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Quantitative corrections to mass sum rules involving baryons containing heavy quarks

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Quantitative corrections are estimated to three of Franklin's sum rules involving the masses of baryons containing at least one charmed quark and to three analogous sum rules for baryons containing at least one bottom quark. The corrections arise from three-body contributions to baryon interaction energies and are calculated from a semiempirical formula for the colormagnetic contributions to baryon masses.

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About 20 years ago, Franklin [1] derived sum rules relating the masses of baryons containing at least one charmed quark. Most of these baryon mass relations have not yet been tested by experimental measurements. Franklin restricted himself to ground-state baryons without radial or orbital excitations, and so do we in this work.

The Franklin sum rules are based on the constituent quark model plus the assumption that the interaction energy of quarks in a baryon consists of only one- and two-body terms. However, according to quantum mechanics, the interaction energy should contain a three-body term even if the forces between quarks consist only of two-body terms [2–5]. These three-body terms in baryons give rise to corrections to Franklin's sum rules. It is the purpose of this Brief Report to estimate the magnitude of these corrections in certain selected cases.

Franklin's sum rules fall into two categories: (i) formulas involving the mass splittings of baryon isospin multiplets, and (ii) formulas relating the masses of different isospin multiplets. In this work we neglect isospin breaking and consider only mass relations of type (ii). Furthermore, our method allows us to calculate corrections to only three of Franklin's formulas in category (ii). These three formulas involve mass differences among baryons with spin 3/2 and spin 1/2 with the same quark content. These mass differences arise from spin-dependent forces and, to a good approximation, are caused by the colormagnetic interaction of QCD [5]. We also obtain corrections to three analogous formulas in which c quarks are replaced by b quarks.

For some baryons, the spin-dependent mass splittings are known from experiment [5]; in other cases the splittings can be estimated from a semiempirical mass formula [6–8] with parameters chosen so as to get a best fit to the observed splittings. In the case in which the splittings are known from experiment, the magnitudes of three-body interaction energies have been deduced [5]. The semiempirical mass formula incorporates some three-body effects as a built-in feature. As a consequence, we can use the semiempirical formula to get a good estimate of the magnitude of the breaking of Franklin's formulas relating the spin splittings in different baryons.

In the following, we let the symbol for a baryon denote its mass. We generally follow the notation of the Particle Data Group [9], with the following amplifications. An asterisk on the symbol for a baryon means that it has spin 3/2. If a baryon contains two quarks of the same flavor, they are the first two. If all three quarks have different flavors, the two lightest are the first two. In the latter case, there are two inequivalent spin-1/2 states. One of these states may be chosen so that the first two quarks have spin 1 and the other so that the first two quarks have spin 0. It has been shown [10] that in the physical baryons the mixing is small between the two ideal states of spin 1/2 defined above, provided the first two quarks are the two lightest ones. If the symbol [9] for the two spin-1/2 baryons is otherwise the same, we put a prime on the state in which the lightest two quarks have spin 1. We let q stand for either a u or d quark.

We neglect isospin breaking terms. Then we can take linear combinations of Franklin's sum rules [1] to obtain the following relations among baryon masses:

$$\Sigma_c^*(qqc) - \Sigma_c(qqc) = \Xi_{cc}^*(ccq) - \Xi_{cc}'(ccq), \tag{1}$$

$$\Omega_c^*(ssc) - \Omega_c(ssc) = \Omega_{cc}^*(ccs) - \Omega_{cc}(ccs), \tag{2}$$

$$\Sigma_c^*(qqc) - \Sigma_c(qqc) + \Omega_c^*(ssc) - \Omega_c(ssc) = 2[\Xi_c^*(qsc) - \Xi_c'(qsc)], \tag{3}$$

where the symbol for a baryon is followed by its quark content in parentheses. In subsequent equations, we omit repeating the quark content of these baryons.

Franklin did not consider sum rules for baryons containing b quarks, as the b quark had not yet been discovered when he wrote his paper. However, as is clear from his paper, equations analogous to (1)–(3) hold for b-quark baryons simply by the replacement of all c quarks by b quarks in the formulas.

In Ref. [5], the following expressions were derived relating the masses of baryons with different spin wave functions but having the same quark content:

$$B^*(123) - B'(123) = 3R(132) + 3R(231), \tag{4}$$

$$2B^*(123) + B'(123) - 3B(123) = 12R(123) \tag{5}$$

where the $R(ijk)$ are three-body colormagnetic interaction energies. If the first two quarks have the same flavor, the prime should be omitted on $B'(123)$ in Eq. (4), and Eq. (5) is absent. Our notation differs from that in Ref. [5] in several respects, including the fact that we use the prime on the baryon whose first two quarks have spin 1, whereas in [5] the prime is used for the baryon whose first two quarks have spin 0. If we have the values of $R(132) + R(231)$ for different baryons, we can use these values to obtain corrections to Eqs. (1)–(3). In order to correct the sum rules (1)–(3), we need Eq. (4) but not Eq. (5).

For some baryons, the values of $R(132) + R(231)$ can be obtained from experiment [5]. For other baryons, the experimental data are not available. In the latter cases, we use a semiempirical mass formula for the spin splittings in baryons in the form given in Ref. [7]. Application of this formula leads to the values of $R(ijk)$ given in Table I. For completeness, we have included the values of $R(123)$ in Table I, even though these values are not necessary for our considerations. The values of the $R(ijk)$ in Table I are rounded to the nearest MeV. We have also included in Table I the values of $R(ijk)$ from Ref. [5] where the data are known.

The interaction energies $R(ijk)$ result from the colormagnetic force between quarks i and j. In deriving his sum rules, Franklin assumed that the $R(ijk)$ depend only on the flavors of the quarks i and j, but not on the flavor of the "spectator" quark k. However, it can be seen from the values in Table I that there is a small dependence on the flavor of the spectator quark. It is this dependence that leads to corrections to Franklin's sum rules. Using the values of $R(ijk)$ from the semiempirical mass formula rather than the rounded values in Table I, we obtain the following formulas instead of Eqs. (1)–(3):

$$\Xi_{cc}^* - \Xi_{cc}' - \Sigma_c^* + \Sigma_c = 15 \text{ MeV}, \tag{6}$$

$$\Omega_{cc}^* - \Omega_{cc} - \Omega_c^* + \Omega_c = 10 \text{ MeV}, \tag{7}$$

$$\Sigma_c^* - \Sigma_c + \Omega_c^* - \Omega_c - 2(\Xi_c^* - \Xi_c') = 0, \tag{8}$$

where we have rounded to the nearest 5 MeV. These formulas give the corrections to Franklin's formulas (1)–(3). We see from the above equations that three-body interaction energies lead to small corrections in two of the sum rules and the third survives essentially unchanged.

The corrections to Franklin's sum rules are still smaller in the case of baryons containing b quarks. Although it may be a long time before mass formulas for baryons containing b quarks can be tested, we nevertheless give these sum rules below:

$$\Xi_{bb}^* - \Xi_{bb}' - \Sigma_b^* + \Sigma_b = 10 \text{ MeV}, \tag{9}$$

$$\Omega_{bb}^* - \Omega_{bb} - \Omega_b^* + \Omega_b = 5 \text{ MeV}, \tag{10}$$

$$\Sigma_b^* - \Sigma_b + \Omega_b^* - \Omega_b - 2(\Xi_b^* - \Xi_b') = 0, \tag{11}$$

again rounding to 5 MeV.

Although the semiempirical mass formula reproduces the observed spin-dependent splittings rather well in baryons containing no heavy quarks, it is not known how accurately it gives the splittings in baryons containing heavy quarks, as the necessary measurements have not yet been carried out. However, even if the formula is in error by 10 MeV or more, Eqs. (6)–(11) should have much smaller errors; we estimate these errors to be less than 5 MeV. The reason is that these equations involve only differences in spin-dependent splittings, and therefore systematic errors in the semiempirical mass formula will tend to cancel.

In conclusion, taking into account three-body interaction energies leads to small, but significant departures from Franklin's baryon sum rules. In principle, these effects can be tested by future measurements of baryon masses.

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Table I. Contributions to baryon masses arising from the colormagnetic interaction. The values of the colormagnetic interaction energy $R(ijk)$ are obtained from a baryon semiempirical mass formula given in Ref. [7]. For comparison, where known, we give in parentheses the values of $R(ijk)$ using experimental data as input with the prescription of Ref. [5].

Quark content	$R(123)$ (MeV)		$R(132)$ (MeV)		$R(231)$ (MeV)	
qqq	49	(49)	49	(49)	49	(49)
qqs	51	(51)	34	(32)	34	(32)
ssq	25		35	(36)	35	(36)
sss	26		26		26	
qqc	54		12		12	
qsc	38		13		10	
ssc	28		11		11	
qqb	55		4		4	
qsb	39		5		4	
ssb	29		4		4	
ccq	5		14		14	
ccs	6		12		12	
qcb	15		5		2	
scb	13		5		3	
bbq	2		6		6	
bbs	2		5		5	