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Electromagnetic Catalysis of a Neutrino Radiative decay or One More Source of Information on the Lepton Mixing Angles?

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Abstract

The radiative decay of ultrarelativistic massive neutrino $\nu_i \to \nu_j \gamma$ is investigated in electromagnetic fields in the framework of the Standard Model with lepton mixing. Estimates of the decay probability and "decay cross-section" for accelerator neutrinos of high energies in the electric field of nucleus permit one to discuss the general possibility of carrying out the neutrino experiment. Such an experiment could give unique information on mixing angles in the lepton sector of the Standard Model which would be almost independent of the specific neutrino masses.

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It is known that neutrino oscillations are the main source of information on the mixing angles and neutrino masses. However limitations on the mixing angles thus obtained rigidly correlate with the neutrino's mass spectrum. In this work we'd like to suggest one more source of information on the lepton mixing angles – namely, the radiative decay of a high-energy neutrino in an external electromagnetic field of a nucleus which has virtually no correlation with the neutrino mass spectrum.

All attempts to find a possible manifestation of lepton mixing have given up to now negative results. It is reasonable to suppose that, because of the insufficiently high precision achieved in experimental studies of neutrino-involving processes, the neutrino mass spectrum appears degenerate (the neutrinos manifest themselves as massless particles). At the same time, with massive neutrino the absence of lepton mixing seems unnatural and is virtually incompatible with attempts to somehow extend the standard model (SM). On the other hand, lepton mixing similar to quark mixing, in itself, does not go beyond the framework of the standard model and may lead to some interesting physical phenomena, such as 1) rare lepton decays with lepton number violation of type $\mu \to e\gamma$, $\mu \to e\gamma\gamma$, $\nu_i \to \nu_j\gamma$ $\nu_i \to \nu_j\gamma\gamma$, 2) neutrino oscillations.

It is known that the massive neutrino's properties are sensitive to the medium it propagates through. It will suffice to mention the well known problem of the solar neutrino and the possibility of solving it, namely, the mechanism of resonance enhancement of neutrino oscillations in substance. Substance is usually considered as medium. We note, however, that medium can also be represented by an external electromagnetic field, which can significantly influence both the properties of the massive neutrino itself and the process of its decay and even induce novel lepton transitions with flavour violation $\nu_i \leftrightarrow \nu_j$ $(i \neq j)$ [1] , forbidden in vacuum.

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In the papers [2-5] we investigated the massive neutrino radiative decay $\nu_i \to \nu_j\gamma$ $(i \neq j)$ in external electromagnetic fields (an uniform magnetic field, a field of intensive electromagnetic wave, a crossed field, a Coulomb field of nucleus). The effect of gigantic enhancement of the decay probability by the external field (electromagnetic catalysis) was discovered. In particular, one of the results, we obtained, is that the probability of the ultrarelativistic neutrino decay in external field is practically independent of the mass of the decaying neutrino, if only the decay channel is open ($m_i^2 > m_j^2$). Here we present the most interesting, in our opinion, result of the high-energy neutrino radiative decay in the Coulomb field of nucleus.

At present the experimentally accessible strengths of electromagnetic fields are significantly below the critical strength ($F/F_e \ll 1, F = B, \mathcal{E},\ F_e = m_e^2/e \simeq 4.41 \cdot 10^{13}\, G$). Because of this, field-induced effects are especially marked in the ultrarelativstic case with the dynamic parameter

$$\chi^2 = e^2(pFFp)/m^6 \tag{1}$$

being not small even for a relatively weak field ($F_{\mu\nu}$ is the external field tensor, p_α is the 4-momentum, m is the mass of the particle). This is due to the fact that in the relativistic particle rest frame the field may turn out of order of the critical one or even higher, appearing very close to the constant crossed field ($\vec{\mathcal{E}} \perp \vec{B},\ \mathcal{E} = B$). Thus, the calculation in the case of constant crossed field is relativistic limit of the calculation in an arbitrary weak field and possesses a great extent of generality. What is why the result we obtained for the amplitude of the ultrarelativstic neutrino radiative decay in the crossed field can be applied to consider the high-energy neutrino radiative decay in the Coulomb field of nucleus. Here the non-uniformity of the electric field $\vec{\mathcal{E}}$ of the nucleus isn't substantial, because the loop process is "local" with the characteristic loop dimension $\Delta x \leq (E_\nu e \mathcal{E})^{-1/3}$ being significantly less than the nucleus size at neutrino energy $E_\nu \geq 100\, GeV$.

There is no need to give the total cumbersome expressions for the propagator and amplitude of the charged fermion in the crossed field. Here we present the main part of the amplitude of neutrino radiative decay $\nu_i(p) \to \nu_j(p')\gamma(q)$ in a physically more interesting case of the ultrarelativistic neutrinos

$$\begin{aligned}
\mathcal{M} &\simeq \frac{e^2 G_F}{\pi^2} \, (\varepsilon^* \tilde{F} p) \left[(1-x) + \frac{m_j^2}{m_i^2}(1+x) \right]^{1/2} \sum_\ell K_{i\ell} K_{j\ell}^* \, J(\chi_\ell), \\
J(\chi_\ell) &= i \int\limits_0^1 dt \; z_\ell \int\limits_0^\infty du \; \exp\left[-i\left(z_\ell u + u^3/3 \right) \right], \qquad\qquad (2) \\
z_\ell &= 4 \left[(1+x)(1-t^2) \left(1 - \frac{m_j^2}{m_i^2} \right) \chi_\ell \right]^{-2/3},
\end{aligned}$$

where $F_{\mu\nu}$, $\tilde{F}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}F_{\alpha\beta}$ are the external electromagnetic field tensor and its dual tensor, $\chi_\ell = \sqrt{e^2(pFFp)}/m_\ell^3$ is the so called dynamic parameter, m_l is the mass of the virtual charged lepton, K is the lepton mixing matrix of Kobayashi-Maskawa type. $x = \cos\theta$, θ is the angle between the vector $\vec{p}$ (momentum of the decaying ultrarelativistic neutrino) and $\vec{q}_0$ (momentum of the photon in the decaying neutrino rest frame). It is worth noting that the amplitude (2) is a sum of three loop contributions ($\ell = e, \mu, \tau$), each one being characterized by its "field form-factor" $J(\chi_\ell)$.

In an electric field the dynamic parameter can be represented as follows:

$$\chi_\ell \simeq \left(\frac{E_\nu}{m_\ell}\right)\left(\frac{e\mathcal{E}}{m_\ell^2}\right)\sin\alpha, \tag{3}$$

where α is the angle between the vector of the momentum $\vec{p}$ of the decaying neutrino and the electric field strength $\vec{\mathcal{E}}$. In a general way, cumbersome numerical calculations are required to find the probability. In the limit of super-high neutrino energies ($E_\nu \geq 1\,TeV$), however, the situation is drastically simplified, as at such neutrino energies the conditions $\chi_e \gg \chi_\mu \gg 1$, $\chi_\tau \ll 1$ are fulfilled in the vicinity of the nucleus.

Therefore, it is sufficient for us to know only the asymptotics of the function $J(\chi)$:

$$\begin{aligned} J(\chi) &= 1 + O(\chi^2), & \chi \ll 1, \\ J(\chi) &= O(\chi^{-2/3}), & \chi \gg 1, \end{aligned} \tag{4}$$

so that the amplitude (2) is dominated by the contribution due to the virtual τ-lepton. The decay probability can be presented in the following form:

$$w \simeq \frac{\alpha}{4\pi}\,\frac{G_F^2}{\pi^3}\,E_\nu e^2\mathcal{E}^2\sin^2\alpha\,\left(1 - \frac{m_j^4}{m_i^4}\right)\,|K_{i\tau}K_{j\tau}^*|^2. \tag{5}$$

We note that there is no suppression associated with the smallness of the mass of the decaying neutrino. The comparison of this expression with the well-known expression for the probability of the radiative decay $\nu_i \to \nu_j\gamma$ of a high-energy neutrino in vacuum (see, for example, [6]) demonstrates the enormous enhancing influence of the external field on the radiative decay

$$R \equiv \frac{w}{w_0} \simeq \frac{512}{9}\left(\frac{m_W}{m_\tau}\right)^4\left(\frac{E_\nu}{m_i}\right)^2\left(\frac{e\mathcal{E}}{m_i^2}\right)^2\sin^2\alpha. \tag{6}$$

As an illustration, let us give a numerical estimate of R in the case of the decay of a neutrino of energy $E_\nu \sim 1\,TeV$ in the vicinity of a nucleus with the atomic number $Z \sim 20$:

$$R \simeq 2 \cdot 10^{+61} \left(\frac{1\,eV}{m_i}\right)^6 \left(\frac{E_\nu}{1\,TeV}\right)^2 . \qquad (7)$$

The neutrino radiative decay in substance must result in γ-quantum of energy $E_\gamma \sim E_\nu$ being observed as the decay product. In experiment, this process would appear as inelastic scattering of the neutrino on the nucleus. Using this expression for the probability and taking the nucleus as a uniformly charged sphere of radius r_N we obtain the following expression for the "cross-section" of the decay of a high energy neutrino in the electric field of a nucleus with the atomic number Z:

$$\begin{aligned} \sigma &\simeq \frac{4}{5} Z^2 \left(\frac{\alpha}{\pi}\right)^3 \frac{G_F^2 E_\nu}{r_N} \left(1 - \frac{m_j^4}{m_i^4}\right) |K_{i\tau} K_{j\tau}^*|^2 \\ &\simeq 10^{-44}\, Z^2 \left(\frac{10^{-12} cm}{r_N}\right) \left(\frac{E_\nu}{1\,TeV}\right) |K_{i\tau} K_{j\tau}^*|^2 \quad (cm^2). \qquad (8) \end{aligned}$$

It is worthwhile noting that the “cross-section” we have presented is, of course, numerically small, but not so small as not to allow discussion concerning the possibility of such an experiment in the future. It is interesting to compare it with the elastic $\nu_\mu e$-scattering cross-section:

$$\sigma^{el}(\nu_\mu e) = G_M^2 m_e E_\nu \frac{Z}{8\pi} \simeq \left(\frac{Z}{100}\right) \left(\frac{E_\nu}{100\,GeV}\right) 10^{-40}\, cm^2 \ ,$$

$\sim 4 \cdot 10^3$ events of which were observed by Charm II Collab. The ratio of these cross-sections does not depend on E_ν and is:

$$\frac{\sigma_Z(\nu_\mu \to \nu_e \gamma)}{\sigma^{el}(\nu_\mu e)} = \frac{1}{70 A^{1/3}} \left(\frac{Z}{100}\right) \left(1 - m_{\nu_e}^4 / m_{\nu_\mu}^4\right) \sin^2 \Theta_{12} \sim 10^{-3} \sin^2 \Theta_{12},$$

where it was put $r_N = r_0 A^{1/3}$, $r_0 \simeq 1/q_0$, $q_0 \simeq 0.15\,GeV$, Θ_{12} is the mixing angle of ν_1 and ν_2. This value of the ratio corresponds to a few events of a hard γ-ray emission by neutrino beams in the conditions of Charm II. However, the number of these events is proportional to E_ν, therefore at larger neutrino beam energy (e.g. at LHC) and with larger detectors the number of emitted high energy photons (with $\omega_\gamma \sim E_\nu$) can be of order of hundreds of thousands per year.

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