2}(M\tau) - (n-1)\delta_4 K_{n+2}(M\tau),
\end{aligned} \tag{2.26}$$

$$
\begin{aligned}
d_n(\tau) \;=\; \tau\Bigg(&-\gamma_2 I_n(M\tau) + \frac{(n+1)}{(n-2)}\gamma_3 I_{n-2}(M\tau) \\
&+\frac{(n-1)}{(n+2)}\gamma_4 I_{n+2}(M\tau) - \delta_2 K_n(M\tau) \\
&+\frac{(n+1)}{(n-2)}\delta_3 K_{n-2}(M\tau) + \frac{(n-1)}{(n+2)}\delta_4 K_{n+2}(M\tau)\Bigg)
\end{aligned}
\tag{2.27}
$$

$$
\begin{aligned}
e_n(\tau) \;=\; \tau\Big(&3\gamma_1 I_n(M\tau) - 2\gamma_2 I_n(M\tau) - \gamma_3 I_{n-2}(M\tau) \\
&-\gamma_4 I_{n+2}(M\tau) + 3\delta_1 K_n(M\tau) - 2\delta_2 K_n(M\tau) \\
&-\delta_3 K_{n-2}(M\tau) - \delta_4 K_{n+2}(M\tau)\Big).
\end{aligned}
\tag{2.28}
$$

We can find the basis functions for vector-like gravitational perturbations in a similar way. Let us suppose that

$$
c_n(\tau) = \varepsilon_1 W_\nu(M\tau)
\tag{2.29}
$$

and

$$
f_n(\tau) = \varepsilon_2 \tau W_\nu(M\tau).
\tag{2.30}
$$

Inserting (2.29)-(2.30) into Eqs. (2.9)-(2.10) one has the system

$$
(\nu^2 - n^2 - 5)\varepsilon_1 + 2(n^2 - 4)\varepsilon_2 = 0,
$$

$$
2\varepsilon_1 + (\nu^2 - n^2 + 3)\varepsilon_2 = 0.
\tag{2.31}
$$

The determinant of the system (2.31) is

$$
(\nu^2 - n^2)^2 - 2(\nu^2 - n^2) - 4n^2 + 1
$$

and its positive roots are $n \pm 1$. For $\nu = n + 1$ one has

$$
\varepsilon_2 = -\frac{1}{(n+2)}\varepsilon_1
$$

and for $\nu = n - 1$ one has

$$\varepsilon_2 = \frac{1}{(n-2)} \varepsilon_1,$$

and correspondingly the functions (2.29)-(2.30) take the form

$$c_n(\tau) = \widetilde{\varepsilon}_1 I_{n+1}(M\tau) + \widetilde{\varepsilon}_2 I_{n-1}(M\tau) + \eta_1 K_{n+1}(M\tau) + \eta_2 K_{n-1}(M\tau), \quad (2.32)$$

$$f_n(\tau) = \tau \left(-\frac{1}{(n+2)} \widetilde{\varepsilon}_1 I_{n+1}(M\tau) + \frac{1}{(n-2)} \widetilde{\varepsilon}_2 I_{n-1}(M\tau) \right.$$
$$\left. -\frac{1}{(n+2)} \eta_1 K_{n+1}(M\tau) + \frac{1}{(n-2)} \eta_2 K_{n-1}(M\tau) \right). \quad (2.33)$$

We have also to find the basis functions for ghosts. The eigenvalue equations for ghosts in the de Donder gauge have the form (see Appendix)

$$\varphi_\mu + \lambda \varphi_\mu = 0$$

and the corresponding fields can be expanded on a family of 3-spheres as

$$\varphi_0(x, \tau) = \sum_{n=1}^{\infty} l_n(\tau) Q^{(n)}(x), \quad (2.34)$$

$$\varphi_i(x, \tau) = \sum_{n=2}^{\infty} \left[m_n(\tau) \frac{\nabla_i Q^{(n)}(x)}{(n^2 - 1)} + p_n(\tau) S_i^{(n)}(x) \right]. \quad (2.35)$$

The functions $l_n(\tau), m_n(\tau)$ and $p_n(\tau)$ can be found similarly to those for harmonics of gravitational perturbations. They have the form

$$l_n(\tau) = \frac{1}{\tau} \Big(\kappa_1 I_{n+1}(M\tau) + \kappa_2 I_{n-1}(M\tau) + \theta_1 K_{n+1}(M\tau) + \theta_2 K_{n-1}(M\tau) \Big), \quad (2.36)$$

$$m_n(\tau) = -(n-1)\kappa_1 I_{n+1}(M\tau) + (n+1)\kappa_2 I_{n-1}(M\tau)$$
$$-(n-1)\theta_1 K_{n+1}(M\tau) + (n+1)\theta_2 K_{n-1}(M\tau), \quad (2.37)$$

$$p_n(\tau) = \vartheta I_n(M\tau) + \rho K_n(M\tau). \quad (2.38)$$

III.   BOUNDARY CONDITIONS

We use the boundary conditions for linearized gravity studied in Ref. [2]. Assuming that the spatial components h_{ij} of perturbations of the gravitational field, and the normal component φ_0 of the ghost field vanish on the boundary, the authors of Ref. [2] obtained boundary conditions for gauge-invariant amplitudes in the form

$$h_{ij}|_{\partial\mathcal{M}} = h_{i0}|_{\partial\mathcal{M}} = \varphi_0|_{\partial\mathcal{M}} = 0, \tag{3.1}$$

$$\left(\frac{\partial h_{00}}{\partial\tau} + \frac{6}{\tau} h_{00} - \frac{\partial(g^{ij} h_{ij})}{\partial\tau} \right)|_{\partial\mathcal{M}} = 0, \tag{3.2}$$

$$\left(\frac{\partial\varphi_i}{\partial\tau} - \frac{2}{\tau}\varphi_i \right)|_{\partial\mathcal{M}} = 0. \tag{3.3}$$

Inserting the expansions (2.2)-(2.4) and (2.34)-(2.35) into (3.1)-(3.3), one finds the following boundary conditions on the basis functions:

$$\left(\frac{da_n(\tau)}{d\tau} + \frac{6a_n(\tau)}{\tau} - \frac{1}{\tau^2} \frac{de_n(\tau)}{d\tau} \right)|_{\partial\mathcal{M}} = 0, \tag{3.4}$$

$$b_n(\tau)|_{\partial\mathcal{M}} = 0, \tag{3.5}$$

$$c_n(\tau)|_{\partial\mathcal{M}} = 0, \tag{3.6}$$

$$d_n(\tau)|_{\partial\mathcal{M}} = 0, \tag{3.7}$$

$$e_n(\tau)|_{\partial\mathcal{M}} = 0, \tag{3.8}$$

$$f_n(\tau)|_{\partial\mathcal{M}} = 0, \tag{3.9}$$

$$k_n(\tau)|_{\partial\mathcal{M}} = 0, \tag{3.10}$$

$$l_n(\tau)|_{\partial\mathcal{M}} = 0, \tag{3.11}$$

$$\left(\frac{dm_n(\tau)}{d\tau} - \frac{2}{\tau}m_n(\tau)\right)|_{\partial\mathcal{M}} = 0, \tag{3.12}$$

$$\left(\frac{dp_n(\tau)}{d\tau} - \frac{2}{\tau}p_n(\tau)\right)|_{\partial\mathcal{M}} = 0. \tag{3.13}$$

Now we can calculate $\zeta(0)$ for the gravitational field on the background which represents the part of 4-dimensional flat Euclidean space bounded by two concentric 3-spheres of radii τ_+ and τ_- respectively. Let us begin with the contribution of the transverse-traceless harmonics of the gravitational field to $\zeta(0)$. On inserting (2.13) into the boundary condition (3.10) one gets the system of equations

$$\alpha_1 I_n^- + \alpha_2 K_n^- = 0,$$

$$\alpha_1 I_n^+ + \alpha_2 K_n^+ = 0, \tag{3.14}$$

where

$$I_n^- \equiv I_n(M\tau_-), I_n^+ \equiv I_n(M\tau_+), K_n^- \equiv K_n(M\tau_-), K_n^+ \equiv K_n(M\tau_+).$$

The condition for the existence of non-trivial solutions of the system (3.14) is the vanishing of its determinant, i.e.

$$I_n^- K_n^+ - I_n^+ K_n^- = 0. \tag{3.15}$$

Eq. (3.15) provides the eigenvalue condition which we need to apply the version of ζ-function technique described in [8-10]. It is obvious that we can neglect the first term on the left-hand side of Eq. (3.15). Then using the power-series expansion for I and K functions [9] one can show that $I_{pole}(0)$

is equal to zero, while using the uniform asymptotic expansions of modified

Bessel functions one can show that $I_{pole}(\infty)$ vanishes as well (cf. [21]). Thus,

we only have to calculate $I_{\log}$. Looking at the uniform asymptotic expansions

for modified Bessel functions one can easily see that the coefficient of $\log M$ in

the determinant (3.15) is equal to (-1). Bearing in mind that the degeneracy

for tensor transverse-traceless harmonics is $2(n^2 - 4)$ [17,23], one gets

$$I_{\log} = \sum_{n=3}^{\infty}(n^2 - 4)(-1) = -\zeta_R(-2) + 4\zeta_R(0) + (-3) = -5,$$

where $\zeta_R(s)$ is the usual Riemann ζ-function, and we have used its well-known

values $\zeta_R(-2) = 0$ and $\zeta_R(0) = -1/2$. Thus, now we can write

$$\zeta(0)_{tensor} = -5. \tag{3.16}$$

Let us now evaluate the contribution from vector harmonics. Inserting

Eqs. (2.32)-(2.33) into the boundary conditions (3.6) and (3.9) one has the

system of equations

$$\widetilde{\varepsilon}_1 I_{n+1}^- + \widetilde{\varepsilon}_2 I_{n-1}^- + \eta_1 K_{n+1}^- + \eta_2 K_{n-1}^- = 0,$$

$$\widetilde{\varepsilon}_1 I_{n+1}^+ + \widetilde{\varepsilon}_2 I_{n-1}^+ + \eta_1 K_{n+1}^+ + \eta_2 K_{n-1}^+ = 0,$$

$$-\widetilde{\varepsilon}_1 \frac{I_{n+1}^-}{(n+2)} + \widetilde{\varepsilon}_2 \frac{I_{n-1}^-}{(n-2)} - \eta_1 \frac{K_{n+1}^-}{(n+2)} + \eta_2 \frac{K_{n-1}^-}{(n-2)} = 0,$$

$$-\widetilde{\varepsilon}_1 \frac{I_{n+1}^+}{(n+2)} + \widetilde{\varepsilon}_2 \frac{I_{n-1}^+}{(n-2)} - \eta_1 \frac{K_{n+1}^+}{(n+2)} + \eta_2 \frac{K_{n-1}^+}{(n-2)} = 0. \tag{3.17}$$

The vanishing of the determinant of the system (3.17) is the eigenvalue

condition for vector harmonics. Taking into account only the dominant terms

of this determinant (i.e. the terms including I^+ and K^-), one can write the eigenvalue condition in the form

$$I^+_{n+1} I^+_{n-1} K^-_{n+1} K^-_{n-1} = 0, \tag{3.18}$$

where we have omitted the unessential common multiplier which does not depend on M. One finds that just as in the case of tensor perturbations

$$I_{pole}(\infty) = I_{pole}(0) = 0$$

and we have to calculate $I_{\log}$. It is easy to see from uniform asymptotic expansions of modified Bessel functions [24] that

$$I_{\log} = \sum_{n=3}^{\infty} (n^2 - 1)(-2) = -2\zeta_R(-2) + 2\zeta_R(0) + 6 = 5.$$

Here, $2(n^2 - 1)$ is the degeneracy of vector harmonics [18,23]. Thus, we have

$$\zeta(0)_{vector} = 5. \tag{3.19}$$

Further to the entangled modes $c_n(\tau)$ and $f_n(\tau)$ where $n = 3, \ldots$ we have also the decoupled mode $c_2(\tau)$ obeying the equation

$$\frac{d^2 c_2(\tau)}{d\tau^2} + \frac{1}{\tau} \frac{dc_2(\tau)}{d\tau} - \frac{9 c_2(\tau)}{\tau^2} - M^2 c_2(\tau) = 0,$$

whose solution can be written as

$$c_2(\tau) = \varepsilon I_3(M\tau) + \eta K_3(M\tau).$$

This function satisfies Dirichlet boundary conditions on the 3-sphere boundaries and the corresponding eigenvalue condition is

$$I^+_3 K^-_3 = 0.$$

Its $I_{\log}$ and correspondingly $\zeta(0)$ is

$$\zeta(0)_{vector\ decoupled} = -3. \tag{3.20}$$

The most complicated task is the calculation of the contribution of scalar harmonics, since in this case we deal with the system of four entangled equations for four basis functions $a_n(\tau), b_n(\tau), d_n(\tau)$ and $e_n(\tau)$. Inserting the expressions for these functions from Eqs. (2.25)-(2.28) into the boundary conditions (3.4), (3.5), (3.7) and (3.8) one finds the following system of equations at each boundary:

$$2\gamma_1(I_n^\pm - M\tau_\pm I_n'^\pm) + 2\gamma_2(I_n^\pm + M\tau_\pm I_n'^\pm) + \gamma_3(6I_{n-2}^\pm + 2M\tau_\pm I_{n-2}'^\pm)$$

$$+\gamma_4(6I_{n+2}^\pm + 2M\tau_\pm I_{n+2}'^\pm) + 2\delta_1(K_n^\pm - M\tau_\pm K_n'^\pm) + 2\delta_2(K_n^\pm + M\tau_\pm K_n'^\pm)$$

$$+\delta_3(6K_{n-2}^\pm + 2M\tau_\pm K_{n-2}'^\pm) + \delta_4(6K_{n+2}^\pm + 2M\tau_\pm K_{n+2}'^\pm) = 0,$$

$$\gamma_2 I_n^\pm + (n+1)\gamma_3 I_{n-2}^\pm - (n-1)\gamma_4 I_{n+2}^\pm$$

$$+\delta_2 K_n^\pm + (n+1)\delta_3 K_{n-2}^\pm - (n-1)\delta_4 K_{n+2}^\pm = 0,$$

$$-\gamma_2 I_n^\pm + \frac{(n+1)}{(n-2)}\gamma_3 I_{n-2}^\pm + \frac{(n-1)}{(n+2)}\gamma_4 I_{n+2}^\pm$$

$$-\delta_2 K_n^\pm + \frac{(n+1)}{(n-2)}\delta_3 K_{n-2}^\pm + \frac{(n-1)}{(n+2)}\delta_4 K_{n+2}^\pm = 0,$$

$$3\gamma_1 I_n^\pm - 2\gamma_2 I_n^\pm - \gamma_3 I_{n-2}^\pm - \gamma_4 I_{n+2}^\pm$$

$$+3\delta_1 K_n^\pm - 2\delta_2 K_n^\pm - \delta_3 K_{n-2}^\pm - \delta_4 K_{n+2}^\pm = 0. \tag{3.21}$$

The determinant of this system of equations is rather a cumbersome one, and we only write down its dominant part, which reads

$$\left(-60n\frac{(n^2-1)}{(n^2-4)}(I_n^+)^2 I_{n-2}^+ I_{n+2}^+\right.$$

$$-\frac{6(n^2-1)}{(n+2)}\,M\tau_+\,(I'_{n-2})^+\,I^+_{n+2}\,(I^+_n)^2$$

$$\left.-\frac{6(n^2-1)}{(n-2)}\,M\tau_+\,(I'_{n+2})^+\,I^+_{n-2}\,(I^+_n)^2\right)$$

$$\times\left(-60n\frac{(n^2-1)}{(n^2-4)}\,(K^-_n)^2\,K^-_{n-2}\,K^-_{n+2}\right.$$

$$-\frac{6(n^2-1)}{(n+2)}\,M\tau_-\,(K'_{n-2})^-\,K^-_{n+2}\,(K^-_n)^2$$

$$\left.-\frac{6(n^2-1)}{(n-2)}\,M\tau_-\,(K'_{n+2})^-\,K^-_{n-2}\,(K^-_n)^2\right) \tag{3.22}$$

Inserting into Eq. (3.22) the expressions for uniform asymptotic expansions of modified Bessel functions [24] we see that in the limit $M \to \infty$ our determinant becomes

$$\frac{144n^2(n^2-1)^2}{(n^2-4)^2}$$

and after taking the logarithm and expanding it in inverse powers of n we only get vanishing contributions to I_{pole}, and hence

$$I_{pole}(\infty) = 0.$$

Using the power series for modified Bessel functions, in an analogous way we see that in the $M \to 0$ limit the determinant (3.22) becomes

$$-\frac{144(n^2-1)^3(n^2-16)}{n^2(n^2-4)^3}.$$

This leads to vanishing contributions to I_{pole} and hence

$$I_{pole}(0) = 0.$$

The calculation of $I_{\log}$ is straightforward and yields

$$I_{\log} = \sum_{n=3}^{\infty}\frac{n^2}{2}(-2) = -\zeta_R(-2) + 5 = 5.$$

Thus, we can write

$$\zeta(0)_{scalar} = 5. \tag{3.23}$$

Now we evaluate the contribution of partially decoupled modes. When $n = 2$ we have the system of equations for $a_2(\tau), b_2(\tau)$ and $e_2(\tau)$, while the mode $d_2(\tau)$ does not exist. Even without writing down explicitly the corresponding determinant, one can show that its dominant part is proportional to the product of two functions I, two functions K and one I' multiplied by M and one K' multiplied by M too. Hence their contribution to $I_{\log}$, which in the case of finite number of degrees of freedom coincides with $\zeta(0)$ (see Ref. [8]), can be easily obtained knowing the uniform asymptotic expansions of I and K and extracting from them the terms proportional to $\log M$ in logarithms of these functions. Thus

$$I_{\log} = n^2/2 \times (-1)|_{n=2} = -2$$

and correspondingly

$$\zeta(0)_{scalar\ n=2} = -2. \tag{3.24}$$

When $n = 1$ only the modes $a_1(\tau)$ and $e_1(\tau)$ survive, and in an analogous way one can show that

$$\zeta(0)_{scalar\ n=1} = 0. \tag{3.25}$$

The only thing which we have to calculate now is the contribution of ghost fields to the full $\zeta(0)$. Inserting the functions $l_n(\tau), m_n(\tau)$ and $p_n(\tau)$ from Eqs. (2.36)-(2.38) into the boundary conditions (3.11)-(3.13) we can get the determinants for the corresponding system of equations. From these

determinants we can obtain, by the method described above, the following contributions to $\zeta(0)$:

$$\zeta(0)_{ghost\ vector} = \frac{1}{2}, \tag{3.26}$$

$$\zeta(0)_{ghost\ scalar} = 0, \tag{3.27}$$

$$\zeta(0)_{ghost\ decoupled} = -\frac{1}{2}. \tag{3.28}$$

Of course, the ghost contributions (3.26)-(3.28) to the full $\zeta(0)$ should be multiplied by -2 [2,21], but this does not affect our result. Thus, by virtue of (3.16), (3.19), (3.20), (3.23)-(3.28) the full $\zeta(0)$ value is given by

$$\zeta(0)_{total} = 0. \tag{3.29}$$

This result obtained by using the ζ-function technique coincides with that obtained by using the covariant Schwinger-DeWitt technique for manifolds with boundaries [2,19]. In fact, on using the covariant technique on the part of flat Euclidean space bounded by two concentric 3-spheres we see that the volume contribution to the A_2 Schwinger-DeWitt coefficient vanishes, while the surface contributions from the two boundaries cancel each other.

IV. CONCLUSIONS

We have shown that, in quantum cosmology, one can obtain unambiguous calculations of one-loop amplitudes providing one studies flat Euclidean backgrounds bounded by two concentric 3-spheres, and providing one takes into account non-physical degrees of freedom and ghost modes. One then

finds that the covariant Schwinger-DeWitt technique, and zeta-function regularization relying on a mode-by-mode analysis of quantized fields, are in agreement.

Moreover, when the background 4-geometry has boundaries, one finds it is no longer true that ghost modes cancel the contribution of non-physical degrees of freedom [21]. To preserve gauge invariance one has thus to deal with physical, non-physical and ghost modes. All these properties have been here proved in the case of pure gravity.

In Ref. [20] it was shown that discrepancies do not occur when massless spin-$\frac{1}{2}$ fields are studied at one-loop about Riemannian 4-geometries with two 3-sphere boundaries. In Ref. [21] we have shown that, on studying vacuum Maxwell theory within the Faddeev-Popov formalism, discrepancies in one-loop calculations are again eliminated providing one takes a flat Euclidean background bounded by two concentric 3-spheres. Here, we have proved that the same property holds in the more involved case of linearized gravity. Interestingly, in our paper the contribution of physical degrees of freedom, i.e. transverse-traceless gravitons, is corrected by the contribution of non-physical degrees of freedom, while the contribution of ghosts on the background under consideration vanishes.

It therefore seems that, on considering Riemannian 4-geometries with two boundaries, covariant or mode-by-mode descriptions of quantum amplitudes are both legitimate, providing one takes into account non-physical degrees of freedom and ghost fields. By contrast, on studying Lorentzian 4-geometries,

the 3+1 decomposition and the extraction of physical degrees of freedom is still valid, providing one can make sense of the corresponding path integral. The Lorentzian regime is in turn more relevant for the description of the time-evolution of the physical universe studied within the framework of the Hartle-Hawking program [25-28].

Last, but not least, we would like to emphasize that the mode-by-mode quantization program may shed new light on modern quantum field theory. We are currently investigating the relation between the $I(M^2, s)$ function defined in Eq. (1.1) [8-11], and the zeta-function. This would enable one to relate gauge invariance of quantum amplitudes in the presence of boundaries to the invariance under homotopy of the residue of a meromorphic function. In physical language, this happens since a change of the gauge-averaging functional leads to a smooth variation of the corresponding matrix of elliptic self-adjoint operators. The residues at the origin of the meromorphic functions occurring in this analysis may be studied by a suitable generalization of the Atiyah-Patodi-Singer theory of Riemannian 4-geometries with boundary [29]. Although it is unclear whether such a research program can be completed, and applied to linearized gravity, it seems to provide a very exciting and deep vision of gauge invariance in quantum field theory.

$$* * *$$

After submitting our paper, we became aware of Ref. [30], where the asymptotic expansion of the heat kernel for elliptic operators on Riemannian 4-manifolds with boundary is studied to improve the analysis in Ref. [19].

In Ref. [31] Moss and Poletti, using the new results in Ref. [30], have re-calculated the conformal anomalies on Einstein spaces with boundary and, in the 1-boundary case (i.e. the disk), they have found agreement with the results obtained in Refs. [5,11,20] for spin-$\frac{1}{2}$ fields, and in Ref. [21] for vacuum Maxwell theory. However, it should be emphasized that the agreement between the covariant Schwinger-DeWitt and the non-covariant mode-by-mode calculations of the covariant Faddeev-Popov path integral is achieved on the disk in the Lorentz gauge only. In this gauge, the second-order differential operator for the electromagnetic field is covariant and does not depend on the choice of 3+1 decomposition. For a more general class of relativistic gauges [21-22], the mode-by-mode analysis of 1-loop quantum amplitudes is gauge-dependent on the disk, while in the 2-boundary case (i.e. the ring), the $\zeta(0)$ value is gauge-independent.

As far as we can see, this last remaining discrepancy seems to point out to serious limitations of the quantum theory when the background 4-geometry does not admit a well-defined 3+1 decomposition. Nevertheless, in the light of the recent literature [30-31], no conclusive argument exists which proves that the gauge-invariance problem can only be addressed in the 2-boundary case. It also appears interesting to compare our mixed boundary conditions for linearized gravity with the boundary conditions studied in Ref. [32].

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APPENDIX

In Sec. II, the eigenvalue equations (2.5)-(2.11) are obtained out of the operator, which is the elliptic operator defined by

$$\equiv g^{\mu\nu}\nabla_\mu\nabla_\nu. \tag{A.1}$$

As stated in Sec. II, covariant differentiation ∇_μ is performed with respect to the flat 4-metric g, in the spherical local coordinates suitable for the description of flat Euclidean 4-space bounded by two concentric 3-spheres. The corresponding perturbation of g is denoted by h, and τ is the Euclidean-

time coordinate. Hence one finds

$$h_{00} = \frac{\partial^2 h_{00}}{\partial \tau^2} + \frac{3}{\tau}\frac{\partial h_{00}}{\partial \tau} + \frac{1}{\tau^2}h_{00|i}{}^{|i} - \frac{4}{\tau^3}h_{0i}{}^{|i} - \frac{6}{\tau^2}h_{00} + \frac{2}{\tau^2}g^{ij}h_{ij}, \qquad (\mathrm{A}.2)$$

$$h_{0k} = \frac{\partial^2 h_{0k}}{\partial \tau^2} + \frac{1}{\tau}\frac{\partial h_{0k}}{\partial \tau} + \frac{1}{\tau^2}h_{0k|i}{}^{|i} - \frac{7}{\tau^2}h_{0k} - \frac{2}{\tau^3}h_{ik}{}^{|i} + \frac{2}{\tau}h_{00,k}, \qquad (\mathrm{A}.3)$$

$$h_{ij} = \frac{\partial^2 h_{ij}}{\partial \tau^2} - \frac{1}{\tau}\frac{\partial h_{ij}}{\partial \tau} + \frac{1}{\tau^2}h_{ij|k}{}^{|k} - \frac{2}{\tau^2}h_{ij} + \frac{2}{\tau}\left(h_{i0|j} + h_{j0|i}\right) + \frac{2}{\tau^2}\,g_{ij}\,h_{00}, \; (\mathrm{A}.4)$$

where a vertical stroke denotes 3-dimensional covariant differentiation on a 3-sphere of unit radius [17-18]. In a similar way, the ghost operators acting on (2.34)-(2.35) are found to be (cf. [21])

$$\varphi_0 = \frac{\partial^2 \varphi_0}{\partial \tau^2} + \frac{3}{\tau}\frac{\partial \varphi_0}{\partial \tau} + \frac{1}{\tau^2}\,\varphi_{0|i}{}^{|i} - \frac{3}{\tau^2}\,\varphi_0 - \frac{2}{\tau^3}\,\varphi_k{}^{|k}, \qquad (\mathrm{A}.5)$$

$$\varphi_i = \frac{\partial^2 \varphi_i}{\partial \tau^2} + \frac{1}{\tau}\frac{\partial \varphi_i}{\partial \tau} + \frac{1}{\tau^2}\,\varphi_{i|k}{}^{|k} - \frac{2}{\tau^2}\,\varphi_i + \frac{2}{\tau}\,\varphi_{0,i}. \qquad (\mathrm{A}.6)$$

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