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Comment on the formation of black holes in nonsymmetric gravity

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Abstract

We critically examine the claim made by Burko and Ori that black holes are expected to form in nonsymmetric gravity and find their analysis to be inconclusive. Their conclusion is a result of the approximations they make, and not a consequence of the true dynamics of the theory. The approximation they use fails to capture the crucial equivalence principle violations which enable the full nonsymmetric field equations to detect and tame would-be horizons. An examination of the dynamics of the full theory reveals no indication that black holes should form. For these reasons, one cannot conclude from their analysis that nonsymmetric gravity has black holes. A definitive answers awaits a comprehensive study of gravitational collapse, using the full field equations.

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I. INTRODUCTION

Anyone that has looked at alternative gravity theories will have been struck by just how ubiquitous black holes are. On closer inspection, it becomes clear why black holes are so hard to get rid of. The experimental success of Einstein's theory in describing weak gravitational fields requires that all alternative theories reduce to Einstein's for weak fields. Indeed, for weak fields this implies that all tenable alternative theories can be recast as Einstein gravity coupled to some effective matter source constructed from the additional gravitational degrees of freedom. Now, we know that a black hole horizon is an entirely regular place so far as local quatities such as curvatures are concerned. For a large black hole the curvatures can be very small at the horizon, and we may conclude that the weak field description of alternative gravity theories should continue to hold. This line of reasoning suggests that the gravitational collapse of a very massive body will proceed in much the same way in an alternative theory as it does in Einstein's theory. For most theories, such as Jordan-Brans-Dicke scalar-tensor theory, this is exactly what is found.

There is, however, one escape clause in this black hole contract. While it is true that a free-fall observer sees nothing special at the event horizon, static observers feel an infinite force. In addition, the redshift between the horizon and any point outside the horizon diverges. If the alternative theory violates the equivalence principle or employs non-local notions, then the horizon can be a very irregular place indeed.

We wish to examine how the preceding considerations apply to nonsymmetric gravity theory (NGT) [1,2]. In earlier work we showed that the unique, static, spherically symmetric vacuum solution for NGT did not describe a black hole [3]. This result held despite the fact that NGT can be recast as Einstein gravity coupled to an effective matter source for weak gravitational fields. Recently, Burko and Ori [4] considered the lowest-order linearisation of NGT about an Einstein gravity background and concluded that black holes will form in NGT. We show that their conclusion follows as an immediate consequence of the approximation they used. We stress that the approximation fails to capture the crucial equivalence principle violations which only occur at higher orders of approximation. If a horizon is present, the higher order terms can dominate. By considering gravitational collapse described by the full NGT field equations we find no reason to expect that black holes will form.

II. THE LINEARISED THEORY

We shall first consider the lowest-order linearised NGT field equations used to study black hole formation in Ref. [4]. To first order, the NGT vacuum field equations linearised about a Einstein gravity (GR) background read [2]

$$R_{\mu\nu} = 0 \,, \tag{1}$$

$$\nabla^\alpha F_{\mu\nu\alpha} + \mu^2 h_{[\mu\nu]} - 4R^{\alpha}{}_{[\mu}{}^{\beta}{}_{\nu]} h_{[\alpha\beta]} = 0 \,, \tag{2}$$

where $F_{\mu\nu\alpha} = h_{\{[\mu\nu],\alpha\}}$ is the field strength formed from the linearised skew metric $h_{[\mu\nu]}$, and μ^2 is a type of cosmological constant. At this order, the NGT field equations are identical to those of a massive, curvature coupled Kalb-Ramond field. The standard no-hair theorems guarantee that a black hole with $h_{[\mu\nu]} = 0$ must form as the result of the gravitational collapse of matter coupled to such a Kalb-Ramond field [5]. The preceding analysis summarises the treatment presented in Ref. [4].

Since violations of the equivalence principle are restricted to benign curvature couplings at this order, the theory has been robbed of its ability to see horizons. It is only at higher orders that interesting violations of the equivalence principle can make themselves felt in NGT, as we shall explain at the end of this section.

The lowest order equations do correctly predict the breakdown of the perturbative treatment for a static $h_{[\mu\nu]}$ on a Schwarzschild background. Of course, this has nothing to do with violations of the equivalence principle as the same goes for a minimally coupled scalar field. The breakdown of the linearised analysis requires that the full field equations be studied. In the case of a scalar field, the pathological behaviour at the horizon persists in the full field equations, turning the horizon into a curvature singularity. Thus, before we even begin to study the collapse of a star coupled to a scalar field we already know what the outcome must be - a black hole with no scalar hair. In contrast, the full NGT field equations reveal that a non-zero static $h_{[\mu\nu]}$ is permitted as curvatures remain small at the Schwarzschild radius and the horizon is destroyed. Clearly, there is no *a priori* reason to exclude the possibility of a static $h_{[\mu\nu]}$ remaining after a star has collapsed.

We now take a closer look at why we expect higher orders in the skew metric expansion to introduce important equivalence principle violations. One example of an effect which only occurs at higher orders involves the volume form. In GR it is always possible to find a coordinate patch in the neighborhood of any regular point in terms of which the volume form is identical to that in Minkowski space. The same is not true in NGT. Importantly, this effect cannot be seen at first order, so it is an example of an equivalence principle violating effect missed in the analysis of Ref. [4]. Another example of an effect missed at lowest order is due to the local anisotropy of spacetime caused by the skew field. This local anisotropy alters the propagation of light [6]. A related, but more serious effect is the modification to the propagation of skew perturbations due to non-linear self-interaction [7]. Because of this effect, skew waves will not follow geodesics of the background geometry in the geometrical optics approximation. This departure from geodesic motion can lead to divergent results at the horizon. Such an effect can be important as non-geodesic trajectories suffer infinite proper accelerations at the event horizon.

We see that the breakdown in the skew perturbation theory can only been expected *when higher-order terms are taken into account and a horizon is present in the background geometry*. Since the first order analysis fails to capture vital features of the full theory, we conclude that the first-order analysis is at best inconclusive, at worst totally misleading. Unfortunately, the NGT field equations are realted non-polynomially to GR so there are an infinite number of higher order terms which must be considered. We would have to prove that divergences do not occur at any order for the first order analysis to be trusted. Clearly, this is an impossible task so a perturbative approach using a GR background must be abandoned. The full field equations must be consulted.

An analogous result has recently been found for string theory in the presence of horizons [8]. The intrinsic non-locality of strings allows them to respond to redshifts. When a horizon is present, the standard low-energy effective action must be modified to include massive, extended string modes. The usual description of low energy string gravity consists of Einstein gravity coupled to a massless dilaton and a massless Kalb-Ramond field. For weak fields, the higher-order massive modes are suppressed and can be neglected. The standard dogma states that the horizon for a large black hole is a weak field region, so we might expect the massless low energy string theory to continue to be valid when a horizon is present. In the case of strings, the standard dogma fails because the theory is non-local. For NGT, the standard dogma fails because the theory incorporates a special kind of equivalence principle violation. It should be mentioned however, that the higher order stringy effects considered in Ref. [8] are intrinsically non-local, and are not expected to impact on the local dynamics of gravitational collapse [9].

III. THE FULL THEORY

Since the linearised treatment cannot be trusted, we must look at the full field equations. While we do not claim that the following argument is rigorous, it does give some idea about what we might expect to find for gravitational collapse described by the full theory.

The static spherically symmetric metric in NGT can be written as

$$g = e^{\nu} dt \otimes dt - \alpha(\nu) d\nu \otimes d\nu - r^2 d\theta \otimes d\theta - r^2 \sin^2\theta d\phi \otimes d\phi + f(\nu) \sin\theta \, d\theta \wedge d\phi \,. \tag{3}$$

The radial variable r is a function of ν. For the vacuum Wyman solution [10] we find that $r(\nu)$ is given implicitly by

$$e^{\nu}(\cosh(a\nu) - \cos(b\nu))^2 \frac{r^2}{2M^2} = \cosh(a\nu)\cos(b\nu) - 1 + s \sinh(a\nu)\sin(b\nu) \,, \tag{4}$$

where

$$a = \sqrt{\frac{\sqrt{1+s^2}+1}{2}} \,, \qquad b = \sqrt{\frac{\sqrt{1+s^2}-1}{2}} \,, \tag{5}$$

and s is a dimensionless constant which varies from body to body. At large r we find

$$r \simeq -\frac{2M}{\nu} \,. \tag{6}$$

It is instructive to consider the massless scalar wave equation in the metric (3). Using the full NGT connection, we find that the monopole mode of a scalar field obeys the relation

$$e^{-\nu} \frac{\partial^2 \Phi}{\partial t^2} - \frac{1}{\alpha} \frac{\partial^2 \Phi}{\partial \nu^2} = 0 \,. \tag{7}$$

When we study this equation on a Wyman background, we see that small Φ perturbations remain small everywhere as the background is everywhere regular. This is true even in the static limit where the wave equation has the explicit solution

$$\Phi = \Phi_0 + \Phi_1 \nu \,. \tag{8}$$

Since the maximum value for ν is roughly π/s, we can always choose Φ_0 and Φ_1 so that Φ is small everywhere. The NGT vacuum solution admits scalar hair. This is in stark contrast to the situation in GR where

$$\Phi = \Phi_0 + \Phi_1 \ln\left(1 - \frac{2M}{r}\right) \ , \tag{9}$$

and we must set $\Phi_1 = 0$ to obtain a solution regular at $r = 2M$.

A similar, but far more complicated, analysis can be made for skew metric perturbations about the metric (3). Indeed, these kind of skew metric perturbations about the static solution have to be included as NGT does not have an analog of Birkoff's theorem. The skew metric perturbations lead to a set of coupled hyperbolic differential equations. This is because a first order skew perturbation excites first order perturbations in the symmetric metric functions. We note that this is in contrast to what we found for skew perturbations about a GR background, where the skew and symmetric sectors remain uncoupled at first order. Despite these technical complications, the physical picture is the essentially the same as what we have described for scalar perturbations. Since the background metric is everywhere regular, small skew perturbations will remain small.

The same holds for skew perturbations about a star described by NGT. The NGT metric inside the star is regular, and it matches smoothly onto an exterior Wyman vacuum solution. Small skew perturbations about the static solution remain small. This continues to be the case if the star undergoes gravitational collapse. The endpoint of collapse is likely to be some kind of matter distribution supported by the repulsive skew fields and matter pressure.

Since skew perturbations on top of the full NGT metric for a collapsing star are expected to remain small, it is difficult to see how a black hole might form. This is because the end-state of spherically symmetric gravitational collapse in GR, the Schwarzschild metric, differs from the NGT vacuum metric by a non-perturbative amount. Thus, we require non-perturbative skew fluctuations to occur if we wish to recover a black hole. As the linearised skew perturbations show no sign of diverging, the required non-perturbative skew fluctuations are ruled out.

IV. CONCLUSIONS

We have pointed out that the lowest order linearisation used in Ref. [4] cannot be trusted when the background geometry contains a horizon. For this reason, we find the analysis in Ref. [4] to be inconclusive, and the claim that black holes form in NGT to be premature. Moreover, a parallel treatment using a NGT background, rather than a GR background, leads to the opposite conclusion - black holes will not form in NGT. The only way to really find out whether or not black holes form in NGT is to solve the collapse problem using the full NGT field equations. Until that is done properly, the jury is out.

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