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# Magnetic Moments of the $SU(3)$ Octet Baryons in the semibosonized $SU(3)$ Nambu-Jona-Lasinio Model

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## Abstract

We calculate the hyperon magnetic moments in the framework of the  $SU(3)$  generalization of the semibosonized Nambu–Jona–Lasinio model for baryons. The effects of symmetry breaking imposed by the non–zero strange quark mass and the influence of the rotational  $1/N_c$  corrections on hyperon magnetic moments are examined.

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## I. INTRODUCTION

Recently it was shown [1] that the correct mass splitting of hyperons are obtained in the SU(3) effective quark-meson theory in the linear approximation with regard to the strange quark mass and soliton angular velocities. In the same approximation the long-standing problem of the underestimate of axial coupling constants  $g_A$  and nucleon magnetic moments in hedgehog models was solved [2–5]. It must be stressed that though the corrections linear in soliton angular velocities in the Skyrme model with explicit vector mesons [6] have the same structure as in the semibosonized Nambu–Jona–Lasinio model (NJL), the origin and the size of the corrections are much different. The success of the perturbation theory in the SU(3) NJL offers a systematic study of each correction to different baryonic observables. In this letter we study such corrections to the magnetic moments of the octet baryons due to the non-zero strange quark mass in order  $O(m_s)$  and due to the rotation of the soliton in order  $O(1/N_c)$  systematically. In order to calculate the baryon one-current matrix elements we use the NJL model for baryons. This model has been introduced a decade ago by Kahana, Ripka and Soni [7,8] and Birse and Banerjee [9]. The original version of the model suffered however from vacuum–instability paradox [10–12]. Later a new version of the model free of that paradox has been suggested [13] as following in the low-momenta limit from the instanton picture of QCD. According to ref. [13] the contents of QCD at low-momenta come to dynamically massive quarks interacting with pseudoscalar fields whose kinetic energy appears only dynamically through quark loops. The basic quantities of the model, viz. the momentum-dependent quark mass  $M(p)$  and the intrinsic ultra-violet cut-off have been also estimated in ref. [13] through the  $\Lambda_{QCD}$  parameter. An immediate implication of this low-momenta theory is the Chiral Quark–Soliton Model identified with the semibosonized Nambu–Jona–Lasinio Model for baryons of ref. [14], which is in the same spirit of refs. [7]– [9] but without the above-mentioned vacuum instability. According to the model the nucleon can be viewed as bound states of  $N_c$  ( $=3$ ) ”valence” quarks kept together by a strong hedgehog-like pion field whose energy coincides by definition with the aggregate energy of quarks from the negative Dirac sea. Such a semi-classical picture of the nucleon gets a justification in the limit  $N_c \rightarrow \infty$  – in line with more general arguments by Witten [15]. Roughly speaking, the model interpolates between the naive valence quark model of baryons and the Skyrme model. A non-trivial self-binding configuration of the pion and quark fields has been first found in ref. [14]. Subsequent numerical studies of the self-binding configuration have been done by many authors using different methods [16]– [18] and have found that results are rather close to the original ones [14]. Meanwhile, in ref. [16] a detailed quark-soliton theory of nucleons has been developed, including a collective-quantization procedure to deal with the rotational excitations of the quark-pion soliton. (The quantization of the otherwise static solution is necessary to obtain physical baryon states with definite quantum numbers). It enables one to go into detailed calculations of the  $N$  and  $\Delta$  properties, such as form factors,  $N - \Delta$  splitting, magnetic moments, axial constants, etc. A convenient numerical technique to calculate the ”one-current” quantities introduced in ref. [16] has been developed by Bochum group [19] and Wakamatsu and Yoshiki [22], based on the discretized Kahana – Ripka plane-wave basis [8], [12].

Many physical processes (semileptonic decays, electromagnetic transitions, the one-current baryon matrix element:

$$\langle B_2 | \bar{\psi} \Gamma \hat{O} \psi(x) | B_1 \rangle, \quad (1)$$

where  $\Gamma = (\gamma_\mu, \gamma_\mu \gamma_5, \sigma_{\mu\nu}, \gamma_5)$  is a particular Dirac matrix depending on the  $O$  is a  $SU(3)$  flavor matrix. For element in Eq. (1) with

$$\Gamma = \gamma_\mu, \quad \hat{O} = \frac{1}{2} \lambda^3 + \frac{1}{2\sqrt{3}} \lambda^8 \quad (2)$$

is relevant to the electromagnetic form factors of the octet baryons (magnetic moments, e.m. radii, etc.). This particular matrix element is a subject of the present paper.

One can relate the hadronic matrix element Eq. (1) to a correlation function:

$$\langle 0 | J_{B_1}(\vec{x}, T) \bar{\psi} \Gamma O \psi J_{B_2}^\dagger(\vec{y}, 0) | 0 \rangle \quad (3)$$

at large Euclidean time  $T$ . The baryon current  $J_B$  can be constructed from quark fields,

$$J_B = \frac{1}{N_c!} \epsilon^{i_1 \dots i_{N_c}} \Gamma_{JJ_3 I I_3 Y}^{\alpha_1 \dots \alpha_{N_c}} \psi_{\alpha_1 i_1} \dots \psi_{\alpha_{N_c} i_{N_c}} \quad (4)$$

$\alpha_1 \dots \alpha_{N_c}$  are spin-isospin indices,  $i_1 \dots i_{N_c}$  are color indices, and the matrices  $\Gamma_{JJ_3 I I_3 Y}^{\alpha_1 \dots \alpha_{N_c}}$  are chosen in such a way that the quantum numbers of the corresponding current are equal to  $JJ_3 I I_3 Y$ . The general expression for the matrix elements Eq. (1) was derived in Ref. [20] with linear  $m_s$  corrections taken into account:

$$\begin{aligned} \langle B_2 | \bar{\psi} \Gamma O \psi(x) | B_1 \rangle &= -N_c \int d^3 x e^{iqx} \int \frac{d\omega}{2\pi} \text{tr} \langle x | \frac{1}{\omega + iH} \gamma_4 \Gamma \lambda^A | x \rangle \\ &\times \int dR \Psi_{B_2}^\dagger(R) \Psi_{B_1}(R) \frac{1}{2} \text{tr}(R^\dagger \lambda^A R O) \\ &+ iN_c \int d^3 x e^{iqx} \int \frac{d\omega}{2\pi} \text{tr} \langle x | \frac{1}{\omega + iH} \gamma_4 \lambda^A \frac{1}{\omega + iH} \gamma_4 \Gamma \lambda^B | x \rangle \\ &\times \int dR \Psi_{B_2}^\dagger(R) \Psi_{B_1}(R) \frac{1}{2} \text{tr}(R^\dagger \lambda^A R \hat{m}) \frac{1}{2} \text{tr}(R^\dagger \lambda^B R O), \end{aligned} \quad (5)$$

where  $q \ll M_N$  is the momentum transfer,  $\Psi_B(R)$  are the rotational wavefunctions of the baryon with strange quark mass taken into account, and  $\lambda^A = (\sqrt{\frac{2}{3}} 1, \lambda^a)$ ,  $\lambda^a$  are Gell-Mann matrices. In Eq. (5) a regularization is not shown for simplicity (see Ref. [21] for details). Recently [3–5] the rotational  $1/N_c$  corrections for matrix elements of vector and axial currents were derived, general expression (without regularization) for these corrections has a form:

$$\begin{aligned} \Delta_{\Omega^1} \langle B_2 | \bar{\psi} \Gamma O \psi(x) | B_1 \rangle &= iN_c \int d^3 x e^{iqx} \int \frac{d\omega}{2\pi} \int \frac{d\omega'}{2\pi} P \frac{1}{\omega - \omega'} (I^{-1})_{aa'} \\ &\times \text{tr} \langle x | \frac{1}{\omega + iH} \lambda^{a'} \frac{1}{\omega' + iH} \gamma_4 \Gamma \lambda^b | x \rangle \\ &\times f^{abc} \int dR \Psi_{B_2}^\dagger(R) \Psi_{B_1}(R) \frac{1}{2} \text{tr}(R^\dagger \lambda^c R O) \\ &+ N_c \int d^3 x e^{iqx} \int \frac{d\omega}{2\pi} \text{tr} \langle x | \frac{1}{\omega + iH} \lambda^a \frac{1}{\omega + iH} \gamma_4 \Gamma \lambda^b | x \rangle \\ &\times \int dR \Psi_{B_2}^\dagger(R) \frac{1}{2} \{ \text{tr}(R^\dagger \lambda^b R O), \hat{\Omega}^a \}_+ \Psi_{B_1}(R). \end{aligned} \quad (6)$$

Where  $I_{ab}$  is a matrix of moments of inertia for the soliton,  $\hat{\Omega}^a$  is an operator of angular velocities acting on angular variables  $R$  (details can be found in [1]). In what follows we shall use these expressions to calculate hyperon magnetic moments. Using the general expressions Eq. (5) and Eq. (6) for the one current baryonic matrix elements, we can express the magnetic moments of the  $SU(3)$  octet baryons in linear approximation in  $m_s$  and  $1/N_c$  via a few quantities  $v_i$  depending on the concrete dynamics of the quark chiral soliton.

$$\begin{aligned} \mu_B &= v_1 \langle B | D_{Q_3}^{(8)} | B \rangle + \frac{v_2}{N_c} d_{ab3} \langle B | D_{Q_a}^{(8)} \cdot \hat{J}_b | B \rangle \\ &+ m_s \left[ (v_3 d_{ab3} + v_4 S_{ab3} + v_5 F_{ab3}) \cdot D_{Q_a}^{(8)} D_{8b}^{(8)} | B \right] \end{aligned} \quad (7)$$

here we have introduced  $SU(2)_T \times U(1)_Y$  invariant tensors

$$\begin{aligned} d_{abc} &= \frac{1}{4} \text{tr}(\lambda_a \{\lambda_b, \lambda_c\}_+), \\ S_{ab3} &= \frac{1}{\sqrt{3}} (\delta_{a3} \delta_{b8} + \delta_{b3} \delta_{a8}), \end{aligned}$$

and

$$F_{ab3} = \frac{1}{\sqrt{3}} (\delta_{a3} \delta_{b8} - \delta_{b3} \delta_{a8}), \quad (8)$$

where  $Q = \frac{1}{2}\lambda^3 + \frac{1}{2\sqrt{3}}\lambda^8$  is the charge operator. The rotational wavefunctions  $|B\rangle$  entering these formulae must be calculated in the linear approximation in  $m_s$ . The  $v_i$  are quantities depending on the concrete dynamics of the quark chiral soliton (they are independent of the hadrons involved). These dynamic quantities have a general structure like:

$$\sum_{m,n} \langle n | O_1 | m \rangle \langle m | O_2 | n \rangle f(E_n, E_m, \Lambda), \quad (9)$$

here  $O_i$  are spin-isospin operators changing the grand spin of states  $|n\rangle$  by 0 or 1, and the double sum runs over all eigenstates of the quark hamiltonian in the soliton field. The numerical technique for calculating such a double sum has been developed in [1,19,22]. Before we numerically calculate the magnetic moments let us estimate the importance of  $1/N_c$  corrections and relative size of subleading  $O(m_s/N_c)$  corrections. To this end we use a dynamically independent relations between magnetic moments arising from the ‘‘hedgehog’’ symmetry of the model. Hyperon magnetic moments are parametrized (in our approximation) by six parameters ( $v_1, v_2, v_3, v_4, v_5$  and one parameter is contained in the rotational wavefunctions). Looking upon them as free parameters, we obtain the relations between the hyperon magnetic moments and the magnetic moment of  $\Sigma^0 \Lambda$  transition :

$$\mu_{\Sigma^0} = \frac{1}{2} (\mu_{\Sigma^+} + \mu_{\Sigma^-}), \quad (10)$$

$$\begin{aligned} \mu_{\Lambda} &= \frac{1}{12} (-12\mu_p - 7\mu_n + 7\mu_{\Sigma^-} + 22\mu_{\Sigma^+} + 3\mu_{\Xi^-} + 23\mu_{\Xi^0}) \\ &\times (1 + O(\frac{m_s}{N_c}) + O(m_s^2)) \end{aligned} \quad (11)$$

$$\mu_{\Sigma^0 \Lambda} = -\frac{1}{\sqrt{3}} (-\mu_n + \frac{1}{4} (\mu_{\Sigma^+} + \mu_{\Sigma^-}) - \mu_{\Xi^0} + \frac{3}{2} \mu_{\Lambda}) \cdot (1 + O(\frac{m_s}{N_c}) + O(m_s^2)), \quad (12)$$

and one additional relation if we neglect rotational  $1/N_c$  corrections, *i.e.* put  $v_2 = 0$  in Eq.(7):

$$\mu_{\Xi^0} = (-3\mu_p - 4\mu_n + 4\mu_{\Sigma^-} + \mu_{\Sigma^+} + 3\mu_{\Xi^-}) \cdot (1 + O(\frac{1}{N_c}) + O(m_s^2)). \quad (13)$$

Let us note that the analogous relations between hyperon magnetic moments was obtained by Adkins and Nappi [23] but they did not take into account mass corrections to the rotational baryon wave functions and neglected  $1/N_c$  corrections. The first relation Eq.(10) is trivially fulfilled. It is an isospin relation and so has no corrections in both  $1/N_c$  and  $m_s$ . The next two relations Eq. (11) and Eq. (12) empirically gives:

$$-(0.613 \pm 0.004) = -(0.402 \pm 0.10) \quad (14)$$

and

$$-(1.61 \pm 0.08) = -(1.48 \pm 0.03) \quad (15)$$

respectively. Whereas the fourth relation Eq. (13) gives:

$$-(1.250 \pm 0.015) = -(4.8 \pm 0.2) \quad (16)$$

We see that the fourth relation Eq. (13) where we neglect  $1/N_c$  corrections is badly reproduced by experiment whereas the first three ones (10,11,12) seems to be successful. The explanation of this difference lies in different large  $N_c$  properties of the relations. These relations have, in principle, corrections of order  $O(1/N_c)$ ,  $O(m_s/N_c)$  and  $O(m_s^2)$ , but in (10,11,12) all corrections proportional to any power of  $1/N_c$  are cancelled. Hence the relations eqs. (10,11,12) are satisfied to accuracy of the order  $O(m_s/N_c)$ , while the eq.(13) is satisfied with the accuracy of  $O(1/N_c)$ . From these estimates one can conclude that corrections of order  $O(1/N_c)$  to magnetic moments numerically are large whereas those of the order  $O(m_s/N_c)$  can be relatively small. Such kind of estimates give us a lower limit for the systematic errors of computations in any “hedgehog” model for baryons due to the neglect of the non-computed  $O(m_s/N_c)$  and  $O(m_s^2)$  corrections, because any “hedgehog” model respects the eqs. (10,11,12) which are deviated from the experiment by about 15 %. Hence such kind of models can not reproduce the experimental data of magnetic moments better than the above-mentioned limit of 15%. We shall see that in the NJL model the accuracy of computation of hyperon magnetic moments very close to its upper limit.

In order to calculate Eq. (7) numerically, we follow the well-known Kahana and Ripka method [8]. In table 1 we show the dependence of the magnetic moments of the SU(3) octet baryons on the constituent quark mass in the chiral limit ( $m_s = 0$ ). Both of the leading order and the rotational  $1/N_c$  corrections tend to decrease as the constituent quark mass  $M$  increases. In this limit the  $U$ -spin symmetry is not broken, so that we have the relations

$$\begin{aligned} \mu_p &= \mu_{\Sigma^+}, & \mu_n &= \mu_{\Xi^0}, \\ \mu_{\Sigma^-} &= \mu_{\Xi^-}, & \mu_{\Sigma^0} &= -\mu_{\Lambda}. \end{aligned} \quad (17)$$

Compared to the SU(2) results, the prediction of the SU(3) model ( $m_s = 0$ ) for the nucleon is different and seems to be better. It is due to the fact that in our approach a nucleon possesses polarized hidden strangeness [27,28]. Technically the difference arises due to contributions of the soliton kaon cloud and due to different collective coordinates quantization rules.

The rotational  $1/N_c$  corrections are equally important to the other octet members as shown in Table 1. As a result, the total rotational  $1/N_c$  corrections contribute to the magnetic moments around 50%.

The symmetry breaking terms, proportional to  $m_s$ , lift the  $U$ -spin symmetry. The  $m_s$  corrections arise due to explicit dependence of the baryon matrix elements on the strange quark mass (second term of eq (5) and due to the dependence of the solitonic rotational wave functions on  $m_s$  (details see in Refs. [27,24]). The latter correction appears in each column of Table 2 and it is equally important as the former one.

It is interesting to compare the NJL model with the Skyrme model, since these two models are closely related. As Ref. [4] already made a comparison between the NJL model and the Skyrme model in case of the  $g_A$ . Apparently both models have the same collective operator structures (see Eq. (7)), but the origin of parameters  $v_i$  given in Eq. (7) is quite distinct each other. In the NJL model, the coefficients  $v_i$  include the contribution which arises from the noncommutivity of the collective operators [3] while it is absent in the Skyrme model, since the lagrangian of the Skyrme model is local in contrast to that of the semibosonized NJL. The coefficient  $v_2$  comes from the pseudoscalar mesons dominated by the induced kaon fluctuations. It is interesting to note that the Skyrme model needs explicit vector mesons in addition to pseudoscalar ones [6] in order to achieve the algebraic structure of the collective hamiltonian as it is obtained in the semibosonized NJL model with pseudoscalar mesons alone. Due to the introduction of vector mesons, it is inevitable to import large numbers of parameters into the Skyrme model.

In Fig. 1 we show how much the predicted magnetic moments deviate from the experimental data. On the whole, the magnetic moments are in a good agreement with the experimental data within about 15%.

In summary, we have studied the magnetic moments of the  $SU(3)$  octet baryons in the framework of the semibosonized  $SU(3)$  NJL model, taking into account the rotational  $1/N_c$  corrections and linear  $m_s$  corrections. The only parameter we have in the model is the constituent quark mass  $M$  which is fixed to  $M = 420$  MeV by the mass splitting of the  $SU(3)$  baryons. We have shown that the NJL model reproduces the magnetic moments of the  $SU(3)$  octet baryons within about 15 %. The accuracy we have reached is more or less the upper limit which can be attained in any model with “*hedgehog symmetry*”.

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## TABLES

TABLE I. The dependence of the magnetic moments of the SU(3) octet baryons on the constituent quark mass  $M$  without  $m_s$  corrections:  $\mu(\Omega^0)$  corresponds to the leading order in the rotational frequency while  $\mu(\Omega^1)$  includes the subleading order.

Baryon	370 MeV		420 MeV		450 MeV		Exp
	$\mu_B(\Omega^0)$	$\mu_B(\Omega^1)$	$\mu_B(\Omega^0)$	$\mu_B(\Omega^1)$	$\mu_B(\Omega^0)$	$\mu_B(\Omega^1)$	
$p$	1.0	2.50	0.92	2.20	0.88	2.08	2.79
$n$	-0.75	-1.71	-0.69	-1.50	-0.66	-1.42	-1.91
$\Lambda$	-0.38	-0.85	-0.34	-0.75	-0.33	-0.71	-0.61
$\Sigma^+$	1.0	2.50	0.92	2.20	0.88	2.08	2.46
$\Sigma^0$	0.38	0.85	0.34	0.75	0.33	0.71	—
$\Sigma^-$	-0.25	-0.79	-0.23	-0.70	-0.22	-0.66	-1.16
$\Xi^0$	-0.75	-1.71	-0.69	-1.50	-0.66	-1.42	-1.25
$\Xi^-$	-0.25	-0.79	-0.23	-0.70	-0.22	-0.66	-0.65

TABLE II. The magnetic moments of the SU(3) octet baryons predicted by our model are compared with the evaluation from the Skyrme model of Park and Weigel [6]. The experimental values are taken from Ref.[26]. The constituent quark mass is fixed as  $M = 420$  MeV. The  $\mu_B(\Omega^1, m_s)$  include subleading orders in  $\Omega$  and  $m_s$ , which are our final values.

Baryons	$\mu_B(\Omega^0, m_s^0)$	$\mu_B(\Omega^1, m_s^0)$	$\mu_B(\Omega^1, m_s^1)$	Park & Weigel	Exp.
$p$	0.93	2.22	2.30	2.36	2.79
$n$	-0.80	-1.60	-1.66	-1.87	-1.91
$\Lambda$	-0.32	-0.73	-0.76	-0.60	-0.61
$\Sigma^+$	0.92	2.21	2.34	2.41	2.46
$\Sigma^0$	0.29	0.71	0.74	0.66	—
$\Sigma^-$	-0.34	-0.79	-0.85	-1.10	-1.16
$\Xi^0$	-0.68	-1.49	-1.59	-1.96	-1.25
$\Xi^-$	-0.21	-0.68	-0.67	-0.84	-0.65
$ \Sigma^0 \rightarrow \Lambda $	0.66	1.35	1.44	1.74	1.61